2016 12th World Congress on Intelligent Control and Automation (WCICA)
June 12-15, 2016, Guilin, China

Nonlinear Model Predictive Controller Design for Air System Control of a Gasoline Engine

You Li, Yunfeng Hu, Shuwen Wang, Hong Chen

Abstract— **In this paper, a control strategy based on nonlinear model predictive control (NMPC) for air system of gasoline engine is proposed in order to meet the engine torque demand. At first, according to mean value thought and modular modeling methodology, a control-oriented model is developed. Secondly, the desired engine torque is translated into desired intake manifold pressure by look-up table. And an NMPC controller is designed to achieve the tracking of intake manifold pressure with regulating the throttle opening, so that the output torque can attain the desired values. Ultimately, to verify the effectiveness and reliability of the designed control strategy, not only the mean value model but also the commercial high-precision engine model is applied and the performance of NMPC controller is shown by the simulation results.**

I. INTRODUCTION

WITH the rapid development of the automotive industry, the problems of environmental pollution and energy shortage are becoming severer, leading to increasingly rigorous emission regulations and fuel consumption limits, and thus the higher requirements for engine control have been put forward [1]. The customers demand that the engine should be capable of favorable power performance which results in vehicle's good drivability. Thereby torque-based engine control system has become an inevitable trend of modern engine control [2][3]. The torque demand for vehicle operating is highly considered and expected to be met. Air system control is one of the most important control issues to achieve this control target, and it determines some critical engine outputs that affect the engine dynamics and produced power.

Accurate mathematical model is the basis of air system control. Generally, researchers adopt mean value modeling [4] and Hybrid modeling method [5]. The mean value modeling can describe the engine dynamic characteristics while it don't need too much complex calculations, which has been the most commonly used for controller design. Many applications of mean value model to engine control have arisen, e.g. [6][7][8], of which a component based modeling methodology has been utilized in paper [7] and it is proved to facilitate the combination and update of submodels. Considering these advantages, the mean value modeling and component based thought are applied in this paper.

The air system control on engine has become one of the most attractive directions of research in automotive control field. A control strategy that includes a Parallel Distributed Compensation Fuzzy controller regulating the mass flow rates and intake manifold pressure to follow the reference is proposed [9]. The feedback linearization technique has been applied to air and charging model-based multivariable control and the optimization of engine emissions and improvement of transient performance can be achieved [10]. A control approach for the airpath of a SI Engine with VVT in [11] is able to manage the air-fuel ratio and engine output torque through motion planning of air system.

In this study, the driver's desire is focused on, and we attempt to realize the engine torque demand. As we know, the gasoline engine is regarded as a quite nonlinear and coupling system. Actuator is physically saturated and the engine should operate in a safe state, which lead to the imperative constraints on system input and output. Based on the above considerations, a model-based control method, the nonlinear model predictive control (NMPC), can be a favorable candidate. A mean-value model is utilized to predict the future behavior of engine in MPC, and this model of controlled plant with certain specific constraints can be formulated in this case by nonlinear form [12]. Through solving the receding horizon optimal control problem obtained by the control aims and demands, NMPC tries to compute an optimal control input, and meanwhile it can address actuators and state constraints well.

This paper is summarized as follows: a control oriented nonlinear model of a gasoline engine is developed in Section II. In Section III A control strategy to satisfy the torque demand is presented and the design of NMPC controller is introduced. The validation of the designed controller with two simulation models is reported in Section IV. In Section V, the conclusion of this paper is summed up.

II. GASOLINE ENGINE SYSTEM MODELING

A. Model Description

To facilitate the design of the controller, it is necessary to establish a relatively effective mathematical model. In this paper, the engine air system model can describe the whole process of the air flowing from ambient to cylinders and reveal relationships between variables using mathematical expressions. Initially the air goes through throttle in the inlet path, and the throttle opening can directly change the amount of mass flow. Then the fresh air passes intake manifold

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This work is supported by National Natural Science Foundation (NNSF) of China(No.61374046; 6151001021) and Jilin Provincial Education Department Project(No.2015-485).

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978-1-4673-8414-8/16/$31.00 ©2016 IEEE

and ultimately flows into cylinders, and the mass flow into cylinders can be governed by the engine speed and manifold pressure. The diagram of a gasoline engine model is demonstrated in Fig. 1. Based on the structure of this system and the thought of mean value, modular modeling methodology has been applied in the whole modeling process. And thus the gasoline air system model consists of throttle, intake manifold and cylinder three submodels [8].

