Fault Estimation and Accommodation for Linear MIMO Discrete-Time Systems

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***Abstract*—In this brief, a methodology for detection and accommodation of actuator faults for a class of multi-input–multi-output (MIMO) stochastic systems is presented. First, a new real-time fault estimation module that estimates the actuator effectiveness is developed. The actuator fault diagnosis is based on the estimation of the state vector. Under some conditions, the stochastic system is transformed into two separate subsystems. One of them is not affected by actuator faults, so a reduced order Kalman filter can be used to estimate its states. The other, whose states are measurable, is affected by the faults. Then, the output of the nominal controller is reconfigured to compensate for the loss of actuator effectiveness in the system. Simulation results of a helicopter in vertical plane is presented to demonstrate the performance of the proposed fault-tolerant control scheme.**

***Index Terms*—Fault accommodation, fault estimation, flight control, linear stochastic systems, uncertainty.**

I. Introduction

CONTROL engineers are faced with increasingly complex systems where dependability considerations are sometimes more important than performance. Sensor, actuator or process (plant) failures may drastically change the system behavior, resulting in performance degradation or even instability. Thus, fault tolerance is essential for modern, highly complex control systems. Fault tolerant control (FTC) systems are needed in order to preserve or maintain the performance objectives, or if that turns out to be impossible, to assign new (achievable) objectives so as to avoid catastrophic failures [25]. In general, fault tolerance can be achieved in two ways [30]: 1) passively, using feedback control laws that are robust with respect to possible systems faults, or 2) actively, using a fault detection and isolation (FDI) and accommodation technique.

During the last decade, different approaches for dealing with this problem have been reported. Most of them belong to the following categories: pseudo-inverse [11], adaptive control systems [3], [8], [26], eigenstructure assignment [16], multiple-model methods [4], [29], H^{∞} control [27], model-matching [14], and compensation via additive input design [21]. The survey papers [2], [23] and the most recent bibliographical review [30] give the state of the art in the field of reconfiguration and FTC. Up to now, most of the existing literature treats FDI problem [1], [5], [10], [12] and FTC problem separately. There are just a few papers that provide integrated FDI and FTC schemes [4], [7], [24], [28] for fault accommodation and [9], [20] for integration of control and fault detection. In [7] and [24], fault detection and accommodation was investigated using learning methods. The main idea behind this approach is to monitor the physical system for any nonnominal behavior in its dynamics using nonlinear modeling techniques such as neural networks. Boskovic *et al.* [4] modeled the flight control effector failures. It showed that the resulting representation leads naturally to a multiple model formulation of the corresponding control problem that can be solved using a multiple model adaptive reconfigurable control approach. In [28], the estimation of the control effectiveness is formulated as an augmented state filter problem with the control effectiveness factors being modeled as the augmented bias states.

Our work belongs to the fault accommodation category. The significance of this brief is that it provides a simple and effective integrated fault estimation and accommodation scheme to handle loss of actuator effectiveness. Compared to the existing work already reported in the literature, the contribution/novelty of this brief are in the following three aspects. 1) A new method for the rapid and accurate estimation of actuator faults is proposed. 2) The controller reconfiguration scheme based on the fault estimation is simple and easy to implement. 3) Both process noise and sensor noise are considered in the fault estimation and accommodation, which makes the proposed approach practical for real systems.

This brief deals with the discrete-time actuator fault diagnosis design for the estimation of multiple and abrupt actuator failures. The original system is decomposed into two subsystems. The first subsystem is decoupled from actuator faults, so a reduced order Kalman filter can be designed under certain conditions. The second one is affected by the fault, but its states can be measured. By using the estimate of the states, the fault can be approximated from the second subsystem. The proposed fault diagnosis design is then applied in a fault-tolerant controller design which estimates the actuator faults online. Output of the nominal controller is reconfigured to compensate for the actuator faults. This fault-tolerant control approach is based on the online estimation of the actuator fault and the continuous modification to the nominal control law. The performance of the fault-tolerant controller is tested on a helicopter example.

The focus on actuator faults may appear to make the topic of this brief relatively narrow; however, in many applications (such as aerospace) it is well known that most sensor faults are handled locally via hardware redundancy and voting strategies [22], and plant faults are relatively rare. Thus, actuator faults deserve great attention for safe and reliable operation of the system.

