Electr Eng
DOI 10.1007/s00202-012-0272-4

ORIGINAL PAPER

# Modeling of stochastic behavior of plug-in hybrid electric vehicle in a reactive power market

**Hassan Feshki Farahani · H. A. Shayanfar · M. S. Ghazizadeh**

Received: 18 March 2012 / Accepted: 6 November 2012


**Abstract** This paper presents a stochastic reactive power market which incorporates plug-in hybrid electric vehicles (PHEVs) in the reactive power market. The uncertainty of PHEVs as well as synchronous generators in the form of system contingencies are explicitly considered in the stochastic market-clearing scheme. The capability curve of PHEV is extracted and after that the expected payment function of PHEV is proposed based on its capability curve. In the proposed stochastic reactive power market, the Monte-Carlo simulation is used to generate random scenarios. The stochastic market-clearing procedure is then implemented as a series of deterministic optimization problems (scenarios) including non-contingent scenario and different post-contingency states. The objective function of each scenario is total payment function of PHEVs and synchronous generators which refers to the payment paid to the generators and PHEV owners for their reactive power compensation. A typical 17-node microgrid is used in the case study.

**Keywords** Plug-in hybrid electric vehicle (PHEV) · Stochastic reactive power market · Total payment function (TPF) · PHEV capability curve

H. F. Farahani (✉)
Department of Electrical Engineering,
Science and Research Branch,
Islamic Azad University, Tehran, Iran
e-mail: hfeshki@yahoo.com

H. A. Shayanfar
Center of Excellence for Power Systems Automation and Operation, Electrical Engineering Department,
Iran University of Science and Technology (IUST),
Tehran, Iran

M. S. Ghazizadeh
Power and Water University of Technology, Tehran, Iran

## 1 Introduction

Plug-in hybrid electric vehicle (PHEV) technology is a promising solution that can lead to decrease greenhouse gas emissions, air pollution in urban areas and totally it can be considered as environmentally clean technology [1]. Recent studies have shown the effect of the PHEVs on the grid. In [2], vehicle to grid (V2G) is considered to evaluate the amount of real power transmitted between vehicle and grid in charge and discharge modes. The vehicle owner profit is calculated for three types of vehicles and it is used for regulation, connection to grid in peak load interval, and reserve as well. V2G is used to provide the reactive power compensation in [3,4] wherein the effects of the loss are ignored.

Several research works illustrate the potential impacts/ benefits of connection of PHEVs integration into the grid. In [5], the impact of electricity rates on distribution load shapes with PHEV penetration. The model of load demand due to EV battery charging in distribution systems is modeled in [6]. A real-time wide area controller is designed to improve the stability of the power system with PHEVs [7]. The impact of charging PHEVs on a residential distribution grid is studied in [8]. The integration of PHEVs/PEVs into power grids is evaluated based on different charging scenarios [9]. The work in [10] deals with the impact of PHEVs on the power system from different viewpoints, focusing on the infrastructure of the network. Dyke et al. [11] established a series of well-defined EV loads to analyze their electrical energy usage and storage in the electrified road transportation. In [12], the effect of penetration level of PEV on the grid investment and energy losses is assessed. The integration of EVs into power systems is studied considering technical operation of grid in the market [13].

From the above mentioned papers, it is concluded that all aspects of the V2G applications are considered. However,

Published online: 24 November 2012

few works consider the potential of V2G as a reactive power compensator to participate in the reactive power market. It is noted that the reactive power compensation of V2G does not degrade the battery life of vehicle as compared to the peak power shaving by V2G. Since the reactive power can be provided by DC link capacitor and therefore the battery is not engaged in reactive power transfer [3,4]. Accordingly, the owners of PHEVs can take this advantage of electric vehicles and enrol them in the reactive power ancillary service. Whether or not the PHEVs are incorporated in reactive power is based on the decision of the PHEVs owner and market condition. However, the incorporation of PHEVs in the reactive power market in fact increases the source of reactive power compensator in the system which leads to increase the reactive reserve of system and consequently enhances the voltage stability of grid that relieves the concern of system operator about the system security.

Reactive power plays an important role in power system reliability and security. It maintains voltage profile and improves real power transfer capability [14,15]. The synchronous generators are a primary source of reactive power in electric power systems but the ISO can provide reactive power from the other sources such as PHEVs. Consequently, PHEVS' characteristics and limitations are of major importance on their generation. Recently, there has been a significant interest shown in reactive power as one of several ancillary services required to ensure system reliability and security.

Zhong et al. [16–19] have designed a competitive reactive power market. The other research works [14,20–22], took into account voltage security in the reactive power pricing. In Ref. [21], a two-level framework is proposed for the operation of a competitive reactive power market taking into account system security aspects. The first level, i.e., procurement, is on a seasonal basis while the second level, i.e., dispatch, is close to real-time operation. However, the seasonal reactive power market encounters problems discussed in Refs. [14,22]. So, these two works proposed day-ahead reactive power market instead of long-term-based reactive power market.

Recently, few studies have undertaken the challenge of optimization problems for power system studies faced with uncertainty. A stochastic formulation for reactive power market is proposed in [23]. The unit commitment problem has been solved by developing a chance-constrained program to the stochastic problem formulation [24]. In [25], a stochastic multiobjective framework is proposed for day-ahead joint market clearing.

The main contribution of this paper with respect to [14,22,23] and the others works in the area is presenting a stochastic framework for the clearing of reactive power market including PHEVs. The uncertainty of PHEVs is modeled based on the percentage of PHEV being at trip at the interested hour. Also, the malfunction of PHEV is considered in the unavailability of PHEV.

In order to incorporate PHEV in the reactive power market, the capability of PHEV is also extracted and after that the expected payment function (EPF) of PHEV is proposed so that the owner of PHEV can offer in the stochastic reactive power market. The proposed stochastic framework includes two steps.

The remainder of this paper is organized as follows: in Sect. 2, the capability curve of PHEVs is proposed. The problem of deterministic reactive power market clearing is formulated in the form of a mixed integer non-linear programming (MINLP) problem in the Sect. 3. In next section, the deterministic MINLP formulation is extended to the stochastic framework considering PHEVs and generators uncertainties. In Sect. 5, the validity of the proposed stochastic reactive power market-clearing scheme is studied based on 17-node microgrid. Some relevant conclusions are drawn in the Sect. 6.

## 2 The capability curve of PHEV

Different structures for charging and discharging parts have been used [3,26]. In this section, a typical circuit which is generally used in PHEV structure is considered as shown in Fig. 1. This circuit consists of three parts: AC/DC converter (rectifier and inverter), DC/DC converter (buck and boost converter) and battery. AC/DC converter operates as rectifier in charging mode and as inverter in discharging mode. The DC/DC converter operates as a buck converter in charging mode and boost converter in discharging mode.

In the charging/discharging mode, PHEV can absorb/inject active power from/to the grid. At the same time, PHEV can inject/absorb reactive power to/from the grid. In other words, the PHEV owners can be considered as a participant of reactive power market and regulate grid voltage in the form of reactive power ancillary service. For this approach it is needed that the PHEV owners offer in the reactive power market. Therefore, the PHEV owners, like synchronous generators and other participants of reactive power market, should offer their price components based on the EPF of PHEV.

In order to obtain the capability curve of PHEV, it is necessary to consider the limitations exist during injection/absorption of power to/from the grid.