B. System Modeling

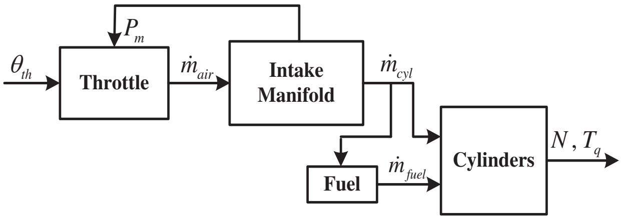

Fig. 1. The schematic of a gasoline engine

1) Throttle Air Flow Model: The function of this model is to describe the mass flow through the throttle. According to the standard orifice model for compressible gas flow, the condition on each side of valve and the throttle opening can determine the air flow status. Therefore the expression of this model is given as follow:

$$\dot{m}_{air} = \frac{P_{atm}}{P_m}\sqrt{\frac{T_{ref}}{T_{atm}}}A_e(\theta_{th})\psi(\frac{P_m}{P_{atm}}) \tag{1}$$

where

$$\psi(\frac{P_m}{P_{atm}}) = \begin{cases} \sqrt{\frac{(\frac{P_m}{P_{atm}})^{\frac{2}{k}} - (\frac{P_m}{P_{atm}})^{\frac{k+1}{k}}}{(\frac{2}{k+1})^{\frac{2}{k-1}} - (\frac{2}{k+1})^{\frac{k+1}{k-1}}}} & \frac{P_m}{P_{atm}} > \gamma, \\ 1, & \frac{P_m}{P_{atm}} \leq \gamma, \end{cases} \tag{2}$$

$$\gamma = (\frac{2}{k+1})^{\frac{k}{k-1}}$$

where P_m represents the intake manifold pressure; P_{atm} is the ambient temperature. This product-term indicates the effect of the ratio of upstream pressure to downstream pressure on the mass flow through throttle. T_{atm} is the ambient temperature.

The A_e is a throttle air mass flow characteristic coefficient, and it presents the effective opening area of the valve and is calculated as a function with respect to valve's angle:

$$A_e(\theta_{th}) = \frac{\dot{m}_{air,max}}{2} \cdot (1 - \cos(2\theta_{th})) \tag{3}$$

This term is on the basis of the maximum mass flow rate $\dot{m}_{air,max}$; θ_{th} is the throttle opening in degrees.

2) Dynamics of the Intake Manifold: This model presents the the pressure dynamics of intake manifold. It is on the basis of ideal gas law and conservation of mass. The air mass flowing into and out of the manifold volume are represented by $\dot{m}_{air}$ and $\dot{m}_{cyl}$ respectively and the air mass flow $\dot{m}_a$ in manifold can be obtained:

$$\dot{m}_a = \dot{m}_{air} - \dot{m}_{cyl}. \tag{4}$$

According to the ideal gas state equation $PV = mRT$, we have:

$$m_a = \frac{P_m V_m}{RT_m}, \tag{5}$$

After the derivation of above equation and the change of temperature in manifold can be ignored, the dynamic of the manifold pressure is given by the following expression:

$$\dot{P}_m = \frac{RT_m}{V_m}(\dot{m}_{air} - \dot{m}_{cyl}). \tag{6}$$

Where R denotes the ideal gas constant. T_m is the intake manifold temperature. And V_m denotes the intake manifold volume.

3) Cylinders Air Mass Flow: The following equation determines the characteristic of air mass flow filling into the cylinders. Here the engine is regarded as a volumetric pump. And the typical calculation formula of the aspirated air mass derived from the empirical equation can be written as follows:

$$\begin{aligned} \dot{m}_{cyl} &= \rho_m \cdot \dot{V} \\ &= \rho_m \cdot \eta_{vol}(P_m, N) \cdot V_{disp} \cdot \frac{N}{120} \\ &= \frac{P_m}{RT_m} \cdot \eta_{vol}(P_m, N) \cdot V_{disp} \cdot \frac{N}{120}. \end{aligned} \tag{7}$$

where ρ_m is the density of charging air. N denotes the engine speed and V_{disp} represents the engine displacement. η_{vol} is the volumetric efficiency, and it is obtained by MAP form. Therefore the air filling into cylinders is largely determined by the intake manifold pressure as well as engine speed.

