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Research article

Adaptive fuzzy funnel congestion control for TCP/AQM network<sup>☆</sup>Kun Wang<sup>a</sup>, Yang Liu<sup>a</sup>, Xiaoping Liu<sup>b</sup>, Yuanwei Jing<sup>a,\*</sup>, Siying Zhang<sup>a</sup><sup>a</sup> College of Information Science and Engineering, Northeastern University, Shenyang, Liaoning 110004, PR China<sup>b</sup> Faculty of Engineering, Lakehead University, Thunder Bay, ON P7B 5E1, Canada

## HIGHLIGHTS

- A new generalized AIMD network model with input saturation and external disturbances is proposed by combining the two existing models for Transmission Control Protocol/Active Queue Management (TCP/AQM) network.
- A novel congestion control method is presented to solve a tracking problem by using funnel control, adaptive backstepping, and fuzzy logic systems.
- The steady and transient state performances of the tracking error can satisfy the design requirements.

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## ABSTRACT

In this paper, a performance constraint control problem is considered for Transmission Control Protocol/Active Queue Management (TCP/AQM) network with external disturbance and input saturation. On the basis of backstepping-like design procedure and fuzzy approximation technique, an adaptive fuzzy controller with prescribed constraint is achieved to ensure that the transient and steady state performances of the tracking errors can be satisfied. The stability analysis proves that all the signals in the closed-loop system are semi-globally, uniformly and ultimately bounded. The simulation results clarify the feasibility and effectiveness of the proposed approach.

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## 1. Introduction

In recent years, the congestion control of transmission control protocol (TCP) network has become an important research topic. TCP congestion control can prevent network collapse, avoid lock-out behavior and effectively reduce the probability of control-loop synchronization. Meanwhile, the active queue management (AQM) mechanism implemented in the router nodes has significant development, for instance, it can improve network utilization, reduce packet drops and keep the best-effort service with low-delay [1]. Therefore, the combination of AQM and congestion mechanism of TCP was regarded as an effective approach to deal with network congestion problems. During the past decades, there are many AQM schemes that have been proposed. The first AQM scheme was called random early detection (RED) [2] where the probability of dropping packets was calculated by the average queue length. In what follows, adaptive RED [3], nonlinear RED [4] and other algorithms such as, BLUE [5], BLACK [6], and YELLOW

[7] were proposed. While, the most of the above-mentioned results were heuristic, which made the parameters tuning very difficult. Due to the establishment of the fluid model [8,9] for congestion control process in TCP networks, control-theory-based approaches can be used to solve the network congestion problem. Based on these models, several control techniques, such as, PI [10], PD [11], PID [12], robust control [13] and fuzzy logic systems [14], have been designed to deal with network congestion issues. Besides, except for the above schemes, there still exist a class of AQM algorithms called deterministic optimization approach (DOA), such as adaptive virtual queue (AVQ) [15], stabilized virtual buffer (SVB) [16] and random early marking (REM) [17]. The network can be driven to an optimal operating point with the aid of DOA.

In the practical industrial process, the system state or output may be restricted due to the limitations of the system itself or the extraneous interference [18–20]. For example, the dynamics of robots should be limited to avoid certain possible damages for the human or robot itself based on [19]. For a crane system [20], its displacement  $S$  is usually required to keep a region, such as  $|S| < \chi$ . Therefore, the constraint control problem is paid attention by numerous scholars. One of the main approaches was called funnel control (FC) which was first proposed in 2002 [21]. In the following, the funnel control was extensively applied to various

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nonlinear systems [22–24]. Moreover, the FC was combined with dynamic surface control (DSC) to tackle the adaptive tracking control problem for a kind of nonlinear dynamic systems in [25] where a novel funnel variable was first introduced. In order to avoid the non-differentiable problem, a new constraint variable was proposed in [26], which can guarantee that the prescribed transient and steady state behaviors of the error were satisfied. Authors in [27] focused on the adaptive control problem for nonlinear systems with unmodeled dynamics by FC. The tracking control problem of uncertain multi-input multi-output (MIMO) systems was investigated in [28] where a new low-complexity controller was designed. Authors in [29] extended FC to multi-agent systems with communication limitations to achieve the consensus for each agents. Besides, this technique has also been applied to some practical systems, such as exothermic chemical reactor models [30], air-breathing hypersonic vehicles [31], rigid revolute joint robotic manipulators [32], nonlinear coupled (rigid) robotic systems [33], nonlinear servomechanisms [34], however, there has no report on TCP/AQM system until now.