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II. Actuator Fault Diagnosis Design: An Ideal Case

Consider the following linear stochastic system with actuator fault:

$$x(k+1) = Ax(k) + Bu(k) + Ef(k) + \omega(k) \quad (1)$$
$$y(k) = Cx(k) + \upsilon(k) = [0 \quad I]x(k) + \upsilon(k) \quad (2)$$

where $x(k) \in \mathbf{R}^n$ is the state vector, $u(k) \in \mathbf{R}^m$ is the control input vector, $y(k) \in \mathbf{R}^r$ is the measured output vector, $f(k) \in \mathbf{R}^q$ is the vector function to model the actuator fault, the process noise $\omega(k)$ and sensor noise $\upsilon(k)$ are zero mean random sequences with covariance matrices $S = S^T > 0$ and $Q = Q^T > 0$, respectively, and A, B, E, and C are real constant matrices of appropriate dimensions. The pair (A, B) is controllable, the pair (A, C) is observable. Note that the special form of the matrix C is not a restriction. If C is of full row rank, $C = [0 \ C_1], C_1$ is a $r \times r$ nonsingular matrix, then there exists a similarity transformation $x = [\begin{smallmatrix} I_{n-r} & 0 \\ 0 & C_1 \end{smallmatrix}]^{-1}\bar{x}$ that can bring the output equation into the desired form.

Remark 1: In this brief, only actuator fault estimation and accommodation are considered. In fact, since $f(k)$ in (1) could be time-varying, it can also represent component fault. As for sensor fault case, if its model is available, it can be reformulated as actuator fault. However, due to changes on both dimension and distribution of "actuator fault," the proposed diagnostic method in this brief might not be used directly.

For the purpose of estimating the actuator fault $f(k)$, the (1)–(2) are rewritten as

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} x(k) + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} u(k) + \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} f(k) + \begin{bmatrix} \omega_1(k) \\ \omega_2(k) \\ \omega_3(k) \end{bmatrix} \quad (3)$$

$$y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 0 & I_{r-q} & 0 \\ 0 & 0 & I_q \end{bmatrix} x(k) + \begin{bmatrix} \upsilon_1(k) \\ \upsilon_2(k) \end{bmatrix} \quad (4)$$

where $x_1(k) \in \mathbf{R}^{n-r}, x_2(k) \in \mathbf{R}^{r-q}, x_3(k) \in \mathbf{R}^q$. Notice that $\mathbf{E}(x_2(k)) = \mathbf{E}(y_1(k))$ and $\mathbf{E}(x_3(k)) = \mathbf{E}(y_2(k))$, where $\mathbf{E}(\cdot)$ stands for the expectation operator. Therefore, only $x_1(k)$ needs to be estimated in mean sense. At first, the following assumption is made.

Assumption 1: $\text{Rank}(CE) = q$ and E_3 is nonsingular.

Remark 2: Since C is of full row rank, the first part in Assumption 1 means that the effects of the actuator faults are independent. With regard to the second part in Assumption 1, if the output matrix is already of the form $[0, \ I]$, we can rearrange the matrix $[\begin{smallmatrix} E_2 \\ E_3 \end{smallmatrix}]$ such that the latter q rows become nonsingular. It is acknowledged that such a structural assumption may be restrictive in a small number of practical systems.

Define

$$\bar{A}_1 \triangleq A_1 - E_1E_3^{-1}A_3 \quad (5)$$
$$\bar{A}_2 \triangleq A_2 - E_2E_3^{-1}A_3 \quad (6)$$
$$\bar{B}_1 \triangleq B_1 - E_1E_3^{-1}B_3 \quad (7)$$
$$\bar{B}_2 \triangleq B_2 - E_2E_3^{-1}B_3 \quad (8)$$
$$\bar{A}_1 \triangleq [\bar{A}_{11} \quad \bar{A}_{12} \quad \bar{A}_{13}] \quad (9)$$
$$\bar{A}_2 \triangleq [\bar{A}_{21} \quad \bar{A}_{22} \quad \bar{A}_{23}] \quad (10)$$

with

$$\bar{A}_{11} \in \mathbf{R}^{(n-r)\times(n-r)} \quad \bar{A}_{12} \in \mathbf{R}^{(n-r)\times(r-q)} \quad \bar{A}_{13} \in \mathbf{R}^{(n-r)\times q}$$
$$\bar{A}_{21} \in \mathbf{R}^{(r-q)\times(n-r)} \quad \bar{A}_{22} \in \mathbf{R}^{(r-q)\times(r-q)} \quad \bar{A}_{23} \in \mathbf{R}^{(r-q)\times q}.$$

The following theorem presents a method to estimate the state x_1 of the system described by (3)–(4).