To simplify fuel injection model, it is assumed that the air-fuel ratio has been at theoretically ideal value, and the fuel injected flow can be defined by: $\dot{m}_{fuel} = \frac{1}{\lambda_s}\dot{m}_{cyl}$. Here λ_s is equal to 14.7.

What's more, the engine produced torque is modeled by a look-up table, as a function of the intake manifold pressure P_m, engine speed N and fuel flow $\dot{m}_{fuel}$. And it is given as follows:

$$T_q = \Phi_{T_q}(\dot{m}_{fuel}, N, P_m) \tag{8}$$

C. Model Validation

For the sake of verification of the established control-oriented model, a commercial engine simulation software enDYNA® has been utilized to evaluate the accuracy of this simplified engine system. The same input operating signals, namely throttle opening and engine speed are fed into the above simplified model and enDYNA model at the same time, which is shown in Fig. 2. After that, three dominant output signals of the new established model are compared with that of enDYNA model respectively under the same working conditions. The validation results are given in Fig. 3.

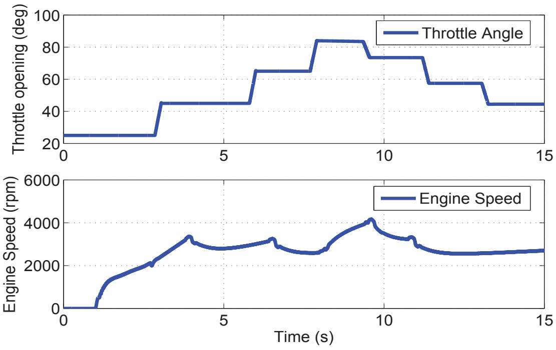

Fig. 2. The input signals: Throttle opening and Engine speed

The comparison of the mass flow through the throttle is illustrated in the top figure of Fig. 3. The contrastive profile of the intake manifold pressure is given in the medial one and the calibration of the air mass flow into cylinders is shown as the bottom figure. The output curves of the new established model and enDYNA model are represented by dashed line and solid line respectively.

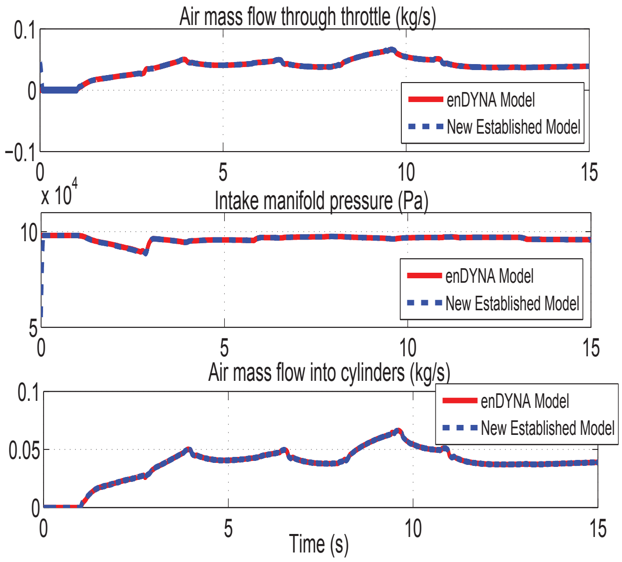

Fig. 3. The contrastive profiles of air mass flow passing the throttle, intake manifold pressure and air mass flow into cylinders respectively

It is easily visible that the outputs of the above new established model can keep close to the enDYNA model' curves, and both results always have the same variation tendency. The difference between them remains within a small region. Therefore this new established model has a good precision and it can fully reflect the dynamic performance of the actual gasoline engine. Besides this model can provide a feasible model for the design of controller.

III. CONTROL SYSTEM DESIGN

A. System State Expression

From the modeling process mentioned before, a control-oriented air system model can be obtained. By considering the equation (1) to (7), a state-space nonlinear model is summarized as the following form:

$$\dot{P}_m = \sigma\alpha f(P_m)(1 - cos(2\theta_{th})) - \sigma\beta g(P_m). \tag{9}$$

where $\alpha = \frac{\dot{m}_{air,max}}{2} \cdot \sqrt{\frac{T_{ref}}{T_{atm}}} \cdot \frac{P_{atm}}{P_{ref}}$, $\beta = \frac{V_{disp}}{120RT_m}$, $\sigma = \frac{RT_m}{V_m}$. $f(P_m)$ represents the product factor $\psi(\frac{P_m}{P_{atm}})$ in equation (1). $g(P_m) = P_m \cdot N \cdot \eta_{vol}(P_m, N)$.