The capability curve of PHEV is shown in Fig. 2. As shown in this figure, the output of PHEV is limited by four curves, i.e., curve $A$, $B$, $(C_1, C_2)$, $(D_1, D_2)$. Curve A is related to the maximum transferable apparent power between PHEV and grid. The formulation of curve $A$ can be written as:

$$P^2 + \left(Q - \frac{V_s^2}{X_c}\right)^2 = \left(\frac{V_s V_P}{X_c}\right)^2 \tag{1}$$

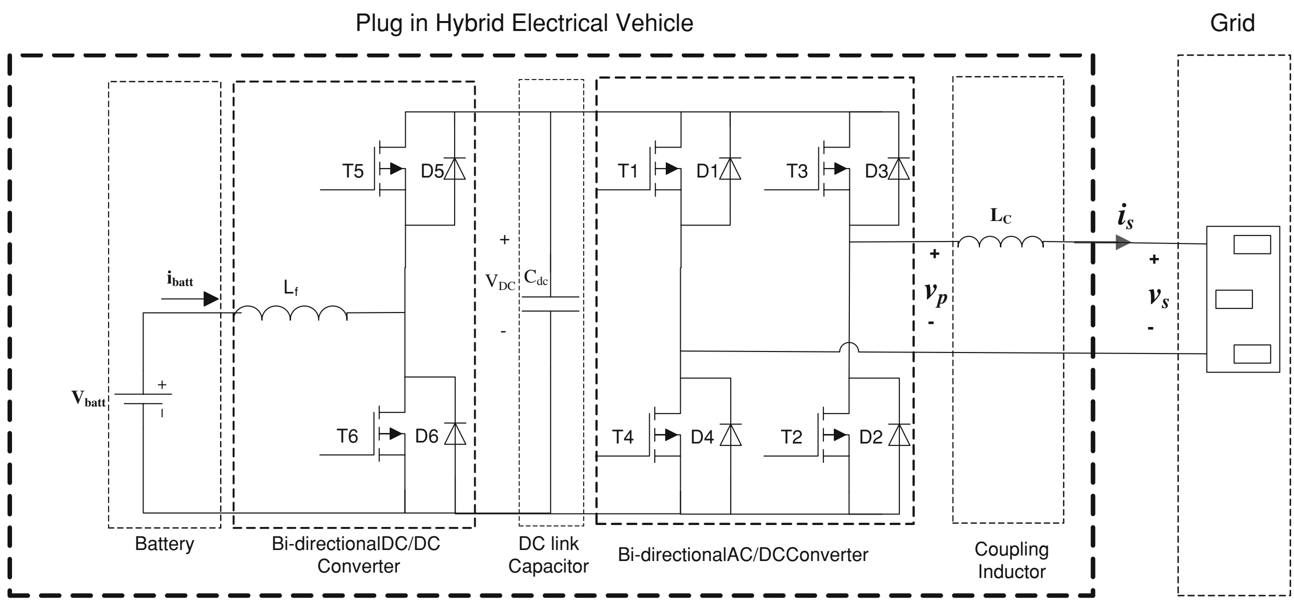



**Fig. 1** Charging and discharging circuit of PHEV

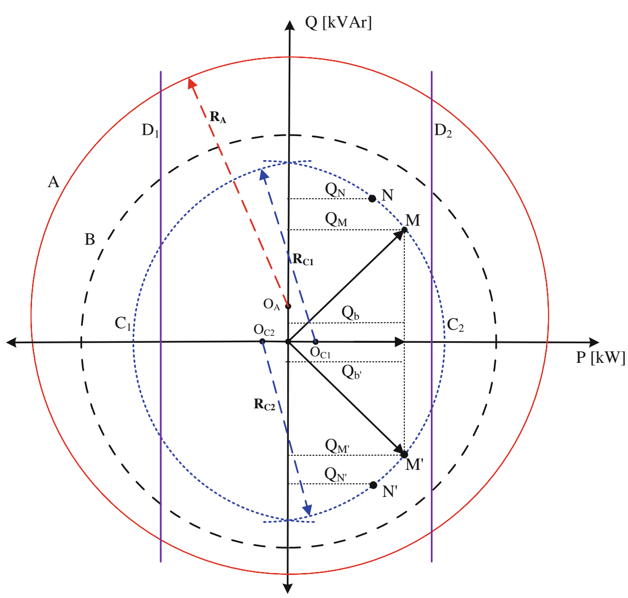



**Fig. 2** The capability curve of PHEV

where, $P$ and $Q$ are active and reactive power entered to grid, $V_s$ and $V_p$ are grid and inverter output voltage respectively and $X_c$ is the reactance of coupling inductor. Equation (1) is a circle with radius $(V_s V_P / X_c)$ and center $O_A(0, V_s^2/X_c)$.

The other limitation curve is coupling inductor loss shown with curve $B$ in Fig. 2 that this curve can be formulated as:

$$P^2 + Q^2 = \frac{P_{L_C} \times V_s^2}{R_C} \tag{2}$$

Where, $R_C$ and $P_{L_C}$ are resistance and losses of coupling inductor. Equation (2) is a circle with radius $\sqrt{(P_{L_C} \times V_s^2)/R_C}$ and center (0, 0).

The other limitation curve is related to inverter maximum output power in kVA, curves $C_1$ and $C_2$, due to the inverter switch parameters such as nominal current, voltage and operating frequency. For a lossless inverter, with nominal power equal to $S_n$, the limitation curve will be a circle with radius $S_n$ and center (0,0) as the following equation.

$$P^2 + Q^2 = S_n^2 \tag{3}$$

However, in real case the inverter includes losses (about 10 %). So, to consider inverter loss ($P_{loss}$) Eq. (3) is modified to (4) as follows:

$$(P \pm P_{loss})^2 + Q^2 = S_n^2 \tag{4}$$

where, '−' and '+' are related to charging and discharging mode, respectively. These two curves are circles with radius $S_n$ and centre of $O_{C1}$ $(+P_{loss}, 0)$ for charging mode, and $O_{C2}(-P_{loss}, 0)$ for discharging mode.

Comparing with maximum apparent power limit curve ($A$), the boundaries of the inverter output power limit ($C_1$ and $C_2$) are much lower than the boundaries of maximum apparent power limit. This matter is mainly for the switch limit of inverters.

The last limit curve is due to the maximum active power of PHEV injected/absorbed to/from the grid which are two vertical lines $D_1$ and $D_2$ shown in Fig. 2. These limits are determined by the PHEV owner considering the conditions of market and energy prices. For instance, in the hours that the energy price is low, the PHEV owner is interested to increase its transaction with the grid and therefore increase its maximum and minimum active power.

The capability curve of PHEV in Fig. 2 shows that PHEV can be operated in four quadrant. According to this figure, $Q_b$ is supposed to be the reactive power that must be generated or absorbed by the PHEV without any remuneration. If the operating point lies inside the limiting curve, e.g., $(P_M, Q_b)$, then the PHEV can increase its reactive generation from $Q_b$ to $Q_M$ without requiring adjustment of $P_M$. However, this will result in the increased loss of PHEV charger, hence, increase the cost of losses. For any of four quadrants regions, if the PHEV operates on the limiting curve (inverter maximum output power limit), any increase in $Q$ will require a decrease in $P$ to adhere to the inverter output power limit. Consider the operating point $M$ on the curve defined by $(P_M, Q_M)$. If more reactive power is required from the PHEV, for example $Q_N$, the operating point requires shifting back along the curve to point $(P_N, Q_N)$, where $P_N < P_M$. This indicates that the PHEV has to reduce its active power output to adhere to the inverter output power limit when higher reactive power is demanded. The lost in the revenue of the PHEV due to the reduced production of active power is termed lost opportunity cost (LOC) which is a significant issue in reactive power market production cost. This matter is valid for all four quadrants of PHEV. In other words, apart from the operating region of PHEV (i.e., quadrant I, II, III, IV), the abovementioned matter should be considered so that the owner of PHEV could participate in the reactive power market at any operating region with enough economic incentive.

The other important fact about PHEV is its fast dynamic response to compensate the required reactive power of grid in the case of contingencies. This fast dynamic behavior can make the PHEV comparable with synchronous generator in terms of dynamic voltage regulation but with lower operation cost. The advantage of PHEV is helpful to improve the voltage stability margin of system. The owner of PHEV can absorb active power from the grid but at the same time inject reactive power to the grid. Or even it can have a reactive power transaction with the grid without any active power transaction. In other words, the incorporation of PHEV in the reactive power market can implicitly increase the reactive power reserve of grid and consequently improve the system voltage stability margin [14,15,22,23].

## 3 Deterministic reactive power market

In this section, at first, the expected payment function (EPF) of PHEV is proposed. The reactive power capability curve of a PHEV is shown in Fig. 2. Based on the explanation in the previous section, five operating regions for a PHEV on the reactive power coordinate can be defined as shown in Fig. 3. In the boundary ($Q_{b'}$ to $Q_b$), the PHEV should operated based on the requirement of system and the ISO decision without any operating payment. A PHEV operated in this boundary is paid only the availability payment. Accordingly the EPF of PHEV in this region is as follows:

$$EPF_0 = \text{availability cost} = a_0 \tag{5}$$

where, $\alpha_0$ is availability price offer in dollars.