In real TCP/AQM networks, the uncertainty is inevitable owing to [35–38], however, it is well known that the fuzzy logic systems and neural networks are the efficient tools to address the uncertain items of nonlinear systems [39–41]. Meanwhile, authors in [42] proposed two adaptive fuzzy output feedback control approaches for a class of uncertain stochastic nonlinear strict-feedback systems. Fuzzy logic systems were used to model unknown nonlinear systems, and a nonlinear fuzzy state observer was designed to estimate the unmeasured states. A robust adaptive fuzzy control approach was developed in [43] for a class of MIMO nonlinear systems with modeling uncertainties and external disturbances by using both the approximation property of the fuzzy logic systems and the backstepping technique. In [44], the complex issue of backstepping was solved by virtue of command filtered method, meanwhile, the command filtering technique can overcome the shortcoming of the dynamic surface control. Moreover, just one adaptive parameter was required to tune, which reduced computation burden. Furthermore, the fuzzy control methods have been applied to TCP/AQM systems to eliminate congestion situation. An AQM algorithm was proposed in [45] to overcome the drawbacks of conventional PID and TCP congestion control mechanism was regarded as the input-rate. By considering asynchronous grades of membership between the fuzzy controller and the T-S fuzzy systems, authors in [46] proposed a new T-S fuzzy control scheme instead of the traditional control approach to improve the TCP network performance. In [47], a perturbed T-S fuzzy model was built to represent the transmission network system and the congestion controller was obtained by linear matrix inequality (LMI). So far, to the best of the authors' knowledge, there is no results on the network congestion control with adaptive fuzzy control, funnel control theory and backstepping technique.

Although there are some results on the controller design by using the adaptive funnel control. However, whether the funnel control can be applied to TCP/AQM network successfully or not is a challenge because the environment of Internet is complex. Thus, this paper investigates an adaptive fuzzy output feedback control design with funnel control for TCP/AQM network nonlinear systems. Compared to the existing literature, the main contributions of this paper are given as follows

(1) Inspired by the above discussions, a new generalized AIMD network model with input saturation and external disturbances is proposed by combining the models used in [8,9,48,49] of the revision. The introduced model can be used to describe the real network more widely.

(2) This paper is the first to investigate congestion case appeared in TCP/AQM network with the help of fuzzy logic systems

and funnel control. By virtue of FC, the transient and steady state performances of the tracking error can be guaranteed, and all the signals in the closed-loop system are semi-globally uniformly and ultimately bounded.

The remainders of this paper are structured as follows. A network model and certain necessary definitions and preliminaries are presented in Section 2. The controller design and main results are formulated in Section 3. Section 4 gives a simulation study to verify the effectiveness of the proposed method. Finally, a conclusion is provided in Section 5.

## 2. System model and preliminaries

### 2.1. TCP/AQM system model

It follows from [49] that the following TCP/AQM network is considered in this paper.

$$\begin{cases} \frac{dW(t)}{dt} = \frac{\mu}{R(t)} - \frac{2(1-\eta)}{1+\eta} W(t) \frac{W(t)}{R(t)} p(t) + w(t) \\ \frac{dq(t)}{dt} = \begin{cases} \frac{N(t)W(t)}{R(t)} - C(t) + w(t), & q(t) > 0 \\ \max \left\{ \frac{N(t)W(t)}{R(t)} - C(t) + w(t), 0 \right\}, & q(t) = 0 \end{cases} \end{cases} \quad (1)$$

where  $W \in [1, W_{\max}]$ ,  $q \in [0, q_{\max}]$  and  $R(t)$  denote the average size of window (packets), the average of queue length and the round-trip time, respectively;  $N(t)$ ,  $C(t)$  and  $T_p$  represent the number of TCP sessions, the queue capacity (packets/s) and the deterministic delay;  $p(t)$  is the dropped/marked probability of a packet and  $w(t)$  is unresponsive flow, which is regarded as the external disturbance. In addition, other parameters of the system (1) satisfy  $\eta \in [0, 1]$ ,  $p \in [0, 1]$  and  $R(t) = q(t)/C(t) + T_p(s)$ .

**Remark 1.** Compared with the model in [9], the introduced model (1) can be used to describe the real network more widely because the system (1) is the same with TCP model in [9] if the parameters  $\mu$  and  $\eta$  are set to 1 and 0.6.

Similar to [36], the rate  $r(t)$  is adopted in this work by

$$r(t) = \frac{W(t)}{R} \quad (2)$$

which implies that (1) can be transformed to (3) with (2).

$$\begin{cases} \dot{r}(t) = \frac{\mu N(t)}{R^2(t)} - \left[ \frac{\mu N(t)}{R^2(t)} + \frac{2(1-\eta)r^2(t)}{(1+\eta)N(t)} \right] p(t) + w(t) \\ \dot{q}(t) = r(t) - C + w(t) \end{cases} \quad (3)$$

To apply backstepping technique, let  $x_1 = q(t) - q_d$ ,  $x_2 = r(t)$  and  $u(t) = p(t)$  with  $q_d$  being the desired queue length, then the system (3) will be written to (4).

$$\begin{cases} \dot{x}_1 = x_2 - C + w(t) \\ \dot{x}_2 = f(x) + g(x) \text{sat}(v(t)) + w(t) \\ y = x_1 \end{cases} \quad (4)$$

where,

$$f(x) = \frac{\mu N}{R^2}, \quad g(x) = -f(x) - \frac{2(1-\eta)(x_2(t) + C)^2}{(1+\eta)N},$$

$$u(t) = \text{sat}(v(t)) = \begin{cases} 1, & v(t) \geq 1 \\ v(t), & 0 \leq v(t) \leq 1 \\ 0, & v(t) \leq 0. \end{cases}$$