According to the model equation, we choose the intake manifold pressure P_m as system state, and the throttle opening θ_{th} as control input. The intake manifold pressure P_m can be measured by sensors, and this variable directly reflects the air flow into cylinders, thereby revealing the engine torque demand indirectly. Thus it can be considered as the system output as well. And we have:

$$x = [P_m]^T, \; u = [\theta_{th}]^T, \; y = [P_m]^T.$$

Based on the continuous state-apace equation (9), Eulers method is utilized for discretization, and the discretized air system model can be defined for controller design:

$$P_m(k+1) = P_m(k) + h(P_m(k), \theta_{th}(k)) \cdot \Delta t = F(P_m(k), \theta_{th}(k)) \tag{10}$$

where Δt is the sample time. Now the NMPC algorithm can be applied.

B. Control Strategy

It should be known that the drivers demand for vehicle power and ride comfort has a crucial influence on the amount of engine torque. Hence the control objective is to attain the torque demand, namely to enable the produced torque to reach the desired value. To achieve this goal, the desired torque T_q^{sp} needs to be transformed into the desired intake manifold pressure P_m^{sp} by two steps. In order to reach the expected pressure profile, nonlinear model predictive control is adopted to find a optimal throttle opening so as to regulate the engine outputs. And the proposed control strategy for air system of gasoline engine based on NMPC is shown as Fig. 4.

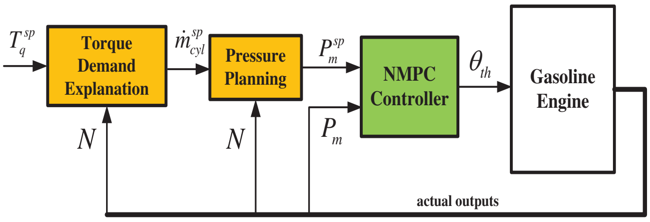

Fig. 4. The control strategy of the gasoline engine air system

1) Torque Demand Explanation: At the first step, air flow charging into cylinders is regarded as a function of the produced torque and engine speed, which can be written as following formula and also represented by a map shown in Fig. 5:

$$\dot{m}_{cyl}^{sp} = f_{map}(T_q^{sp}, N) \tag{11}$$

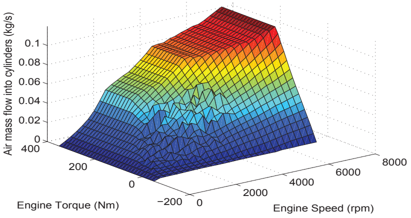

Fig. 5. The map of Torque Demand Explanation

By the above method, the desired engine torque T_q^{sp} can be transformed into the desired air flow aspirated into cylinder $\dot{m}_{cyl}^{sp}$ through looking up table.

2) Pressure Planning: At the second step, an inverse formula that is derived based on equation (7) can be obtained, which is described as follows:

$$P_m^{sp} = \Psi^{-1}(\dot{m}_{cyl}^{sp}, N) = \Psi^{-1}(f_{map}(T_q^{sp}, N), N) \tag{12}$$

Hence the the desired air mass aspirated into cylinder $\dot{m}_{cyl}^{sp}$ can be converted into the desired intake manifold pressure P_m^{sp} in this way.

C. NMPC Controller Design

On the principle of basic thought of the model predictive control, at the current time instant k the dynamic behavior of system in the time domain $[k, k+1]$ can be predicted. In this paper, prediction horizon is chosen as N_p, and control horizon is defined as N_u. The relation $N_u \leq N_p$ for the both horizons should be satisfied. After the time instant N_u, the control input remains unchanged: $u(k + N_u - 1) = u(k + Nu) = \cdots = u(k + N_p - 1)$. Hence the prediction output function at discrete time instant k is written by equation (13):

$$Y_{N_p}(k+1|k) = \begin{bmatrix} y_c(k+1|k) \\ y_c(k+2|k) \\ \vdots \\ y_c(k+N_p|k) \end{bmatrix} \tag{13}$$