In region I ($Q_{b'}$ to $Q_{M'}$) and region II ($Q_b$ to $Q_M$) because of generating/absorbing reactive power, the losses of PHEV increase and therefore, it can expect to be paid for its service. Thus, the EPF of PHEV, besides availability component, will contain the cost of loss component. It is noted that the losses of region II are different from those of region I. The losses of PHEV in regions I, II are a quadratic function of PHEV reactive power generation/absorption which is discussed with more details in Appendix. Accordingly, the losses cost in region I and II are considered as $(m_1 Q)$ and $(m_2 Q)$, respectively. Therefore, the EPF of PHEV in these regions are

$$EPF_I = \text{availability cost} + \text{losses cost} = a_0 + \int_{Q_b}^{Q_M} (m_1 \cdot Q)\, dQ \tag{6}$$

$$EPF_{II} = \text{availability cost} + \text{losses cost} = a_0 + \int_{Q_{b'}}^{Q_{M'}} (m_2 \cdot Q)\, dQ \tag{7}$$

$m_1$ is cost of losses price offer for operating in reactive power absorption in region I ($Q_{b'} \leq Q \leq Q_{M'}$) in \$/kVAr–h/kVAr–h, $m_2$ is cost of loss price offer for operating in reactive power injection in region II ($Q_b \leq Q \leq Q_M$) in \$/kVAr–h/kVAr–h (Fig. 3).

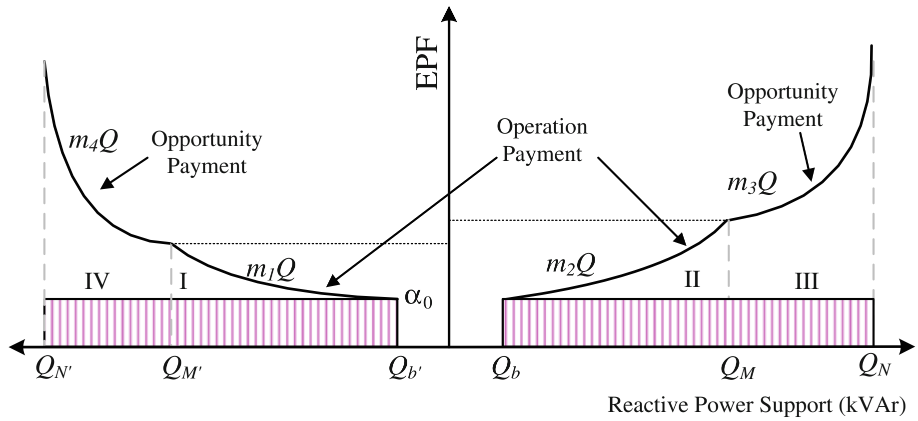
In region III ($Q_M$ to $Q_N$), and region IV ($Q_{M'}$ to $Q_{N'}$), the PHEV is managed to reduce its active power to generate the required reactive power. Thus, the PHEV incurs loss of revenue cost and consequently, the EPF will contain all components of cost (i.e., availability cost, cost of loss and opportunity cost). The LOC of PHEV is a quadratic function of its reactive power generation/absorption which is discussed with more details in Appendix. Accordingly, the EPF of PHEV in this region will be as follows:

$$\begin{aligned} \text{EPF}_{\text{III}} &= \text{availability cost} + \text{losses cost} + \text{LOC cost} \\ &= a_0 + \int_{Q_b}^{Q_M} (m_2 Q)\, dQ + \int_{Q_M}^{Q_N} (m_3 Q)\, dQ \end{aligned} \tag{8}$$

$$\begin{aligned} \text{EPF}_{\text{IV}} &= \text{availability cost} + \text{losses cost} + \text{LOC cost} \\ &= a_0 + \int_{Q_{b'}}^{Q_{M'}} (m_1 Q)\, dQ + \int_{Q_{M'}}^{Q_{N'}} (m_4 Q)\, dQ \end{aligned} \tag{9}$$

Where, $m_3$ is opportunity price offer for operating in region ($Q_M \leq Q \leq Q_N$) in \$/kVAr–h/kVAr–h and $m_4$ is

**Fig. 3** Reactive power offer structure of PHEV




opportunity price offer for operating in region ($Q_{M'} \leq Q \leq Q_{N'}$) in \$/kVAr–h/kVAr–h (Fig. 3).

According to the classification of reactive power production cost, an offer structure is formulated mathematically as [18,27]:

$$\mathrm{EPF}_j = a_{0j} + \int_{Q_{b'}}^{Q_{M'}} (m_{1j} Q_j)\,\mathrm{d}Q_j + \int_{Q_b}^{Q_M} (m_{2j} Q_j)\,\mathrm{d}Q_j + \int_{Q_M}^{Q_N} (m_{3j} Q_j)\,\mathrm{d}Q_j + \int_{Q_{M'}}^{Q_{N'}} (m_{4j} Q_j)\,\mathrm{d}Q_j \quad (10)$$

where $a_{0j}, m_{1j}, m_{2j}, m_{3j}$ and $m_{4j}$ are the bid values of the $j$th provider for the reactive power market. The cost of loss and also the opportunity cost are a quadratic function of $Q$ (Fig. 6 in Appendix).

The reactive power is settled based on the minimization of total payment function (TPF) paid to the participants of reactive power market. In other words, the objective function of this minimization problem is the EPF of PHEVs plus the EPF of synchronous generators that should be minimized. Therefore, the total payment will depend on the market price of the five components of the bid prices offered by the PHEVs and four components of the bid prices offered by synchronous generators. The TPF is formulated as follows:

$$\begin{aligned}
\mathrm{TPF} &= \mathrm{TPF}_G + \mathrm{TPF}_P \\
&= \sum_{i \in \mathrm{gen}} \begin{bmatrix} \rho_0 Q^i_{G\max} W^i_0 - \rho_1 W^i_1 Q^i_{1G} + \rho_2 W^i_2 \left(Q^i_{2G} - Q^i_{G\mathrm{base}}\right) \\ + \rho_2 W^i_3 \left(Q^i_{3G} - Q^i_{G\mathrm{base}}\right) + \frac{1}{2}\rho_3 W^i_3 \left[\left(Q^i_{3G}\right)^2 - \left(Q^i_{AG}\right)^2\right] \end{bmatrix} + \sum_{j \in \mathrm{PHEV}} \\
&\Bigg( a^j_0 W^j_0 + \overbrace{\frac{1}{2}\rho_1 W^j_1 \left(\left(Q^j_{1P}\right)^2 - \left(Q^j_{b'}\right)^2\right) + \frac{1}{2}\rho_1 W_{4j} \left(\left(Q^j_{4P}\right)^2 - \left(Q^j_{b'}\right)^2\right)}^{\text{Losses Payment regions I, V}} \\
&+ \overbrace{\frac{1}{2}\rho_2 W^j_2 \left(\left(Q^j_{2P}\right)^2 - \left(Q^j_b\right)^2\right) + \frac{1}{2}\rho_2 W^j_3 \left(\left(Q^j_{3P}\right)^2 - \left(Q^j_b\right)^2\right)}^{\text{Losses Payment regions II, III}} \\
&+ \overbrace{\frac{1}{2}\rho_4 W^j_4 \left(\left(Q^j_{4P}\right)^2 - \left(Q^j_{M'}\right)^2\right)}^{\text{LOC Payment region IV}} + \overbrace{\frac{1}{2}\rho_3 W^j_3 \left(\left(Q^j_{3P}\right)^2 - \left(Q^j_M\right)^2\right)}^{\text{LOC Payment region III}} \Bigg)
\end{aligned} \quad (11)$$

where $\rho_0$, $\rho_1$, $\rho_2$, $\rho_3$, and $\rho_4$ are the market-clearing prices (MCPs) of offer prices of market participants for $a_0, m_1, m_2, m_3$ and $m_4$, respectively, which are accepted in the reactive power market. The discussion for TPF of synchronous generator ($\mathrm{TPF}_i$) is the same as [14,18]. However, $\mathrm{TPF}_j$ deserves more explanation. According to (11), the PHEV owner is paid for losses payment as it enter to region I, IV for reactive power absorption, and region II, III for reactive power production. Despite the losses of synchronous generator (which is a linear function), the losses payment of PHEV is quadratic function of PHEV reactive power output as taken in the Appendix. The LOC payment of PHEV is similar to that of synchronous generator which is a quadratic function of its reactive power output [18,22].