Due to the relationship between the applied control  $u(t)$  and the control input  $v(t)$  has a sharp corner when  $v(t) = 0$  or

$v(t) = 1$ . Thus, backstepping technique cannot be directly applied. According to [50], the saturation can be approximated by a smooth function defined as

$$h(v) = \frac{1}{2} \tanh(v(t) - \frac{1}{2}) + \frac{1}{2} \quad (5)$$

Then, the saturation function can be rewritten as:

$$\text{sat}(v) = h(v) + p(v) = \frac{1}{2} \tanh(v(t) - \frac{1}{2}) + \frac{1}{2} + p(v) \quad (6)$$

where  $p(v) = \text{sat}(v) - h(v)$  is a bounded function in time and its bound can be obtained as

$$|p(v)| = |\text{sat}(v) - h(v)| = \frac{1}{2} \left( 1 - \tanh \frac{1}{2} \right) = A_1 \quad (7)$$

Note that when  $v(t)$  changes from 0 to 1, the bound  $p(v)$  increases from 0 to  $A_1$  and when  $v(t)$  is outside this range, the bound  $p(v)$  decreases from  $A_1$  to 0.

**Remark 2.** It is worth noting that the parameters  $N, C, T_p$  can be regarded as constants in a relative long time due to [9–13].

The control objectives of this paper are described as follows: (1) the tracking error  $x_1 = q(t) - q_d$  satisfies the prescribed performance; (2) all the signals of the closed-loop system are bounded.

### 2.2. Preliminaries

In this subsection, some basic knowledge are first given. According to [22–27], a funnel boundary function  $F_\psi(t)$  is defined as follows:

$$F_\psi(t) = (p_0 - p_\infty)e^{-\beta t} + p_\infty \quad (8)$$

where  $p_0$  is the initial value,  $\beta$  is the convergence rate,  $p_0, p_\infty, \beta$  are positive design parameters.

To achieve the control objectives, an output error transformation is introduced as

$$\xi_1 = \frac{x_1^2}{F_\psi^2 - x_1^2} \quad (9)$$

The time-derivative of  $\xi_1$  can be calculated by

$$\begin{aligned} \dot{\xi}_1 &= \frac{1}{(F_\psi^2 - x_1^2)^2} [2x_1\dot{x}_1(F_\psi^2 - x_1^2) - x_1^2(2F_\psi\dot{F}_\psi - 2x_1\dot{x}_1)] \\ &= \frac{2}{(F_\psi^2 - x_1^2)^2} (x_1\dot{x}_1F_\psi^2 - x_1^2F_\psi\dot{F}_\psi) \\ &= \frac{2F_\psi^2x_1}{(F_\psi^2 - x_1^2)^2} \left( \dot{x}_1 - \frac{x_1\dot{F}_\psi}{F_\psi} \right) \\ &= 2\Gamma_1 \left( x_2 - C + w(t) - \frac{x_1\dot{F}_\psi}{F_\psi} \right) \end{aligned} \quad (10)$$

$$\text{with } \Gamma_1 = \frac{F_\psi^2x_1}{(F_\psi^2 - x_1^2)^2}.$$

**Lemma 1** ([26]). If  $f(Z)$  is defined as a continuous function on a compact set  $\Omega_Z$ , then there exists a fuzzy logic system  $y(Z) = W^T S(Z)$ , for a desired level of  $\varepsilon > 0$ , such that

$$\sup_{x \in \Omega_Z} |f(Z) - W^T S(Z)| \leq \varepsilon \quad (11)$$

where  $W = [w_1, w_2, \dots, w_n]^T$  is the ideal constant weight vector,  $S(Z)$  is the basis function vector, which is expressed as follows:

$$S(Z) = \frac{[s_1(Z), s_2(Z), \dots, s_N(Z)]^T}{\sum_{i=1}^N s_i(Z)} \quad (12)$$

where  $N$  is the number of fuzzy rules and  $s_i(Z)$  are selected as Gaussian functions, that is

$$s_i(Z) = \exp \left[ \frac{-(Z - \varsigma_i)^T (Z - \varsigma_i)}{\eta_i^2} \right], i = 1, 2, \dots, n \quad (13)$$

where  $\varsigma_i = [\varsigma_{i1}, \varsigma_{i2}, \dots, \varsigma_{in}]^T$  is the center vector and  $\eta_i$  is the width of Gaussian function.