Meanwhile, optimal sequence of control input $U(k)$ computed by the controller at instant k is defined as follows:

$$U(k) = \begin{bmatrix} \theta_{th}(k|k) \\ \theta_{th}(k+1|k) \\ \vdots \\ \theta_{th}(k+N_u-1|k) \end{bmatrix} \tag{14}$$

Based on the basic principles and related theories of the model predictive control and derived from the equation (10), the prediction function of system output variable for N_p time steps can be deduced at instant k, which is determined as following equations:

$$\begin{aligned} y_c(k+1|k) &= g_c(P_m(k+1)) \\ &= F(P_m(k), \theta_{th}(k)) \\ y_c(k+2|k) &= g_c(P_m(k+2)) \\ &= F(P_m(k+1), \theta_{th}(k+1)) \\ &= F(F(P_m(k), \theta_{th}(k)), \theta_{th}(k+1)) \\ &\vdots \\ y_c(k+N_p|k) &= g_c(P_m(k), \theta_{th}(k), \theta_{th}(k+1), \cdots, \\ &\qquad \theta_{th}(k+N_u-1)) \end{aligned} \tag{15}$$

Considering the conversion of desired engine torque mentioned before, the desired intake manifold pressure P_m^{sp} as reference profile is derived, and this sequence of system reference output has been defined by:

$$R(k+1) = \begin{bmatrix} P_m^{sp}(k+1) \\ P_m^{sp}(k+2) \\ \vdots \\ P_m^{sp}(k+N_p) \end{bmatrix} \tag{16}$$

In order to ensure the safety of the engine system, it is necessary to confine the intake manifold pressure in a safe range. Besides, due to the physical saturation of the actuator, the throttle opening are bounded within the range of 0° $\sim$ 90°. Taking the above factors into account, the following constraints must be considered for controller design:

$$\begin{gathered} P_{m,min} \leq P_m(k) \leq P_{m,max} \\ 0\,\mathrm{deg} \leq \theta_{th}(k) \leq 90\,\mathrm{deg} \end{gathered} \tag{17}$$

There are two targets that the control system should achieve. One is to enable the output intake manifold pressure to track the desired value, so that the torque demand can be met. The other aim is to ensure that the variation in throttle action is as small as possible, and thus the reaction of actuator gets more close to the actual response characteristics, which can reduce the throttle mechanical loss as well. The two control targets should be described as an optimization problem, that is, to minimize the tracking error of pressure and the difference between two control actions. As a result, the following quadratic performance index $J(Y_{N_p}(k), U(k), N_u, N_p)$ is obtained as the cost function, which is required to be minimized:

$$\begin{aligned} &\min_{U(k)} J(Y_{N_p}(k), U(k), N_u, N_p) \\ &J = \left\|\Gamma_y(Y_{N_p}(k+1|k) - R(k+1))\right\|^2 + \|\Gamma_u \Delta U(k)\|^2 \\ &\textbf{s.t.} \qquad P_{m,min} \leq P_m(k) \leq P_{m,max} \\ &\qquad\qquad 0\,\mathrm{deg} \leq \theta_{th}(k) \leq 90\,\mathrm{deg} \end{aligned} \tag{18}$$

where $\Delta U(k) = U(k+1) - U(k)$.

The term $\left\|\Gamma_y(Y_{N_p}(k+1|k) - R(k+1))\right\|^2$ in cost function indicates the deviation of actual manifold pressure from the desired pressure. And $\|\Gamma_u \Delta U(k)\|^2$ represents the manipulation of throttle opening. Γ_y and Γ_u denote the weighting factor of system output and control input sequence respectively. Γ_y can affect the tracking performance and Γ_u can limit the fluctuation of throttle action. Actually, there exists a contradiction in minimizing both two weighting factors. Thus the compromise between them needs to be taken into consideration in parameters adjustment.

The optimal problem as formulated as equation (18) can be solved through MATLAB NAG toolbox, and optimal sequence of control input is available. Then the first element $\theta_{th}(k)$ of this control input sequence $U(k)$ acts on the engine system as shown in equation (19). The next instant, control horizon will be pushed forward one step, and the prediction optimization process is repeated beginning from the new state $k+1$. By this process, the closed-loop control of engine air system can be achieved.

$$u(k) = [1 \ 0 \ \cdots \ 0]\, U(k) \tag{19}$$

IV. CONTROL PERFORMANCES SIMULATION

A. *Mean Value Model Experiment*

The designed control strategy of air system based on NMPC algorithm will be verified by the established mean value model in Matlab/Simulink. To evaluate the performance of the NMPC controller, the simulation experiment is carried out under the operating condition that engine speed varies, and the variable engine speed is given as Fig. 6. The control input, that is throttle opening, is illustrated in Fig. 7. The tracking profile of intake manifold pressure is shown in Fig. 8. The comparision of expected engine torque with the actual torque is given as Fig. 9.