In (11), $Q^j_{1P}$, $Q^j_{2P}$, $Q^j_{3P}$, and $Q^j_{4P}$ represent the regions ($Q_{b'}$ to $Q_{M'}$), ($Q_b$ to $Q_M$), ($Q_M$ to $Q_N$), and ($Q_{M'}$ to $Q_{N'}$), respectively. $W^j_1$, $W^j_2$, $W^j_3$, and $W^j_4$ are binary variable, showing the compensation region of the PHEV. If the $j$th PHEV is participated in the reactive power market and operated in region ($Q_{b'}$ to $Q_b$), then $W^j_0 = 1$ and $W^j_1 = W^j_2 = W^j_3 = W^j_4 = 0$. If the accepted PHEV is operated in region I ($Q_{b'}$ to $Q_{M'}$) then $W^j_0 = W^j_1 = 1$ and $W^j_2 = W^j_3 = W^j_4 = 0$. If the PHEV is operated in region II ($Q_b$ to $Q_M$) then $W^j_0 = W^j_2 = 1$ and $W^j_1 = W^j_3 = W^j_4 = 0$. If the PHEV is operated in region III ($Q_M$ to $Q_N$) then $W^j_0 = W^j_3 = 1$ and $W^j_1 = W^j_2 = W^j_4 = 0$, and if the PHEV is operated in region IV ($Q_{M'}$ to $Q_{N'}$) then $W^j_0 = W^j_4 = 1$ and $W^j_1 = W^j_2 = W^j_3 = 0$. When the $j$th PHEV is not selected or is selected and operated in one of regions I, II, III, IV then the constraint $W^j_1 + W^j_2 + W^j_3 + W^j_4 \leq W^j_0$ will be satisfied in the equality form (0 = 0 and 1 = 1, respectively).

The objective function of reactive power market clearing, depicted in (11), is subject to equality and inequality constraints, which will be described in Sect. 3. In the next section, PHEV contingencies are modeled in the proposed stochastic framework for reactive power market clearing. The other uncertainty sources of power system including branch contingencies and load variations with presence of PHEVs will be considered in our future work.

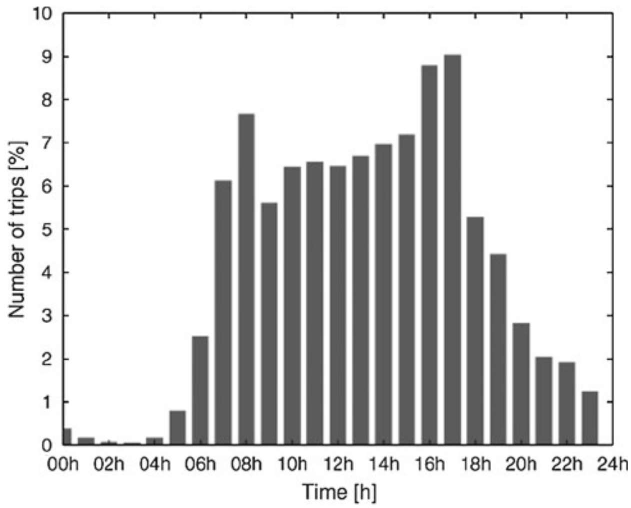



**Fig. 4** Percentage of trips by vehicle each hour [8,29]

## 4 Stochastic reactive power market

The uncertainty of synchronous generator is considered based on its forced outage rate (FOR) [28]. The FOR of synchronous generators can be defined as:

$$\mathrm{FOR}_G = \frac{\lambda_G}{\lambda_G + \mu_G} \tag{12}$$

Where, $\lambda_G$ and $\mu_G$ are failure rate and repair rate of the $G$th generator, respectively.

The unavailability of PHEV is different from that of synchronous generator. The uncertainty of PHEV depends on the failure rate of PHEV component as well as the percentage of the PHEV trip at the interested hour of day.

A typical percentage of trips by vehicle at each hour is shown in Fig. 4 [8,29]. For an interested hour of day, if the percentage of PHEV trip is $u$ % and the PHEV is considered $k$ % reliable, the availability of PHEV at interested hour in percent is

$$A_P = (1-u) \times (1-k) \tag{13}$$

where, $A_P$ is the availability of PHEV. Accordingly the FOR of PHEV will be

$$\mathrm{FOR}_P = 1 - A_P \tag{14}$$

The percentage of trips of PHEV at 7:00 a.m., according to Fig. 4 is about 6 %. If we consider PHEV 95 % reliable, then the availability of PHEV based on (13) is calculated as:

$$\text{Availability of PHEV} = A_P = (1-0.06) \times (1-0.05) = 0.893$$

Accordingly, the FOR of PHEV will be:

$$\mathrm{FOR} = 1 - A_P = 1 - 0.893 = 0.107$$

In spite of the synchronous generator FOR value which is constant, the FOR of PHEV depends on the hour of day as shown in Fig. 4.

The proposed stochastic framework for reactive power market clearing in the presence of PHEVs consists of two steps. In the first step, based on the Monte-Carlo simulation (MCS) technique, some scenarios are generated [23]. In order to decrease computational burden, scenario reduction technique is used in the proposed stochastic market-clearing procedure. The optimization problem of each selected scenario (after using scenario reduction technique) is modeled in the second step and solved in the form of a MINLP problem.

### 4.1 First step: scenario generation and reduction

In the first step, the scenario generation and reduction are discussed. The applied MCS technique is based on the FOR of synchronous generators and PHEVs that are defined in (12) and (14), respectively.

In each scenario of the MCS, a random number between [0, 1] is separately generated for each reactive power provider (i.e., PHEV or synchronous generator), and compared with it's FOR. If the random number produced for a PHEV or generator is greater than its FOR, then the PHEV or generator is available and it can participate in the reactive power market; otherwise, it is out of service and it cannot offer in the reactive power market. This procedure is iterated for all PHEVs and generators and consequently, a number of scenarios are generated. It is suggested that increasing of scenarios to have better modeling of uncertainties but, the computation burden will be increased by increasing of scenarios numbers. Some of scenarios are identical and non-effective, so by applying the scenario reduction technique can reduce the number of scenarios while maintaining a good approximation of the system uncertain behavior. In this paper, the basic idea of scenario reduction is to eliminate a scenario with very low probability and scenarios that are very similar [30]. Based on this approach the most probable contingencies can be extracted for using in the market-clearing procedure. Furthermore, in the generated scenarios, repeated ones should be discarded. The probability of each scenario ($\pi_s$) can be calculated as follows:

$$\pi_s = \prod_{i=1}^{N_G} \left( Z_G^{i,s}(1-\mathrm{FOR}_G^i) + (1-Z_G^{i,s})\,\mathrm{FOR}_G^i \right) \times \prod_{j=1}^{N_P} \left( Z_P^{j,s}(1-\mathrm{FOR}_P^j) + (1-Z_P^{j,s})\,\mathrm{FOR}_P^j \right) \tag{15}$$

where, $Z_G^{i,s}$ and $Z_P^{j,s}$ are binary variable indicating whether the $i$th generator and $j$th PHEV in the $s$th scenario are available and so can offer in the reactive power market ($Z_P^{j,s} = 1$) or not ($Z_P^{j,s} = 0$) that is determined by the MCS for each scenario. The optimization problem of each selected scenario of the first step, after the scenario reduction, has MINLP form, which is solved in the second step. Its details are provided in the next subsections.