Theorem 1: Suppose that $(\bar{A}_{11}, \bar{A}_{21})$ is observable pair and Assumption 1 holds. Suppose further, that $\mathbf{E}[(x_1(0) - \hat{x}_1(0))(x_1(0) - \hat{x}_1(0))^T] \triangleq P(0)$ is given. Then the unbiased minimum variance estimate of $x_1(k)$ is given by

$$\hat{x}_1(k) = \bar{A}_{11}\hat{x}_1(k-1) + \rho(k) + K(k)(\lambda(k) - \bar{A}_{21}\hat{x}_1(k-1)) \quad (11)$$

where $\rho(k), \lambda(k)$ are as defined as follows:

$$\rho(k) \triangleq \bar{A}_{12}y_1(k-1) + \bar{A}_{13}y_2(k-1) + E_1E_3^{-1}y_2(k) + \bar{B}_1u(k-1) \quad (12)$$
$$\lambda(k) \triangleq y_1(k) - E_2E_3^{-1}y_2(k) - \bar{A}_{22}y_1(k-1) - \bar{A}_{23}y_2(k-1) - \bar{B}_2u(k-1) \quad (13)$$

and $K(k)$ is a Kalman filter gain defined as

$$K(k) = \bar{A}_{11}P(k)\bar{A}_{21}^T\left[\bar{A}_{21}P(k)\bar{A}_{21}^T + \bar{S}\right]^{-1} \quad (14)$$

where the error covariance matrix $P(k) \triangleq \mathbf{E}[(x_1(k) - \hat{x}_1(k))(x_1(k) - \hat{x}_1(k))^T]$ is updated by

$$P(k+1) = \bar{A}_{11}P(k)\bar{A}_{11}^T + \bar{Q} - K(k)\left[\bar{A}_{21}P(k)\bar{A}_{21}^T + \bar{S}\right]K^T(k) \quad (15)$$

with

$$\bar{S} \triangleq T_1 \begin{bmatrix} S & 0 \\ 0 & Q \end{bmatrix} T_1^T \quad (16)$$
$$\bar{Q} \triangleq T_2 \begin{bmatrix} S & 0 \\ 0 & Q \end{bmatrix} T_2^T \quad (17)$$
$$T_1 \triangleq [I \quad 0 \quad -E_1E_3^{-1} \quad -\bar{A}_{12} \quad -\bar{A}_{13} - E_1E_3^{-1}] \quad (18)$$
$$T_2 \triangleq [0 \quad I \quad -E_2E_3^{-1} \quad I + \bar{A}_{22} \quad -\bar{A}_{23} - E_2E_3^{-1}]. \quad (19)$$

Proof: Similarly as in [13], by premultiplying

$$\begin{bmatrix} I & 0 & -E_1E_3^{-1} \\ 0 & I & -E_2E_3^{-1} \\ 0 & 0 & I \end{bmatrix} \tag{20}$$

into (3), one can obtain

$$\begin{aligned} &\begin{bmatrix} x_1(k+1) - E_1E_3^{-1}y_2(k+1) \\ y_1(k+1) - E_2E_3^{-1}y_2(k+1) \\ y_2(k+1) \end{bmatrix} \\ &= \begin{bmatrix} A_1 - E_1E_3^{-1}A_3 \\ A_2 - E_2E_3^{-1}A_3 \\ A_3 \end{bmatrix} x(k) + \begin{bmatrix} B_1 - E_1E_3^{-1}B_3 \\ B_2 - E_2E_3^{-1}B_3 \\ B_3 \end{bmatrix} u(k) \\ &+ \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} f(k) + \begin{bmatrix} \omega_1(k) - E_1E_3^{-1}\omega_3(k) \\ \omega_2(k) - E_2E_3^{-1}\omega_3(k) \\ \omega_3(k) \end{bmatrix} \\ &+ \begin{bmatrix} -E_1E_3^{-1}\upsilon_2(k+1) \\ \upsilon_1(k+1) - E_2E_3^{-1}\upsilon_2(k+1) \\ \upsilon_2(k+1) \end{bmatrix}. \end{aligned} \tag{21}$$