### 3. Main result

#### 3.1. Adaptive fuzzy controller design

In this section, the main result will be shown by using backstepping technique and fuzzy control. Firstly, the following transformation of coordinates is defined by

$$\begin{cases} z_1 = \xi_1 \\ z_2 = x_2 - \alpha_1 \end{cases} \quad (14)$$

with  $\alpha_0 = q_d$ . By differentiating (14), together with (4) and (8), it yields

$$\begin{cases} \dot{z}_1 = 2\Gamma_1(x_2 - C + w(t) - (x_1\dot{F}_\psi/F_\psi)) \\ \dot{z}_2 = f(x) + g(x)v + w(t) - \dot{\alpha}_1 \end{cases} \quad (15)$$

**Step 1:** Choose the following Lyapunov function candidate

$$V_1 = \frac{1}{4}z_1^2 + \frac{1}{2\gamma_1}\tilde{\theta}_1^2 \quad (16)$$

where,  $\tilde{\theta}_1 = \theta_1 - \theta_1^*$ ,  $\theta_1^*$  is the estimation of  $\theta_1$  with  $\theta_1 = \|W_1\|^2$  and  $\gamma_1$  is a positive design parameter.

The time-derivative of  $V_1$  is given by

$$\dot{V}_1 = \frac{1}{2}z_1\dot{z}_1 - \frac{1}{\gamma_1}\tilde{\theta}_1\dot{\theta}_1^* = \xi_1(\bar{f}_1 + \Gamma_1x_2) - \frac{1}{\gamma_1}\tilde{\theta}_1\dot{\theta}_1^* \quad (17)$$

$$\text{with } \bar{f}_1 = -\Gamma_1C + \Gamma_1w(t) - \Gamma_1\frac{x_1\dot{F}_\psi}{F_\psi}.$$

Due to  $z_2 = x_2 - \alpha_1$ ,  $\dot{V}_1$  can be calculated to

$$\dot{V}_1 = z_1(\bar{f}_1 + \Gamma_1\alpha_1) + z_1\Gamma_1z_2 - \frac{1}{\gamma_1}\tilde{\theta}_1\dot{\theta}_1^* \quad (18)$$

In this paper, the fuzzy logic system can be employed to approximate the unknown function  $\bar{f}_1$  owing to Lemma 1, that is

$$\bar{f}_1 = W_1^T S_1(Z_1) + \delta_1(Z_1), |\delta_1(Z_1)| \leq \lambda_1 \quad (19)$$

where  $\delta_1(Z_1)$  is the approximation error and  $\lambda_1 > 0$ .

As a result, the following inequality can be derived with the help of Young's inequality.

$$\begin{aligned} z_1\bar{f}_1 &= z_1W_1^T S_1(Z_1) + z_1\delta_1(Z_1) \\ &\leq \frac{1}{2a_1^2}z_1^2 \|W_1\|^2 S_1^T(Z_1)S_1(Z_1) + \frac{1}{2}a_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}\lambda_1^2 \\ &= \frac{1}{2a_1^2}z_1^2\theta_1 S_1^T(Z_1)S_1(Z_1) + \frac{1}{2}a_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}\lambda_1^2 \end{aligned} \quad (20)$$

where  $\theta_1 = \|W_1\|^2$ ,  $\alpha_1$  is a positive design parameter.

The first virtual controller can be designed as

$$\alpha_1 = -\frac{x_1(F_\psi^2 - x_1^2)}{F_\psi^2} \left( k_1 + \frac{1}{2} + \frac{1}{2a_1^2}\theta_1 S_1^T(Z_1)S_1(Z_1) \right) \quad (21)$$

where  $k_1$  is the design parameter.

Substituting (19) and (20) into (18), we can get

$$\begin{aligned} \dot{V}_1 &\leq \frac{1}{2a_1^2} z_1^2 \theta_1^T S_1^T(Z_1) S_1(Z_1) + \frac{1}{2} a_1^2 + \frac{1}{2} z_1^2 + \frac{1}{2} \lambda_1^2 \\ &\quad - z_1^2 \left( k_1 + \frac{1}{2} + \frac{1}{2a_1^2} \theta_1^T S_1^T(Z_1) S_1(Z_1) \right) + z_1 \Gamma_1 z_2 - \frac{1}{\gamma_1} \tilde{\theta}_1 \dot{\theta}_1^* \\ &= -k_1 z_1^2 + \frac{1}{2} (a_1^2 + \lambda_1^2) + \frac{1}{2a_1^2} z_1^2 \tilde{\theta}_1^T S_1^T(Z_1) S_1(Z_1) \\ &\quad - \frac{1}{\gamma_1} \tilde{\theta}_1 \dot{\theta}_1^* + z_1 \Gamma_1 z_2 \end{aligned} \quad (22)$$

Therefore, the adaptive law can be defined by

$$\dot{\theta}_1^* = \frac{\gamma_1}{2a_1^2} z_1^2 S_1^T(Z_1) S_1(Z_1) - \sigma_1 \theta_1^* \quad (23)$$

where  $\sigma_1$  is the design parameter.