### 4.2 Second step: reactive power market clearing

In the proposed stochastic optimization framework for reactive power market clearing, the Objective Function (OF) is to minimize the expected value of TPF of reactive power providers that are PHEVs and synchronous generators. Accordingly, the OF can be written as the following equation:

$$\begin{aligned} ObjectedFunction = &\sum_{s=1}^{N_s} \pi_s \left( \sum_{j=1}^{N_P} Z^{j,s} \left( \rho_0^s W_0^{j,s} \right.\right. \\ &+ \frac{1}{2}\rho_1^s W_1^{j,s} \left( \left( Q_{1P}^{j,s} \right)^2 - \left( Q_{b'}^{j} \right)^2 \right) + \frac{1}{2}\rho_1^s W_4^{j,s} \left( \left( Q_{4P}^{j,s} \right)^2 - \left( Q_{b'}^{j} \right)^2 \right) \\ &+ \frac{1}{2}\rho_4^s W_4^{j,s} \left( \left( Q_{4P}^{j,s} \right)^2 - \left( Q_{M'}^{j} \right)^2 \right) + \frac{1}{2}\rho_2^s W_2^{j,s} \left( \left( Q_{2P}^{j,s} \right)^2 - \left( Q_{b}^{j} \right)^2 \right) \\ &+ \frac{1}{2}\rho_2^s W_3^{j,s} \left( \left( Q_{3P}^{j,s} \right)^2 - \left( Q_{b}^{j} \right)^2 \right) + \frac{1}{2}\rho_3^s W_3^{j,s} \left.\left( \left( Q_{3P}^{j,s} \right)^2 - \left( Q_{M}^{j} \right)^2 \right)\right) \\ &+ \sum_{i=1}^{N_G} Z^{i,s} \left( \rho_0^s W_0^{i,s} - \rho_1^s W_1^{i,s} Q_{1G}^{i,s} + \rho_2^s W_2^{i,s} \left( Q_{2G}^{i,s} - Q_{Gbase}^{i} \right) \right. \\ &\rho_2^s W_3^{i,s} \left( Q_{3G}^{i,s} - Q_{Gbase}^{i} \right) + \frac{1}{2}\rho_3^s . W_3^{i,s} \left.\left.\left( \left( Q_{3G}^{i,s} \right)^2 - \left( Q_{AG}^{i} \right)^2 \right)\right)\right) \end{aligned} \quad (16)$$

where, $i$, $j$ and $s$ refer to PHEV number and scenario number, respectively. $Q_{1P}^{j,s}$, $Q_{2P}^{j,s}$, $Q_{3P}^{j,s}$, $Q_{4P}^{j,s}$, $Q_{1G}^{i,s}$, $Q_{2G}^{i,s}$, $Q_{3G}^{i,s}$, $W_{1P}^{j,s}$, $W_{2P}^{j,s}$, $W_{3P}^{j,s}$, $W_{4P}^{j,s} W_{1G}^{i,s}$, $W_{2G}^{i,s}$, $W_{3G}^{i,s}$ $\rho_0^s$, $\rho_1^s$, $\rho_2^s$, $\rho_3^s$, and $\rho_4^s$ are as described for (11) except that in the stochastic framework, they are also a function of the scenario number $s$. Also, $Z_G^{i,s}$, $Z_P^{j,s}$, and $\pi_s$ are defined in (15).

The FOR of PHEVs or generators has a fundamental role in the selecting of PHEVs or generators in the first step and the selected PHEVs or generators in this step are participated in the reactive power market based on their offered prices as well as network constraints and operating characteristics of PHEVs and generators. $Z_P^{j,s}$ is the binary variable and $Z_P^{j,s} = 1$ indicated that the PHEVs can offer in the reactive power market. The participation of PHEV in the reactive power market is determined by ISO and if $W_0^{j,s} = 1$, the PHEV can participated in the reactive power market. So, the PHEV may be offered but it cannot be accepted in the reactive power market. In other words, $W_0^{j,s} \leq Z_P^{j,s}$.

The optimization problem (Eq. 16) is subject to following equality and inequality constraints, which should be satisfied for all scenarios selected in the first step after scenario reduction.

– *AC load flow constrains:*

$$\begin{aligned} &\underbrace{Z_G^{i,s} P_G^{i,s}}_{i=m} + \underbrace{Z_P^{j,s} P_P^{j,s}}_{j=m} - P_D^m \\ &\quad = \sum_{n=1}^{NB} V_{m,s} V_{n,s} Y_{m,n} \cos(\theta_{m,s} - \theta_{n,s} - \theta_{m,n}) \\ &m = 1, \ldots, NB \quad s = 1, \ldots, NS \end{aligned} \quad (17)$$

$$\begin{aligned} &\underbrace{Z_G^{i,s} Q_G^{i,s}}_{i=m} + \underbrace{Z_P^{j,s} Q_P^{j,s}}_{j=m} - Q_D^m \\ &\quad = \sum_{n=1}^{NB} V_{m,s} V_{n,s} Y_{m,n} \sin(\theta_{m,s} - \theta_{n,s} - \theta_{m,n}) \\ &\quad m = 1, \ldots, NB \quad s = 1, \ldots, NS \end{aligned} \quad (18)$$

where, $P_D^m$ and $Q_D^m$ are active and reactive power part of load at bus $m$, respectively. $P_G^{i,s}$, $Q_G^{i,s}$, $P_P^{j,s}$, and $Q_P^{j,s}$ are respectively, active and reactive power output of synchronous generators and PHEVs. $V$ and $Y$ refer to voltage magnitude and magnitude of network admittance matrix, respectively. Equations (17) and (18) refer to power flow balance equations for each bus of network in each scenario. It is assumed that in the post-contingency states, spinning and non-spinning reserves substitute lost generating units to balance the power system active demand [31]. These spinning and non-spinning reserve markets together with energy market are cleared prior to the reactive power market. In other words, active power balance of system is not the concern of ISO in the reactive power market.

– The operation constraints of generators:

$$W_0^{i,s}, W_1^{i,s}, W_2^{i,s}, W_3^{i,s} \in \{0, 1\}; \quad i \in \text{the generator index} \quad (19)$$

$$Q_G^{i,s} = Q_{1G}^{i,s} + Q_{2G}^{i,s} + Q_{3G}^{i,s} \quad (20)$$

$$W_1^{i,s} Q_{\min}^i \leq Q_{1G}^{i,s} \leq 0 \quad (21)$$

$$W_2^{i,s} Q_{\text{Gbase}}^i \leq Q_{2G}^{i,s} \leq W_2^{i,s} Q_{AG}^i \quad (22)$$

$$W_3^{i,s} Q_{AG}^i \leq Q_{3G}^{i,s} \leq W_3^{i,s} Q_{BG}^i \quad (23)$$

$$W_1^{i,s} + W_2^{i,s} + W_3^{i,s} \leq W_0^{i,s} \quad (24)$$

It is noted that, $Q_{\text{AG}}$ is the point that synchronous generator enter to LOC region to generate $Q_{\text{BG}}$ reactive power market [18,22].

– The operation constraints of PHEV:

$$W_0^{j,s}, W_1^{j,s}, W_2^{j,s}, W_3^{j,s}, W_4^{j,s} \in \{0, 1\}; \quad j \in \text{PHEV index} \quad (25)$$

$$Q_P^{j,s} = Q_{1P}^{j,s} + Q_{2P}^{j,s} + Q_{3P}^{j,s} + Q_{4P}^{j,s} \quad (26)$$

$$W_1^{j,s} Q_b^j \le Q_{1P}^{j,s} \le W_1^{j,s} Q_M^j \tag{27}$$
$$W_2^{j,s} Q_M^j \le Q_{2P}^{j,s} \le W_2^{j,s} Q_N^j \tag{28}$$
$$W_3^{j,s} Q_{b'}^j \le Q_{3P}^{j,s} \le W_3^{j,s} Q_{M'}^j \tag{29}$$
$$W_4^{j,s} Q_{M'}^j \le Q_{4P}^{j,s} \le W_4^{j,s} Q_{N'}^j \tag{30}$$
$$W_1^{j,s} + W_2^{j,s} + W_3^{j,s} + W_4^{j,s} \le W_0^{j,s} \tag{31}$$

– Constraints related to determination of MCPs of price components in reactive power market:

$$W_0^{i,s} a_0^i \le \rho_0^s \tag{32}$$
$$W_1^{i,s} m_1^i \le \rho_1^s \tag{33}$$
$$\left(W_2^{i,s} + W_3^{i,s}\right) m_2^i \le \rho_2^s \tag{34}$$
$$W_3^{i,s} m_3^i \le \rho_3^s \tag{35}$$
$$W_0^{j,s} a_0^j \le \rho_0^s \tag{36}$$
$$\left(W_1^{j,s} + W_4^{j,s}\right) m_1^j \le \rho_1^s \tag{37}$$
$$\left(W_2^{j,s} + W_3^{j,s}\right) m_2^j \le \rho_2^s \tag{38}$$
$$W_3^{j,s} m_3^j \le \rho_3^s \tag{39}$$
$$W_4^{j,s} m_4^j \le \rho_4^s \tag{40}$$

– PHEV capability curve constraints:

$$\left(P_P^{j,s} + P_{\text{loss}}^{j,s}\right)^2 + \left(Q_P^{j,s}\right)^2 \le \left(S_n^j\right)^2 \quad \text{for charging mode} \tag{41}$$
$$\left(P_P^{j,s} - P_{\text{loss}}^{j,s}\right)^2 + \left(Q_P^{j,s}\right)^2 \le \left(S_n^j\right)^2 \quad \text{for discharging mode} \tag{42}$$
$$P_{a-}^j \le P^{j,s} \le P_{a+}^j \quad \text{Maximum PHEV output power} \tag{43}$$

where, $S_n^j$ is the nominal apparent output power of $j$th PHEV and $P_{a+}^j$ and $P_{a-}^j$ are the maximum discharging and charging active power of PHEV in scenario $s$, respectively which are determined by PHEV owners.