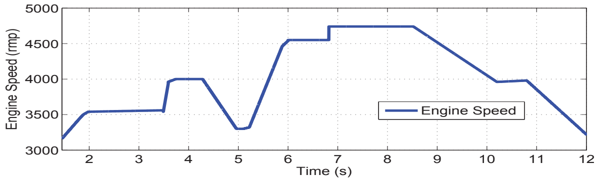

Fig. 6. The variable engine speed

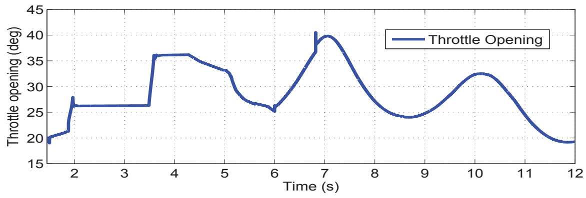

Fig. 7. The resulting throttle opening

It can be seen from the experimental results that the intake manifold pressure and torque produced by gasoline engine can keep up with the desired profiles accurately, so that the driver's torque demand can be met well. Therefore, the effectiveness of designed controller is verified preliminary.

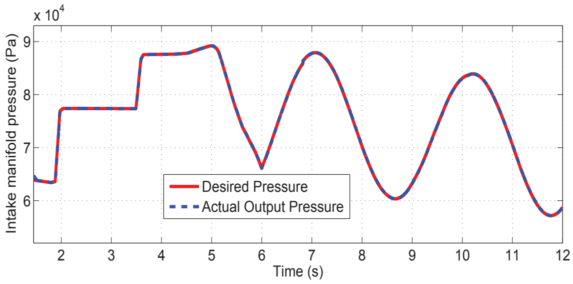

Fig. 8. The tracking curves of intake manifold pressure

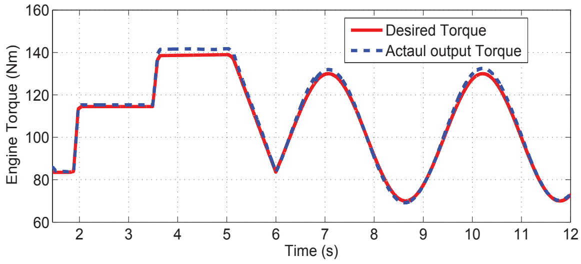

Fig. 9. The torque denmand tracking

B. *Virtual Engine Model enDYNA Experiment*

In order to further test the effectiveness of the designed controller, the above established mean value model will be substituted for high-precision engine model enDYNA. Moreover the simulation verification will be implemented in two different situations where engine speed are constant and variable respectively for comprehensive evaluation.

1) Constant Engine Speed: In this case, the engine operates at a fixed speed: $N = 3000\ rmp$. The throttle opening as output of the NMPC controller is shown in Fig. 10. The tracking curves of desired intake manifold pressure and torque are given in Fig. 11 and Fig. 12. Thereby the controller has a good control effect under the steady state operation.

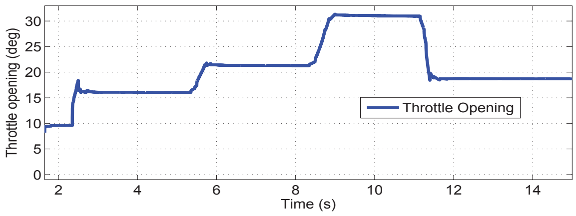

Fig. 10. The output of controller: throttle opening

2) Variable Engine Speed: To further examine the control performance, the validation of this control system will be carried out with the variable speed. The condition of engine speed is set as Fig. 13. This speed data can simulate the normal driving behaviour of the engine on a road. The variation of throttle opening is shown in Fig. 14. The tracking curves of desired intake manifold pressure is illustrated in Fig. 15. The torque denmand tracking is shown as Fig. 16.