Substituting (23) into (22) gives

$$\begin{aligned} \dot{V}_1 &\leq -k_1 z_1^2 + \frac{1}{2} (a_1^2 + \lambda_1^2) + \frac{1}{2a_1^2} z_1^2 \tilde{\theta}_1^T S_1^T(Z_1) S_1(Z_1) \\ &\quad - \frac{1}{\gamma_1} \tilde{\theta}_1 \left( \frac{\gamma_1}{2a_1^2} z_1^2 S_1^T(Z_1) S_1(Z_1) - \sigma_1 \theta_1^* \right) + z_1 \Gamma_1 z_2 \\ &= -k_1 z_1^2 + \frac{1}{2} (a_1^2 + \lambda_1^2) + \frac{\sigma_1}{\gamma_1} \tilde{\theta}_1 \theta_1^* + z_1 \Gamma_1 z_2 \end{aligned} \quad (24)$$

**Step 2:** With  $\theta_2 = \|W_2\|^2$  and  $\theta_2^*$  being the estimation of  $\theta_2$ , the Lyapunov function  $V_2$  can be selected by

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2\gamma_2} \tilde{\theta}_2^2 \quad (25)$$

where  $\tilde{\theta}_2 = \theta_2 - \theta_2^*$  and  $\gamma_2$  is the designed parameter.

Then the time-derivative of  $V_2$  can be calculated as

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2 \dot{z}_2 - \frac{1}{\gamma_2} \tilde{\theta}_2 \dot{\theta}_2^* \\ &= \dot{V}_1 + z_2 (\dot{x}_2 - \dot{\alpha}_1) - \frac{1}{\gamma_2} \tilde{\theta}_2 \dot{\theta}_2^* \\ &= \dot{V}_1 + z_2 (f(x) + g(x)v(t) + w(t) - \dot{\alpha}_1) - \frac{1}{\gamma_2} \tilde{\theta}_2 \dot{\theta}_2^* \end{aligned} \quad (26)$$

Substituting (24) into (26) produces

$$\begin{aligned} \dot{V}_2 &\leq -k_1 z_1^2 + \frac{1}{2} (a_1^2 + \lambda_1^2) + \frac{\sigma_1}{\gamma_1} \tilde{\theta}_1 \theta_1^* + z_1 \Gamma_1 z_2 + z_2 (f(x) \\ &\quad + g(x)v(t) + w(t) - \dot{\alpha}_1) - \frac{1}{\gamma_2} \tilde{\theta}_2 \dot{\theta}_2^* \\ &= -k_1 z_1^2 + \frac{1}{2} (a_1^2 + \lambda_1^2) + z_2 (\bar{f}_2 + g(x)v(t)) \\ &\quad - \frac{1}{\gamma_2} \tilde{\theta}_2 \dot{\theta}_2^* + \frac{\sigma_1}{\gamma_1} \tilde{\theta}_1 \theta_1^* \end{aligned} \quad (27)$$

with  $\bar{f}_2 = z_1 \Gamma_1 + f(x) + w(t) - \dot{\alpha}_1$ .

Similar to Step 1,  $\bar{f}_2$  can be approximated by the fuzzy logic system  $W_2^T S_2(Z_2)$ . Then, for any given  $\lambda_2 > 0$ ,

$$\bar{f}_2 = W_2^T S_2(Z_2) + \delta_2(Z_2), |\delta_2(Z_2)| \leq \lambda_2 \quad (28)$$

According to Young's inequality, the following inequality can be derived.

$$\begin{aligned} z_2 \bar{f}_2 &= z_2 W_2^T S_2(Z_2) + z_2 \delta_2(Z_2) \\ &\leq \frac{1}{2a_2^2} z_2^2 \theta_2 S_2^T(Z_2) S_2(Z_2) + \frac{1}{2} a_2^2 + \frac{1}{2} z_2^2 + \frac{1}{2} \lambda_2^2 \end{aligned} \quad (29)$$

where  $\theta_2 = \|W_2\|^2$ ,  $\alpha_2$  is positive design parameter.

The second adaptive law is given as follows:

$$\dot{\theta}_2^* = \frac{\gamma_2}{2a_2^2} z_2^2 S_2^T(Z_2) S_2(Z_2) - \sigma_2 \theta_2^* \quad (30)$$

where  $\sigma_2$  is the designed parameter.

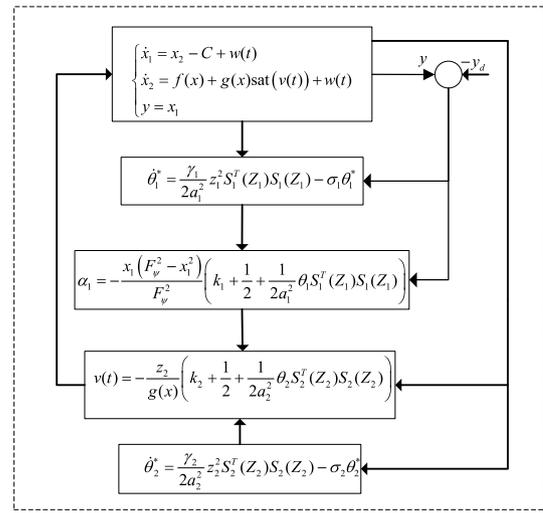


Fig. 1. The block diagram of control system.