Using the definitions in (5)–(10), the first and second block row of (21) can be rewritten as

$$x_1(k+1) = \bar{A}_{11}x_1(k) + \rho(k+1) + \bar{\omega}(k) \tag{22}$$

$$\lambda(k+1) = \bar{A}_{21}x_1(k) + \bar{\upsilon}(k) \tag{23}$$

where $\bar{\omega}(k)$ and $\bar{\upsilon}(k)$ are linear combinations of $\omega(k)$ and $\upsilon(k)$, which are given, respectively, by

$$\begin{aligned} \bar{\omega}(k) \triangleq\ & \omega_1(k) - E_1E_3^{-1}\omega_3(k) - \bar{A}_{12}\upsilon_1(k) - \bar{A}_{13}\upsilon_2(k) \\ & - E_1E_3^{-1}\upsilon_2(k+1) \end{aligned} \tag{24}$$

$$\begin{aligned} \bar{\upsilon}(k) \triangleq\ & \omega_2(k) - E_2E_3^{-1}\omega_3(k) - \bar{A}_{22}\upsilon_1(k) + \upsilon_1(k+1) \\ & - \bar{A}_{23}\upsilon_2(k) - E_2E_3^{-1}\upsilon_2(k+1) \end{aligned} \tag{25}$$

and $\rho(k+1), \lambda(k+1)$ are as defined in (12) and (13), respectively.

A filter to estimate the state x_1 is given by (11). Denote the estimation error $\tilde{e}_1 \triangleq x_1 - \hat{x}_1$, then the estimation error dynamics is modeled as follows:

$$\tilde{e}_1(k) = (\bar{A}_{11} - K\bar{A}_{21})\tilde{e}_1(k-1) + \bar{\omega}(k-1) - K\bar{\upsilon}(k-1). \tag{26}$$

From (24)–(26), as in [5], one can conclude that the estimation error $e_1(k)$ will converge to zero in the mean sense if all the eigenvalues of the matrix $\bar{A}_0 \triangleq (\bar{A}_{11} - K\bar{A}_{21})$ are placed within the unit circle. This completes the proof of the Theorem 1. □

According to Theorem 1 and (2), the estimate of the state for the linear stochastic system as defined in (1)–(2) is given by

$$\hat{x}(k) = \begin{bmatrix} \hat{x}_1(k) \\ y(k) \end{bmatrix} \tag{27}$$

where $\hat{x}_1(k)$ is given by (11).

From Assumption 1 and (21), the actuator fault can be estimated as follows:

$$\hat{f}(k-1) = (E_3)^{-1}[y_2(k) - A_3\hat{x}(k-1) - B_3u(k-1)] \tag{28}$$

where $\hat{x}(k-1)$ is given by (27). □

Remark 3: From (28), it can be seen that the faulty actuator signal at time instant k can be estimated only after the measurements from time instant $(k+1)$ become available. This is also clear from (1)–(2): it simply means that there is a one time-step delay in the fault estimation, which would be quite acceptable for practical applications.

Remark 4: A good feature of the method proposed in this brief is that it not only enables fault detection, but also provides the shape of the fault, which will be used for fault accommodation later. As the design is based on the reduced order Kalman filter, this method is easy to implement and is able to give rapid and accurate identification in the mean sense.

III. Fault-Tolerant Controller Design

In this section, we consider fault-tolerant controller design for loss of actuator effectiveness which is known to be a very important type of fault in certain applications, particularly in aerospace-related fields. Thus, to deal with such kind of faults has attracted the interest of a large number of researchers [4], [26], [28]. Loss of actuator effectiveness can be modeled by the matrix $E = -B$ and $q = m$ such that the function $f(k) = R(k)u(k) \in \mathbf{R}^m$, i.e., the actuator failures are modeled in a multiplicative form. In this case, the matrix $R(k) \in \mathbf{R}^{m \times m}$ is a diagonal matrix with time-varying and continuous elements $r_i(k), i = 1, 2, \ldots, m$ such that $r_i(k) = 0$ represent the no-fault nominal condition and $0 < r_i(k) \leq 1$ represents the percentage degradation in each actuator input channel i at time instant k. For example, in practical flight situations, actuator failures like the loss of actuator effectiveness due to fin breakage or high angle-of-attack flight conditions might occur. As a result, the performance of the designed nominal controller might degrade or become unstable under these actuator failure conditions.