Accordingly, the results have indicated that whether the engine speed is constant or variable, the actual intake manifold pressure can track the expected profile quickly

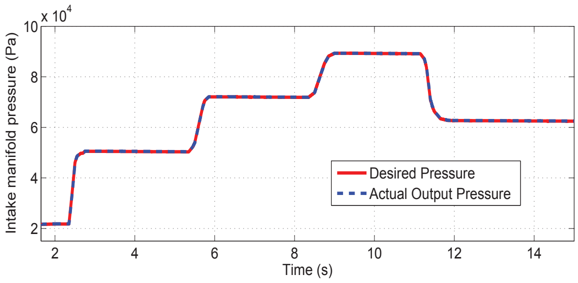

Fig. 11. The intake manifold pressure tracking in enDYNA

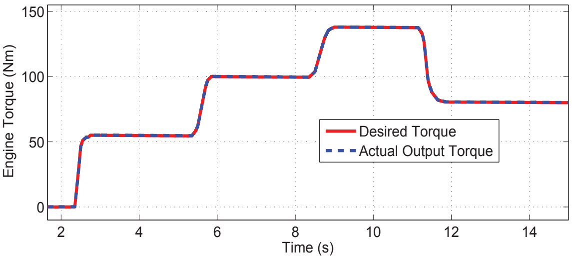

Fig. 12. The engine torque tracking in enDYNA

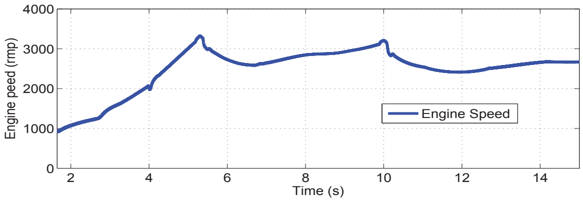

Fig. 13. The variable engine speed

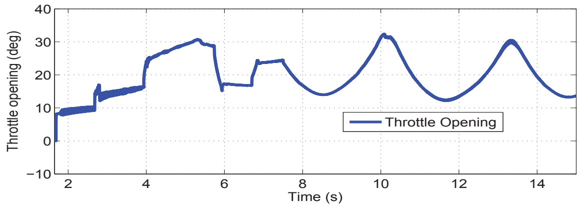

Fig. 14. The output of controller: throttle opening

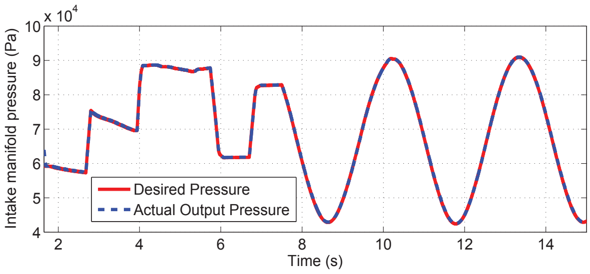

Fig. 15. The intake manifold pressure tracking in enDYNA

and accurately. In addition, the engine produced torque is nearly consistent with the given engine torque demand. The designed NMPC controller can perform good capacity for tracking under steady state or transient condition, and steady state error is quite small. The closed-loop air control system based on NMPC algorithm is able to ensure the torque demand to be met effectively.

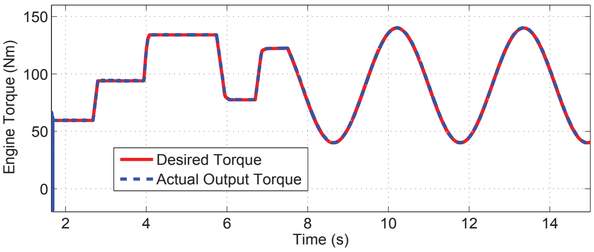

Fig. 16. The engine torque tracking in enDYNA

V. CONCLUSIONS

Firstly, for purpose of control, a mean value model of gasoline engine air system is developed by diving the whole system into three portions. After validation, this simplified model has high accuracy and is able to predict the engine dynamic state accurately. Secondly, a control strategy for air system is designed aiming to meet the torque demand. Through two-step conversions, the torque demand has been interpreted as the desired intake manifold pressure, and an NMPC controller of air system is presented to achieve the satisfactory intake manifold pressure tracking and minimum throttle action while considering constraints. Ultimately, the controller has a good control effect for not only mean value model but also enDYNA model from the simulation results.

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