Substituting (29) and (30) into (27) gives

$$\begin{aligned} \dot{V}_2 &\leq -k_1 z_1^2 + \frac{1}{2} (a_1^2 + \lambda_1^2) + z_2 (\bar{f}_2 + g(x)v(t)) \\ &\quad - \frac{1}{\gamma_2} \tilde{\theta}_2 \dot{\theta}_2^* + \frac{\sigma_1}{\gamma_1} \tilde{\theta}_1 \theta_1^* \\ &= -k_1 z_1^2 + \frac{1}{2} (a_1^2 + \lambda_1^2) + \frac{\sigma_1}{\gamma_1} \tilde{\theta}_1 \theta_1^* + \frac{1}{2a_2^2} z_2^2 \theta_2^T S_2^T(Z_2) S_2(Z_2) \\ &\quad + \frac{1}{2} a_2^2 + \frac{1}{2} z_2^2 + \frac{1}{2} \lambda_2^2 + z_2 g(x)v(t) + \frac{\sigma_2}{\gamma_2} \tilde{\theta}_2 \theta_2^* \end{aligned} \quad (31)$$

So far, the real control input  $v$  is achieved by

$$v(t) = -\frac{z_2}{g(x)} \left( k_2 + \frac{1}{2} + \frac{1}{2a_2^2} \theta_2 S_2^T(Z_2) S_2(Z_2) \right) \quad (32)$$

where  $k_2$  is a positive design parameter.

The block diagram is shown the design procedure of the controller in Fig. 1.

### 3.2. Stability analysis

In this subsection, the stability analysis of the closed-loop system is carried out by using Lyapunov theory.

**Theorem 1.** If the initial condition of tracking error  $x_1(t)$  satisfies  $x_1(t) < F_\psi(0)$ , then the following properties hold:

- (1) All the signals in the closed loop system are semi-globally, uniformly and ultimately bounded.
- (2) The output tracking error  $x_1(t)$  satisfies the prescribed transient and steady state performances.

**Proof.** Substituting (32) into (31), it produces

$$\dot{V}_2 \leq -k_1 z_1^2 - k_2 z_2^2 + \frac{1}{2} \sum_{i=1}^2 (a_i^2 + \lambda_i^2) + \sum_{i=1}^2 \frac{\sigma_i}{\gamma_i} \tilde{\theta}_i \theta_i^* \quad (33)$$

It can be easily deduced from Young's inequality that

$$\tilde{\theta}_i \theta_i^* \leq \tilde{\theta}_i (\theta_i - \tilde{\theta}_i) \leq -\frac{1}{2} \tilde{\theta}_i^2 + \frac{1}{2} \theta_i^2 \quad (34)$$

Substituting (34) into (33) produces

$$\dot{V}_2 \leq -k_1 z_1^2 - k_2 z_2^2 - \sum_{i=1}^2 \frac{\sigma_i}{2\gamma_i} \tilde{\theta}_i^2 + \vartheta \quad (35)$$

with  $\vartheta = \frac{1}{2} \sum_{i=1}^2 (a_i^2 + \lambda_i^2 + \frac{\sigma_i}{\gamma_i} \theta_i^2)$ . Select the design parameters as:

$$k_1 = \frac{1}{4} \sigma_1, k_2 = \frac{1}{2} \sigma_2, \bar{k} = \min[\sigma_1, \sigma_2] \quad (36)$$

Therefore, (35) becomes

$$\dot{V}_2(t) \leq -\bar{k}V_2 + \vartheta \quad (37)$$

Solving inequality (37) gives

$$0 \leq V_2(t) \leq \frac{\vartheta}{\bar{k}} + (V_2(0) - \frac{\vartheta}{\bar{k}})e^{-\bar{k}t} \leq \frac{\vartheta}{\bar{k}} + V_2(0)e^{-\bar{k}t} \quad (38)$$

which implies that  $V_2(t)$  is bounded. Therefore, all the signals in the closed loop system are semi-globally, uniformly and ultimately bounded.

From (16), (25) and (38), we can get

$$\frac{1}{4}z_1^2 = \frac{1}{4} \left( \frac{x_1^2}{F_\psi^2 - x_1^2} \right)^2 \leq V_1(t) \leq V_2(t) \leq \frac{\vartheta}{\bar{k}} + V_2(0)e^{-\bar{k}t} \quad (39)$$

Due to  $\vartheta > 0, \bar{k} > 0$  and  $0 < e^{-\bar{k}t} \leq 1$ , it gives

$$V_2(0)e^{-\bar{k}t} \leq V_2(0) \quad (40)$$

It follows from (39) and (40) that one has

$$\frac{x_1^4}{(F_\psi^2 - x_1^2)^2} \leq \frac{4\vartheta}{\bar{k}} + 4V_2(0) \quad (41)$$