– Generators capability curve constraints[18]:

$$Q_G^{i,s} \le \sqrt{\left(V_t^{i,s} I_a^{i,s}\right)^2 - \left(P_G^{i,s}\right)^2} \tag{44}$$

Capability curve limit (Armature current limit)

$$Q_G^{i,s} \le \sqrt{\left(\frac{V_t^{i,s} E_{af}^{is}}{X_s^{i,s}}\right)^2 - \left(P_G^{i,s}\right)^2} - \frac{\left(V_t^{i,s}\right)^2}{X_s^{i,s}} \tag{45}$$

Capability curve limit (Field current limit)

where, $V_t^{i,s}$, $I_a^{i,s}$, $E_{af}^{i,s}$ and $X_s^{i,s}$ are respectively terminal voltage, steady state armature current, excitation voltage and synchronous reactance of generator at bus $i$ in scenario $s$. More details of synchronous generator capability curve can be found in [14,15,18,22].

– Grid constraints: Line flow and bus voltage magnitude constrains:

$$S_{m,n}^s \le S_{m,n}^{s,\max}, \quad m = 1, \ldots, NB\, n = 1, \ldots, NB \tag{46}$$
$$V_{m,\min}^s \le V_m^s \le V_{m,\max}^s \tag{47}$$

where, $S_{m,n}^{s,\max}$ is the power flow of the line connected to bus $m$ and $n$ of network in scenario $s$ and $V_m^s$ is the voltage of bus $m$ in scenario $s$. Equations (46) and (47) ensure that line flows and bus voltages are in their pre-determined limits.

## 5 Case study

A typical low voltage (LV) network is considered as study case which is shown in Fig. 5 [32,33]. The network comprises three feeders: one feeder serves residential area, the other one is industrial feeder, and the last feeder includes commercial consumers. The power factor of all loads is assumed to be equal to 0.85 lagging.

This network is modified by adding ten PHEVs to the network and eliminating the DGs such as micro turbines (MTs), small wind turbines (WTs) and photovoltaic (PVs) from the first version of the network used in [14,21]. The required data of this network can be found in [14,21]. It is worth to mention that the penetration level of PHEVs in recent power grid is not significant; however, they are growing in the world. For instance, the US government puts a lot of effort into accelerating the introduction and penetration of advanced electric drive vehicles into the market.

To study our proposed stochastic reactive power market framework, only synchronous generator and PHEVs are considered as the participant of reactive power market. In our study the grid is considered as a synchronous generator which submits its four-component price offers, i.e., $a_0$, $m_1$, $m_2$, and $m_3$ in order to participate in the stochastic reactive power market. Also, the owners of PHEVs should offer their five-component offer prices which are $a_0$, $m_1$, $m_2$, $m_3$, and $m_4$. In this study a decoupled reactive power market is considered by the ISO. Therefore, the output of synchronous generator and PHEVs in the energy market is the boundary of participant wherein they entered to LOC region and should be paid for the LOC payment by the ISO if they are accepted in the reactive power market and operated in the LOC region. In other words, the $(P_A Q_A)$ of synchronous generator [18,22] and $(P_M Q_M)$ of PHEVs are determined based on their output in the energy market cleared previously. However, for the sake of simplicity, in our study it is assumed that $Q_{AG} = 0.6 \times Q_{BG}$ and

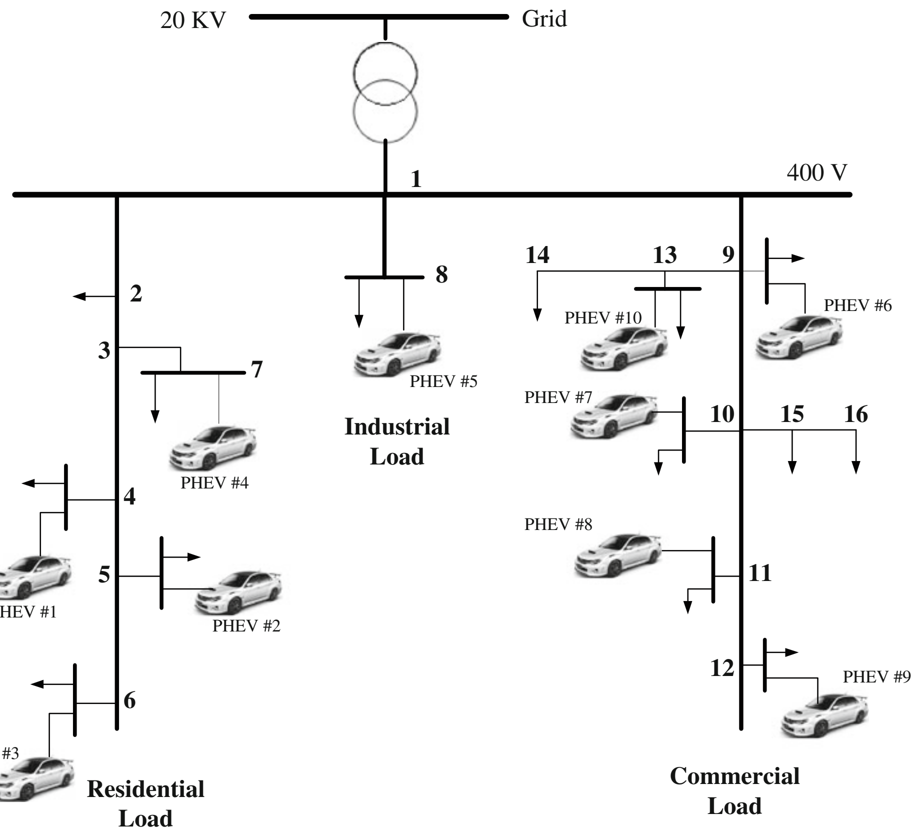



**Fig. 5** The modified version of 17-node LV network

**Table 1** Reactive power offer prices of participant PHEVs and synchronous generator (SG)

| Bus No. | PHEV No. | Components of offer prices | | | | |
|---|---|---|---|---|---|---|
| | | $m_4$ \$/$(kVAr-h)^2$ | $m_1$ \$/$(kVAr-h)^2$ | $a_0$ \$ | $m_2$ \$/$(kVAr-h)^2$ | $m_3$ \$/$(kVAr-h)^2$ |
| 4 | 1 | 0.772 | 0.609 | 0.069 | 0.603 | 0.722 |
| 5 | 2 | 0.869 | 0.811 | 0.048 | 0.726 | 0.743 |
| 6 | 3 | 0.672 | 0.483 | 0.067 | 0.551 | 0.717 |
| 7 | 4 | 0.708 | 0.602 | 0.052 | 0.518 | 0.683 |
| 8 | 5 | 0.898 | 0.537 | 0.063 | 0.459 | 0.858 |
| 9 | 6 | 0.641 | 0.396 | 0.06 | 0.492 | 0.8 |
| 10 | 7 | 0.814 | 0.624 | 0.068 | 0.564 | 0.683 |
| 11 | 8 | 0.769 | 0.433 | 0.066 | 0.368 | 0.663 |
| 12 | 9 | 0.675 | 0.707 | 0.075 | 0.629 | 0.627 |
| 13 | 10 | 0.685 | 0.895 | 0.073 | 0.765 | 0.702 |
| 1 | | | | | | |
| Syn Gen (Grid) | – | – | 0.281 | 0.1 | 0.32 | 0.35 |

$Q_M = 0.7 \times Q_N$. The price offers of reactive power market participants are taken in Table 1 [18,22]. The buses' voltage ranges are 0.95–1.05 per unit. The optimization problem of stochastic reactive power market clearing is in the form of mixed integer non-linear programming (MINLP) that is modeled in GAMS software using DICOPT solver [34]. In case study, both deterministic and stochastic reactive power market are cleared and the results are discussed which are taken in the following.