Let $\hat{R}(k)$ be the estimation of $R(k)$ as determined from the fault estimation design in (28). Suppose that $I - \hat{R}(k)$ is nonsingular, otherwise, the FTC problem has to be set in the reconfiguration frame [25]. To recover the loss of actuator effectiveness, the compensation law $u_R(k)$ is defined as follows:

$$u_R(k) = (I - \hat{R}(k))^{-1}u(k). \tag{29}$$

Using the reconfigured law of (29), the resulting closed-loop system is

$$x(k+1) = Ax(k) + B(I - R(k))(I - \hat{R}(k))^{-1}u(k) + \omega(k). \tag{30}$$

IV. Extension to Unknown Input Case

Many practical systems are subjected to modeling errors or external disturbance. In this section, the results obtained previously are extended to the model with unknown input, as in [17]. The system considered is expressed as

$$x(k+1) = Ax(k) + Bu(k) + Ef(k) + D\zeta(k) + \omega(k) \tag{31}$$

$$y(k) = Cx(k) = [0 \quad I]x(k) + \upsilon(k) \tag{32}$$

where $\zeta(k)$ denotes unknown input vector of the system.

Using the same premultiplying as (20) into (31), one can obtain

$$x_1(k+1) = \bar{A}_{11}x_1(k) + \rho(k+1) + \bar{D}_1\zeta(k) + \bar{\omega}(k) \quad (33)$$
$$\lambda(k+1) = \bar{A}_{21}x_1(k) + \bar{D}_2\zeta(k) + \bar{v}(k) \quad (34)$$

where the definitions of $x_1(k), \rho(k), \lambda(k)$ are the same as those in Section II, and $\bar{D}_1, \bar{D}_2$ are defined by

$$\bar{D}_1 \triangleq D_1 - E_1E_3^{-1}D_3$$
$$\bar{D}_2 \triangleq D_2 - E_2E_3^{-1}D_3.$$

The state of the previous system and the unknown input vector $\zeta(k)$ can be estimated by using augmented Kalman filter (AKF) approach. First, define an enlarged state vector by appending the unknown input vector $\zeta(k)$ to the state

$$\xi(k) \triangleq \begin{bmatrix} x_1(k) \\ \zeta(k) \end{bmatrix}. \quad (35)$$

The augmented system is described as follows:

$$\xi(k+1) = \begin{bmatrix} \bar{A}_{11}x_1(k) + \rho(k+1) + \bar{D}_1\zeta(k) \\ \zeta(k) \end{bmatrix} + \begin{bmatrix} \bar{\omega}(k)) \\ 0 \end{bmatrix} \quad (36)$$
$$\lambda(k+1) = \bar{A}_{21}x_1(k) + \bar{D}_2\zeta(k) + \bar{v}(k)). \quad (37)$$

Under the assumption that the previous augmented system is observable, the estimates of both x_1 and ζ are obtained by applying the standard Kalman filter to it which gives

$$\hat{\xi}(k+1) = \begin{bmatrix} \bar{A}_{11} & \bar{D}_1 \\ 0 & I \end{bmatrix}\hat{\xi}(k) + \begin{bmatrix} \rho(k+1) \\ 0 \end{bmatrix} + L(k)[\lambda(k+1) - [\bar{A}_{21}\ \bar{D}_2]\hat{\xi}(k)] \quad (38)$$

where the filter gain $L(k)$ is given by

$$L(k) = \begin{bmatrix} \bar{A}_{11} & \bar{D}_1 \\ 0 & I \end{bmatrix} P(k)[\bar{A}_{21}\ \bar{D}_2]^T \times \{[\bar{A}_{21}\quad \bar{D}_2]P(k)[\bar{A}_{21}\quad \bar{D}_2]^T + \bar{S}\}^{-1} \quad (39)$$
$$P(k+1) = \begin{bmatrix} \bar{A}_{11} & \bar{D}_1 \\ 0 & I \end{bmatrix} P(k) \begin{bmatrix} \bar{A}_{11} & \bar{D}_1 \\ 0 & I \end{bmatrix}^T + \begin{bmatrix} \bar{Q} & 0 \\ 0 & 0 \end{bmatrix} - L(k)\{[\bar{A}_{21}\quad \bar{D}_2]P(k)[\bar{A}_{21}\quad \bar{D}_2]^T + \bar{S}\}L^T(k). \quad (40)$$

Remark 5: This section extends the proposed method from Section III to deal with unknown input. If other kinds of uncertainties such as modeling errors are considered (entering the system in multiplicative form), the previous augmented system becomes nonlinear. In such cases, extended Kalman filter (EKF) should be used to estimate both the state and unknown parameters. However, as pointed out in [18], for high-order systems, the computational burden for EKF may be substantial. If the system is noise-free or with low-level noise, one can use augmented error technique in adaptive control theory [19] to achieve the estimation. □

Immediately from the model (31), the actuator fault $f(k)$ can be estimated in mean sense as follows:

$$\hat{f}(k-1) = (E_3)^{-1}[y_2(k) - A_3\hat{x}(k-1) - B_3u(k-1) - D_3\hat{\zeta}(k-1)] \quad (41)$$

where $\hat{x}$ and $\hat{\zeta}$ are given by (38).