Then, we can get

$$\left[ 1 - 4 \left( V_2(0) + \frac{\vartheta}{\bar{k}} \right) \right] x_1^4 \leq 4 \left( V_2(0) + \frac{\vartheta}{\bar{k}} \right) (F_\psi^4 - 2F_\psi^2 x_1^2) \quad (42)$$

By choosing the suitable initial conditions and control design parameters, the following result can be reached.

$$1 - 4 \left( V_2(0) + \frac{\vartheta}{\bar{k}} \right) \geq 0 \quad (43)$$

which means that

$$\vartheta \leq \bar{k} \left( \frac{1}{4} - V_2(0) \right) \quad (44)$$

Further, we can obtain

$$4 \left( V_2(0) + \frac{\vartheta}{\bar{k}} \right) (F_\psi^4 - 2F_\psi^2 x_1^2) \geq 0 \quad (45)$$

which indicates that

$$F_\psi^4 - 2F_\psi^2 x_1^2 \geq 0 \quad (46)$$

As a consequence,  $F_\psi^2 \geq 2x_1^2$  holds, which implies that

$$|x_1| \leq \frac{1}{\sqrt{2}} |F_\psi| < |F_\psi| \quad (47)$$

Therefore, the output tracking errors can converge to an arbitrarily small region and satisfy the pre-set performances.

In order to make the system have the better performance, the parameter tuning guideline is given in Remark 3.

**Remark 3.** The tuning parameters in this paper include  $\bar{k}, \gamma_i, a_i$  and  $\lambda_i$ . For ensuring the tracking error to be smaller,  $\bar{k}$  and  $\gamma_i$  should be tuned to be a large value as well as  $a_i$  and  $\lambda_i$  should be modified to be a small one, which is inspired by [51]. However, if the gain  $\bar{k}$  is very large, which leads to a larger control input. As a result, in the real application, a tradeoff between the better tracking performance and control action should be considered.

**Remark 4.** According to (47), it can be easily seen that the transient and steady-state performances of the signal  $x_1$  is satisfied by adjusting the corresponding parameters of the function  $F_\psi$ .

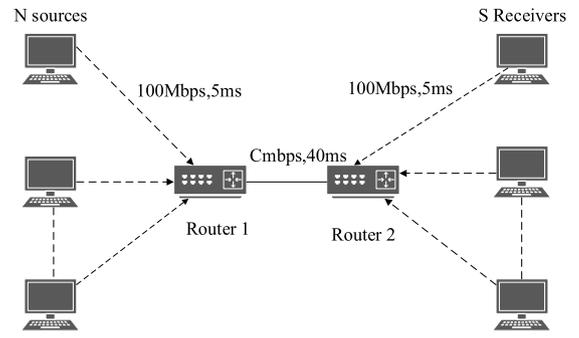


Fig. 2. Structure of computer network.

#### 4. Simulation results

In this section, the proposed approach is verified by using Matlab to simulate a typical dumbbell topology, which is shown in Fig. 1. From Fig. 1, it can be observed that the bottleneck link locates between Router 1 and Router 2, and N sources send data flows to their respective receivers through the bottleneck.

Simulation is carried out by choosing the following parameters.

$$C = 1750 \text{ packets/s}, R = 100 \text{ ms}, q_d = 100 \text{ packets}$$

$$k_1 = 0.5, k_2 = 0.25, a_1 = 1, a_2 = 1, \gamma_1 = 10,$$

$$\gamma_2 = 5, \sigma_1 = 2, \sigma_2 = 0.5,$$

Simulation results are collected in Figs. 3–8. It follows from Fig. 3 that the queue length  $q(t)$  converges to 100, which illustrates that it tracks the desired queue  $q_d$ . Moreover, the trajectory of the rate is shown in Fig. 4, where the trajectory of the signal  $r(t)$  satisfies the target (1) of Theorem 1. Besides, Figs. 5–6 demonstrate the curves of the adaptive laws, from which it can be observed that the adaptive laws are semi-globally, uniformly and ultimately bounded. The control law  $u$  is illustrated in Fig. 7. It can be seen from Fig. 7 that the trajectory is always between 0 and 0.4. To verify the superiority of the proposed scheme, the comparison result between the presented method and PPC [39] on the tracking error is shown in Fig. 8. It can be observed that the tracking error of the proposed method has a faster convergent rate and smaller jitter than that in [39]. To sum up, all the signals in the closed-loop system are semi-globally, uniformly and ultimately bounded based on the above analysis.

**Remark 5.** It follows from the above discussions that the error converges to the preset zone faster than [39]. In addition, it can be seen from Fig. 8 that the pre-given performances with respect to the error is satisfied. Based on the curves of Figs. 1–8, it is a fact that the proposed method is effective.

#### 5. Conclusion

In this paper, an adaptive congestion control scheme has been designed for TCP/AQM nonlinear systems by combining backstepping technique, funnel control and fuzzy logic system. A sufficient condition with respect to the boundedness of the closed-loop system is given, by which it can be verified that all the signals in the closed-loop system are semi-globally, uniformly and ultimately bounded. Besides, by comparing with the existing congestion control methods, a better tracking performance has been achieved. In the future, the proposed algorithm can be applied to solve the congestion control problem for multiple-bottleneck network model.