### 5.1 Deterministic reactive power market

In the deterministic market-clearing procedure, all of PHEVs are in service and therefore can offer their bids in the reactive

**Table 2** Results of the deterministic and stochastic reactive power market clearing

| Bus No. | PHEV No. | Deterministic market | | | | | | Stochastic market | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $W_0$ | $W_2$ | $W_3$ | $Q_{\text{PHEV}}$ (kVAr) | LOC payment (cent) | TPF (cent) | $Q_{\text{PHEV}}$ (kVAr) | LOC payment (cent) | TPF (cent) |
| 4 | 1 | 1 | 1 | 0 | 5.3132 | – | 14 | 4.83 | – | – |
| 5 | 2 | 1 | 0 | 1 | 9.09 | 20.86 | 62.4 | 6.7 | – | – |
| 6 | 3 | 1 | 1 | 0 | 5.3512 | – | 14.24 | 5.21 | – | – |
| 7 | 4 | 1 | 1 | 0 | 6.2982 | – | 19.76 | 5.75 | – | – |
| 8 | 5 | 1 | 1 | 0 | 4.8649 | – | 11.78 | 4.3 | – | – |
| 9 | 6 | 1 | 1 | 0 | 6.7432 | – | 22.66 | 5.37 | – | – |
| 10 | 7 | 1 | 1 | 0 | 6.271 | – | 19.58 | 5.45 | – | – |
| 11 | 8 | 1 | 0 | 1 | 8.43 | 17.94 | 53.69 | 6.86 | – | – |
| 12 | 9 | 1 | 0 | 1 | 7.2408 | 2.895 | 29.09 | 5.87 | – | – |
| 13 | 10 | 1 | 1 | 0 | 6.2684 | – | 19.56 | 5.68 | – | – |
| 1 | | | | | | | | | | |
| Syn Gen (Grid) | – | 1 | 1 | 0 | 38.947 | – | 29.44 | 38.95 | – | – |
| Total | | | | | 104.82 | **41.695** | **296.2** | 94.97 | **154.9** | **437.82** |

power market. The sixth column of Table 2 includes the reactive power output of PHEVs in the deterministic reactive power market clearing. Total reactive power of PHEVs is 104.82 kVAR. In this market, only three PHEVs (PHEV#2, 8 and 9) enter to region III (LOC region) and accordingly the LOC payment is approximately low in the deterministic market. The total TPF and LOC payment in this case are 296.2 and 41.695 cents of dollar respectively, shown in Table 2. It is noted that in the case of incorporating thousands of PHEVs in the reactive power market, there are some aggregators/parking which can collect the produced reactive power of each PHEV and this model can be used for aggregators/parking. Therefore, the aggregators or parking owners can estimate the amount of reactive power generation or absorption in each hour. Consequently, the incorporating of thousands of PHEVs in the reactive power market is applicable (Table 3).

### 5.2 Stochastic reactive power market

In the stochastic market, at first 120 scenarios are generated by using MCS method. This method is used in different works such as [23,25]. The generated scenarios present the probable states of power system. It imposes a high computational burden to clear reactive power market for all of these scenarios. So, the set of generated scenarios is reduced using the scenario reduction technique, described in Sect. 4.1. In the scenario reduction stage, the set is reduced to 28 scenarios after discarding similar ones. After discarding low probable scenarios, the set is more reduced to nine scenarios.

The scenario probability threshold is considered as 0.005; that is, scenarios with the probability lower than this threshold are ignored. For each of the remaining scenarios, the reactive power market clearing is separately run, as described in Sect. 4.2, considering the status of PHEVs in the scenario. The nine remaining scenarios of the proposed stochastic framework include the non-contingent scenario (deterministic market) plus eight post-contingent scenarios. The reactive power output of PHEVs in the stochastic reactive power market clearing is shown in the last column of Table 2 which is equal to 94.97 kVAR. In fact, this value is the expected value of PHEVs' reactive power output (expected value of $Q_P^{j,s}$) over all nine accepted scenarios.

The expected TPF and expected LOC of the stochastic framework are 437.82 and 154.9 cents of dollar, respectively (Table 2). Both of expected TPF and expected LOC in the stochastic reactive power market-clearing framework are greater than the TPF and LOC in deterministic reactive power market-clearing scheme, indicating that considering power system uncertainities lead the stochastic framework to become more expensive than the deterministic scheme. This extra charge can be interpreted as the cost of considering the uncertainties of power system in the clearing of reactive power market. However, the stochastic framework is more realistic than the deterministic scheme, motivating the ISO to clear reactive power market in stochastic framework despite its more TPF value.

## 6 Conclusions

The unavailability of PHEV due to trip and also the FOR of synchronous generators increase the uncertainty of reactive

**Table 3** The results of reactive power market-clearing framework for nine accepted scenarios after scenario reduction

| No. | Scenario number | Out of service PHEVs #PHEV (bus) | Probability ($\pi_s$) | TPF ($) |
|---|---|---|---|---|
| 1 | 1 | – | 0.5025 | 2.962 |
| 2 | 5 | 1 (4) | 0.0547 | 3.453 |
| 3 | 18 | 3 (6) | 0.0409 | 3.655 |
| 4 | 27 | 10 (13) | 0.0456 | 3.457 |
| 5 | 35 | 7 (10) | 0.0443 | 3.464 |
| 6 | 57 | 4 (7) | 0.0527 | 4.069 |
| 7 | 67 | 5 (8) | 0.0309 | 3.273 |
| 8 | 85 | 6 (9) | 0.0464 | 3.688 |
| 9 | 114 | 9 (12) | 0.0374 | 3.324 |

**Table 4** PHEV charger electrical parameters

| Parameter | Description | Value |
|---|---|---|
| $V_{T0}$ | Transistor open-circuit voltage | 0.9 V |
| $V_{D0}$ | Diode open-circuit voltage | 0.75 V |
| $f_s$ | Switching frequency | 10 kHz |
| $R_{T0}$ | Transistor on-state resistance | 52 mΩ |
| $R_{D0}$ | Diode on-state resistance | 30 mΩ |
| $t_{eq}$ | Equivalent time commutation constant | 366 nsec |
| $C_{dc}$ | DC link capacitor | 2.2 mF |
| $L_c$ | Inductance of coupling inductor | 2.5 mH |
| $R_c$ | Resistance of coupling inductor | 0.3 Ω |
| $V_{grid}$ | Grid voltage | 230 V |
| $V_{DC}$ | DC link voltage | 400 V |
| $I_s$ | Grid current | 20 A |
| $S_n$ | Inverter VA | 4.6 kVA |
| $P_n$ | Nominal active power injected/absorbed to /from grid | 3 kW |

power market-clearing procedure. In this paper, the clearing of reactive power market in the presence of PHEVs is studied based on the stochastic and deterministic optimization model using MINLP formulation. For both stochastic and deterministic models, considering technical limit, the PHEV capability curve and after that its EPF is extracted and stochastic reactive power market is cleared based on the minimization of the expected TPF. For stochastic model, different probable scenarios are generated by applying MCSs, to use in the stochastic optimization procedure. The system operator can use this scheme to determine the influence of PHEV and synchronous generator uncertainty on the market-clearing process. The stochastic approach includes better modeling of power system uncertain behavior.

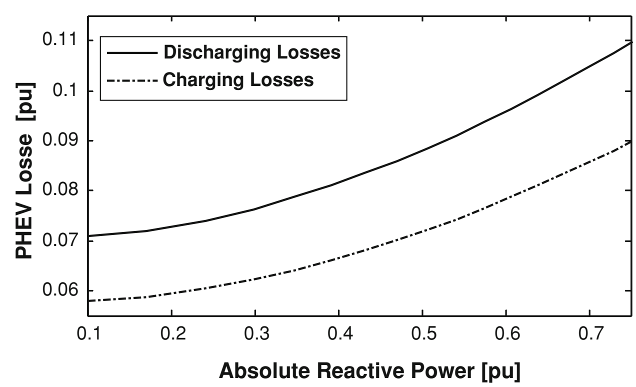



**Fig. 6** Losses of charging and discharging in terms of PHEV reactive power generation/absorption

## Appendix

Losses of PHEV in terms of its reactive power output

To obtain the EPF curve, the losses of typical PHEV are considered and discussed. The parameters for a typical inverter Fig. 1 are listed in Table 4 [35]. Using this table parameters, and also considering PHEV capability curve shown in Fig. 2, the per unit charging and discharging losses of PHEV are obtained in terms of PHEV reactive power output (generation/absorption) which is shown in Fig. 6.