V. Aircraft Example

In this section, the proposed approach is applied to actuator fault estimation and accommodation of a helicopter in vertical plane [15]. The discrete-time system can be described as follows:

$$x(k+1) = \begin{bmatrix} 0.9996 & 0.0003 & 0.0002 & -0.0037 \\ 0.0005 & 0.9900 & -0.0002 & -0.0406 \\ 0.0010 & 0.0037 & 1.0453 & 10.5644 \\ 0.0000 & 0.0000 & 0.0101 & 1.0524 \end{bmatrix} x(k) + \begin{bmatrix} 0.0044 & 0.0018 \\ 0.0353 & -0.0755 \\ -0.0559 & 0.0454 \\ -0.0003 & 0.0002 \end{bmatrix} u(k) + Ef(k) + \omega(k) \quad (42)$$
$$y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k) + v(k) \quad (43)$$

where the state variable vector $x(t) \in R^4$ is composed by the following:

$x_1 = u$ longitudinal velocity;
$x_2 = \omega$ vertical velocity;
$x_3 = \omega_y$ rate of pitch;
$x_4 = \theta$ pitch angle

and the component of command vector is:

u_1 general cyclic command;
u_2 longitudinal cyclic command.

The covariance matrices for sensor and process noise sequences are $Q = 0.2^2 I_{3\times 3}, S = \text{diag}\{0.1^2\ 0.1^2\ 0.01^2\ 0.01^2\}$. The loss of actuator effectiveness in u_1 and u_2 are considered. As a result, E and $f(k)$ in (42) can be described as follows:

$$E = -B$$
$$f(k) = \begin{bmatrix} r_1(k) & 0 \\ 0 & r_2(k) \end{bmatrix} u(k).$$

with $0 \le r_i(k) \le 1$ $(i = 1, 2)$ representing the percentage degradations in actuator input channel i, respectively.

It can be checked that all the assumptions in Theorem 1 are satisfied in this aircraft model. In fact, $\text{rank}(CE) = 2, \bar{A}_{11} = 1.0521, \bar{A}_{21} = 10.5091$. In the simulation, both constant and time-varying actuator faults are created as follows:

$$r_1(t) = \begin{cases} 0, & t < 4\ (\text{s}) \\ 0.4, & 4 \le t \le 10\ (\text{s}) \end{cases}$$
$$r_2(t) = \begin{cases} 0, & t < 2\ (\text{s}) \\ 0.5 + 0.3\sin(\pi t), & 2 \le t \le 10\ (\text{s}). \end{cases}$$