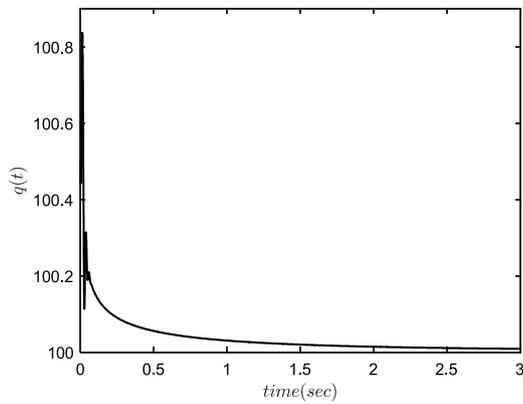


Fig. 3. The queue length  $q(t)$ .

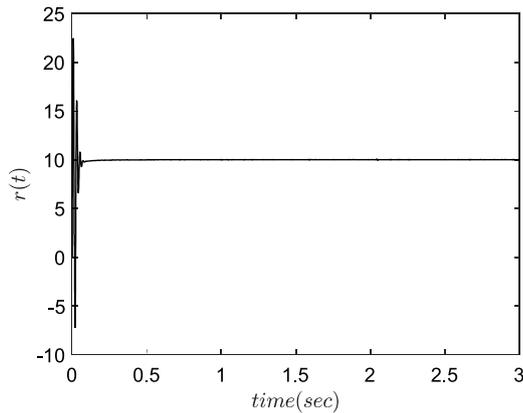


Fig. 4. The sending rate  $r(t)$ .

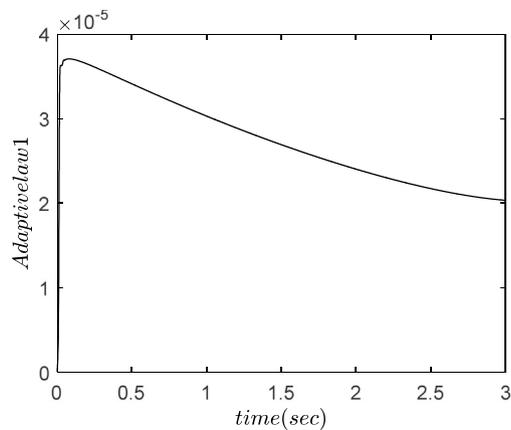


Fig. 5. The trajectory of adaptive law1.

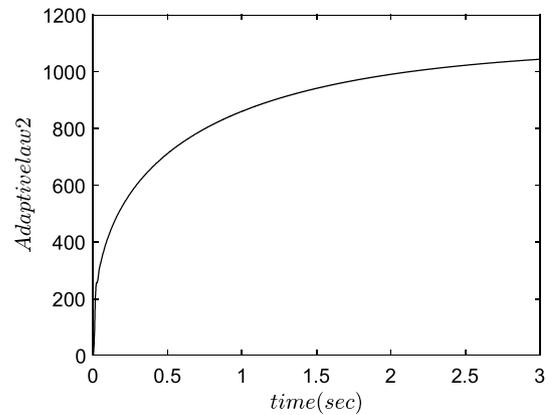


Fig. 6. The trajectory of adaptive law2.

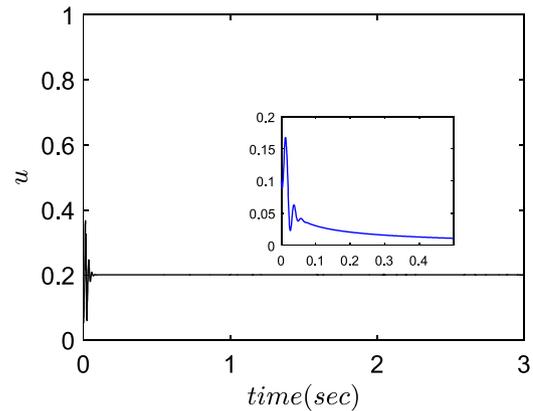


Fig. 7. The control input  $u$ .

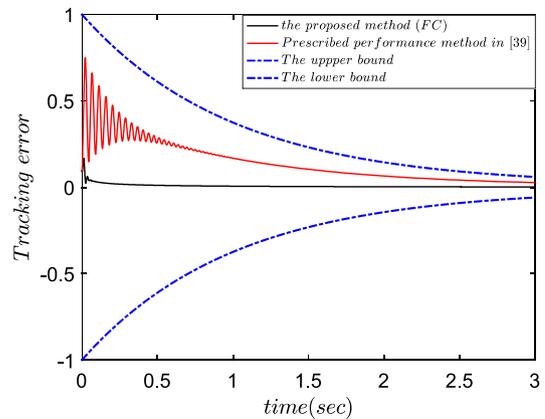


Fig. 8. The comparison result with tracking error.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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