From Fig. 6, it can be observed that the PHEV losses are a quadratic function of its reactive power output in both charging and discharging mode. However, the losses in the discharging mode are greater than that of charging mode which is mainly due to inverter switching losses.

The other important problem is the modeling of LOC of PHEV in terms of its reactive power output. The LOC payment is related to energy price at electricity market. In order to model the LOC payment of PHEV, for 15 different daily price of energy [32], the LOC of PHEV is calculated at different reactive power output of PHEV which is shown in Fig. 7 for reactive power generation and Fig. 8 for reactive power absorption of PHEV.

From Fig. 7, the LOC is a quadratic function of PHEV reactive power output in region III. The quadratic estimation of LOC is also shown in Fig. 7. Accordingly, the LOC of PHEV is considered as quadratic function of PHEV reactive power to extract the EPF of PHEV.

Figure 8 shows the LOC of PHEV in region IV which can be estimated as a quadratic function of PHEV reactive

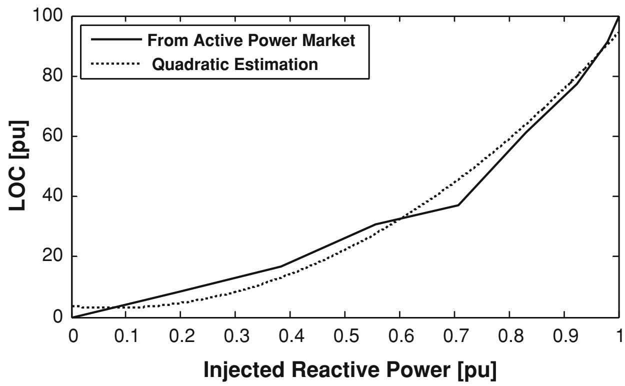



**Fig. 7** LOC of PHEV in terms of its reactive power generation (region III)

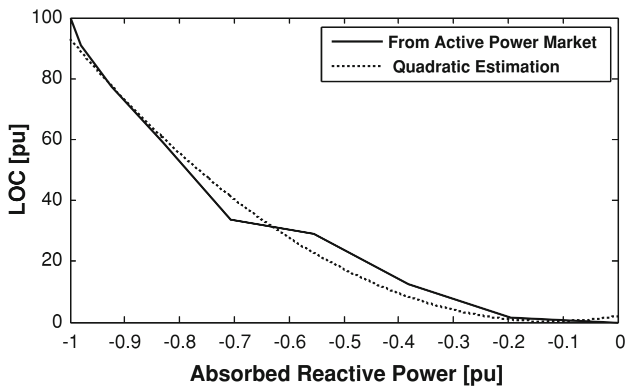



**Fig. 8** LOC of PHEV in terms of its reactive power absorption (region IV)

power absorption. Accordingly, the LOC part of PHEV is considered as quadratic function $Q$ the same as [18].

## References


1. Niknam T et al (2011) Multi-objective daily operation management of distribution network considering fuel cell power plants. In: IET Renewable Power Generation, vol 5. pp 356–367
2. Kempton W, Tomi′c J (2005) Vehicle-to-grid power fundamentals: calculating capacity and net revenue. J Power Sources 268–279
3. Kisacikoglu MC et al (2010) Examination of a PHEV bidirectional charger system for V2G reactive power compensation. In: 25th Annual IEEE applied power electronics conference and exposition (APEC). pp 458–465
4. Kisacikoglu MC et al (2010) Effects of V2G reactive power compensation on the component selection in an EV or PHEV bidirectional charger. In: IEEE energy conversion congress and exposition (ECCE). pp 870–876
5. Shao S et al (2010) Impact of TOU rates on distribution load shapes in a smart grid with PHEV penetration. In: Transmission and distribution conference and exposition, 2010 IEEE PES. pp 1–6
6. Qian K et al (2011) Modeling of load demand due to EV battery charging in distribution systems. IEEE Trans Power Syst 26(2)
7. Mitra P, Venayagamoorthy GK (2010) Wide area control for improving stability of a power system With plug-in electric vehicles. Gener Transm Distrib IET 4:1151–1163
8. Clement K et al (2010) The impact of charging plug-in hybrid electric vehicles on a residential distribution grid. IEEE Trans Power Syst 25
9. Su W, Chow M-Y (2011) Investigating a large-scale PHEV/PEV parking deck in a smart grid environment. In: 43rd North American Power Symposium. MA, Boston
10. Roe C et al (2009) Power system level impacts of PHEVs. In: 42nd Hawaii international conference on system sciences. pp 1–10
11. Dyke KJ et al (2010) The impact of transport electrification on electrical networks. IEEE Trans Ind Electron 57:3917–3926
12. Nde LPF et al (2011) Assessment of the impact of plug-in electric vehicles on distribution networks. IEEE Trans Power Syst 26: 206–213
13. Lopes JAP et al (2011) Integration of electric vehicles in the electric power system. Proc IEEE 99:168–183
14. Rabiee A et al (2009) Multiobjective clearing of reactive power market in deregulated power systems. In: Applied energy. pp 1555–1564
15. Rabiee A et al (2009) Reactive power pricing: problems and a proposal for a competitive market. IEEE Power Energy Mag 7(1):18–32
16. Bhattacharya K, Zhong J (2001) Reactive power as an ancillary service. IEEE Trans Power Syst 16:294–300
17. Zhong J (2006) A pricing mechanism for network reactive power devices in competitive market. In: IEEE Power India Conference
18. Zhong J, Bhattacharya K (2002) Toward a competitive market for reactive power. IEEE Trans Power Syst 17:1206–1215
19. Zhong J et al (2004) Localized reactive power markets using the concept of voltage control areas. IEEE Trans Power Syst 19:1555–1561
20. Chung CY et al (2004) Cost-based reactive power pricing with voltage security consideration in restructured power systems. Electr Power Syst Res 70:85–91
21. El-Samahy I et al (2008) A procurement market model for reactive power services considering system security. IEEE Trans Power Syst 23:137–149
22. Rabiee A et al (2009) Coupled energy and reactive power market clearing considering power system security. Int J Energy Convers Manage 50:907–915
23. Amjady N et al (2009) A stochastic framework for clearing of reactive power market. Energy 1–7
24. Ozturk UA et al (2004) A solution to the stochastic unit commitment problem using chance constrained programming. IEEE Trans Power Syst 19:1589–1598
25. Amjady N et al (2009) Stochastic multiobjective market clearing of joint energy and reserves auctions ensuring power system security. IEEE Trans Power Syst 25(4):1841–1854
26. Zhou X et al (2009) Multi-function bi-directional battery charger for plug-in hybrid electric vehicle application. IEEE Energy Convers Congr Exposition 3930–3936
27. Amjady N et al (2010) Pay-as-bid based reactive power market. Energy Convers Manag 376–381
28. Billinton R, Allan RN (1984) Reliability evaluation of power systems. Pitman Books, New York
29. O. v. o. Nationaal Wetenschappelijke Instituut voor Verkeersveiligheidsonderzoek (SWOV) (2007) http://www.swov.nl.
30. Li T et al (2007) Risk-constrained bidding strategy with stochastic unit commitment. IEEE Trans Power Syst 22:449–458
31. Aghaei J et al (2008) Incorporating power system security into market-clearing of day-ahead joint energy and reserves auctions. European Trans Electrical Power
32. Antonis GT, Hatziargyriou ND (2008) Centralized Control for Optimizing Microgrids Operation. IEEE Trans Energy Convers 23

33. Papathanassiou S et al (2005) A benchmark LV microgrid for steady state and transient analysis. In: CIGRE Symp. Power Syst. Dispersed Generation. Athens, Greece, Apr. 17–20
34. Generalized Algebraic Modeling Systems (GAMS). http://www.gams.com
35. Manigrasso R et al (2008) Inverter loss minimization for a plug-in hybrid veihcle traction drive using DSC control. In: International symposium on power electronics, electrical drives, automation and motion. pp 889–894