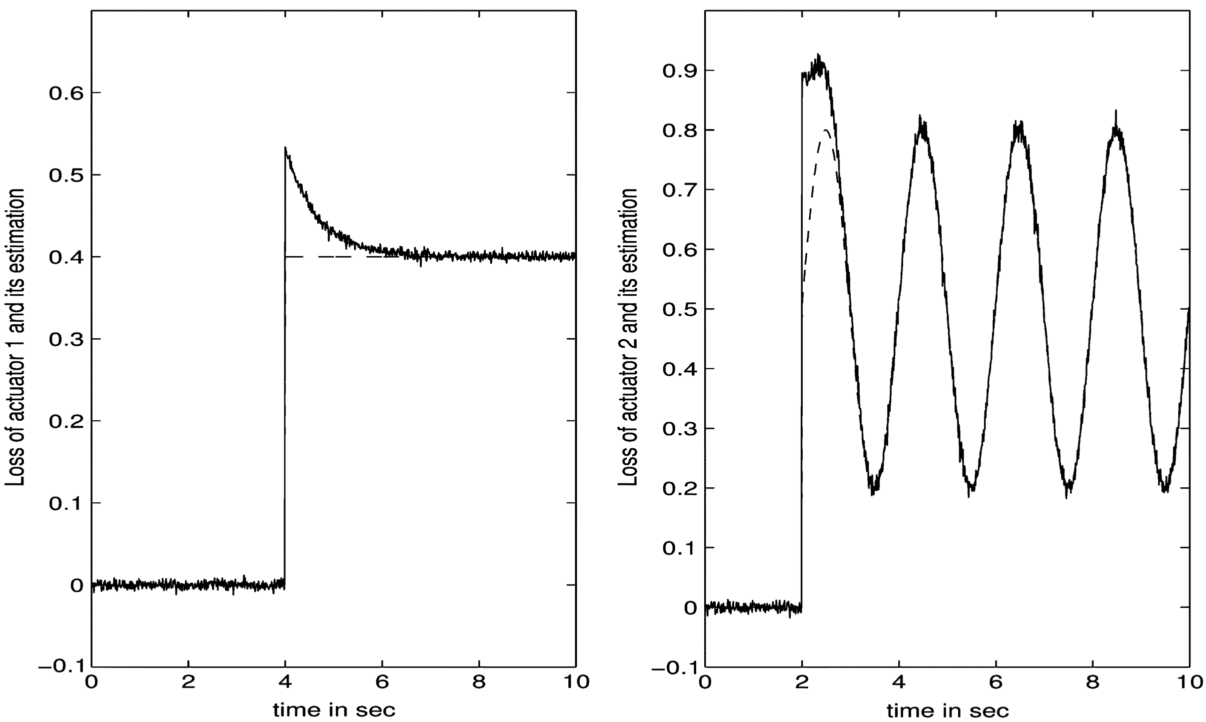

Fig. 1. Loss of actuator effectiveness (Dashed: actual. Solid: estimated).

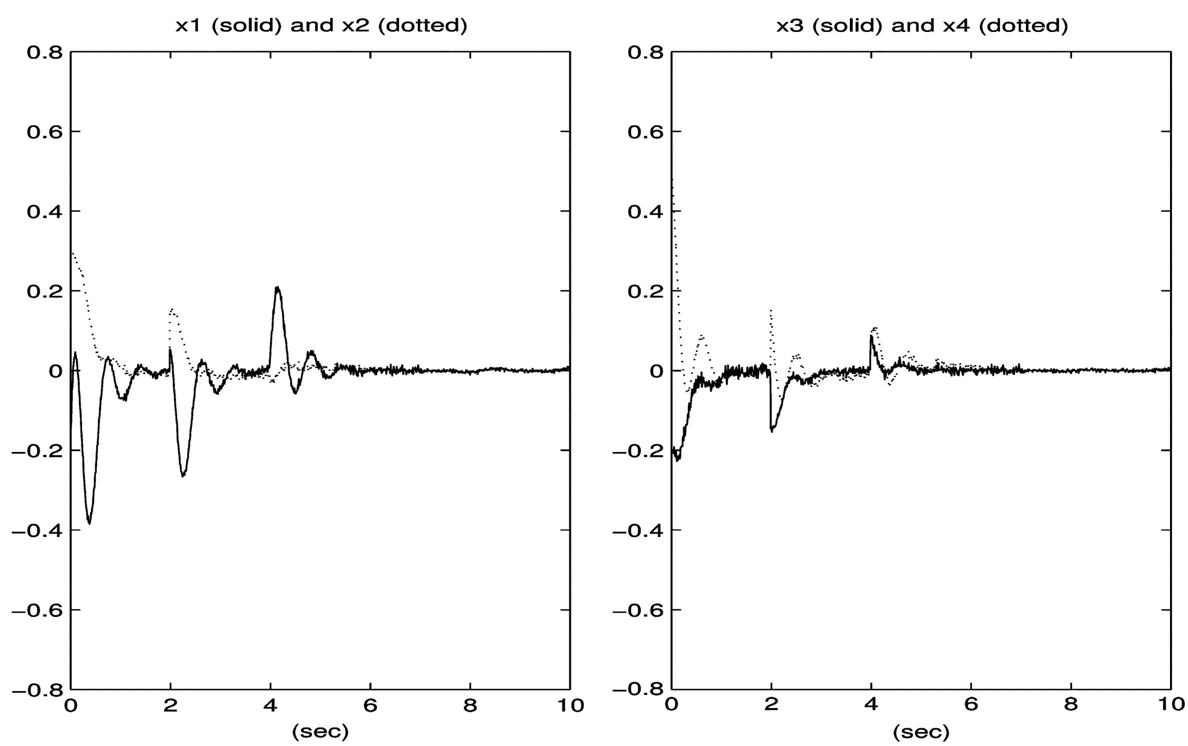

Fig. 2. States trajectories of the closed-loop system.

According to Theorem 1, the state estimation is given by (11) and (27); then the previous actuator faults are estimated by (28) and compensated by (29). The sampling interval in the simulation was taken as $T = 0.01$ s. The result of actuator fault estimation is shown in Fig. 1. It can be seen that both constant and time-varying actuator faults are estimated with satisfactory accuracy. Fig. 2 shows the time response of the closed-loop system. After the actuator faults occur, the dynamic behavior of the closed-loop system recovers quickly with the proposed fault accommodation technique.

To test the proposed method in Section IV, consider the system (42) which is subject to an unknown input. In the

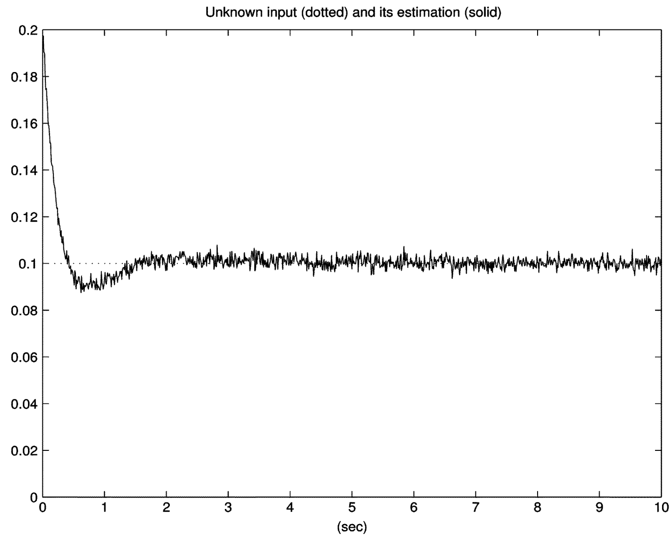

Fig. 3. Estimation of unknown input ζ.

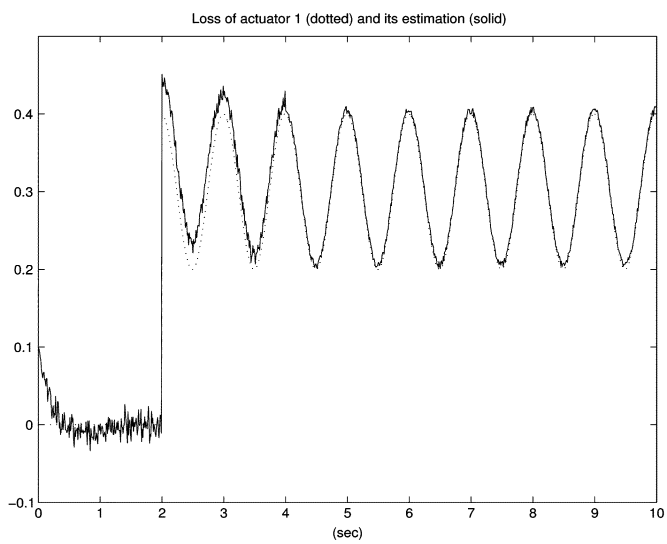

Fig. 4. Estimation of loss of actuator effectiveness r_3.

simulation, $\zeta(k) = 0.1$. The loss of actuator effectiveness in u_1 is created as follows:

$$r_3(t) = \begin{cases} 0, & t < 2 \text{ (s)} \\ 0.3 + 0.1\cos(2\pi t), & 2 \le t < 10 \text{ (s)}. \end{cases}$$

The results of estimations of actuator fault and unknown input are shown in Figs. 3 and 4, respectively. It can be seen that both of them converge to their actual values in mean sense. It is clear that the reconfigured controller design is simple (see Section III) as long as the fault is successfully estimated; therefore, and due to lack of space, we show plots of only the estimation results for the unknown input case.

VI. CONCLUSION

A new actuator fault diagnosis technique capable of estimating and compensating for actuator faults (in the presence of process noise and measurement noise) has been presented in this brief. Key advantages of this method are 1) its ability to handle multiple, simultaneous and abrupt actuator faults, and 2) rapid, accurate estimation and compensation of the faults.

The proposed fault-tolerant design scheme reconfigures the nominal controller output to compensate for actuator faults. The simulation results based on the helicopter model show good performance of the proposed technique. The diagnosis (detection/estimation) technique proposed in this brief is applicable to a large class of faults, but the compensation technique can work accurately for actuator faults only. In future work, we plan to utilize the diagnosis results for more FTC designs that can handle faults other than actuator malfunctions. Possible extensions to nonlinear systems and implementation on an experimental aircraft system will also be investigated.

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