Fuzzy-Model-Based Sliding Mode Control of Nonlinear Descriptor Systems

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***Abstract*—This paper addresses the problem of sliding mode control (SMC) for a type of uncertain time-delay nonlinear descriptor systems represented by T–S fuzzy models. One crucial contributing factor is to put forward a novel integral fuzzy switching manifold involved with time delay. Compared with previous results, the key benefit of the new manifold is that the input matrices via different subsystems are permitted to be diverse, and thus much more applicability will be achieved. By resorting to Frobenius' theorem and double orthogonal complement, the existence condition of the fuzzy manifold is presented. The admissibility conditions of sliding motion with a strictly dissipative performance are further provided. Then, the desired fuzzy SMC controller is synthesized by analyzing the reachability of the manifold. Moreover, an adaptive fuzzy SMC controller is also proposed to adapt the input saturation and the matched uncertainty with unknown upper bounds. The feasibility and virtue of our theoretical findings are demonstrated by a fuzzy SMC controller implementation for a practical system about the pendulum.**

***Index Terms*—Dissipativity, fuzzy descriptor systems, fuzzy switching manifold, sliding mode control (SMC), time delay.**

I. INTRODUCTION

DESCRIPTOR systems, also known as singular systems or semistate systems, can naturally depict a variety of practical systems, including circuits systems, power systems, and economic systems [1]. Except for the finite dynamic modes existed in state-space systems, the descriptor systems also contain impulse modes and nondynamic modes, which make their analysis more complicated [2]. Due to their tighter expressions for representing actual independent parametric perturbation, descriptor systems have been adopted to design controllers and filters in various areas of research for theoretical development as well as for real applications (see [3]–[8]). It is worth mentioning that due to the difficulties in constructing appropriate Lyapunov functions and the high complexity in analyzing the existence and uniqueness of the solution, most of the existing theoretical results are only applicable to linear descriptor models, while most practical descriptor systems are highly complicated. A feasible idea to overcome this issue is to represent a highly nonlinear descriptor system by the T–S fuzzy descriptor model, which is composed of a group of linear descriptor submodels [9]. In this case, a highly nonlinear descriptor system can be analyzed by well-established linear descriptor system theory. Moreover, it is demonstrated that the T–S fuzzy descriptor model can effectively reduce the number of linear models in comparison with fuzzy model based on state-expression [10]–[19]. In view of these significant advantages, many researchers have been devoted to research on fuzzy descriptor systems (see [20]–[25]). For instance, Lin *et al.* [20] studied a class of time-delay fuzzy descriptor systems, and stability/stabilization conditions were offered by resorting to Lyapunov function. The problem about nonfragile stabilization with guaranteed cost performance was investigated in [22] for a type of uncertain nonlinear delayed descriptor systems depicted by T–S fuzzy models, and the desired fuzzy feedback controller was designed via parallel distributed compensation (PDC) scheme. In [25], a systematic way to generalize the controller design for discrete-time T–S fuzzy descriptor models was provided by using delayed Lyapunov functions, and the well-known Finsler's lemma was used to avoid the explicit substitution of the closed-loop dynamics.

Sliding mode control (SMC) is well recognized as a famous nonlinear control strategy, which contains a variety of prominent superiorities such as fast responses and strong robustness [26]–[33]. Specially, it can compensate the matched uncertainty completely in case that the closed-loop system is driven into the sliding mode phase. During the past few decades, considerable research efforts have been devoted to SMC of descriptor systems. For instance, the problem of SMC with passivity of singular time-delay systems was studied in [34], where normal/adaptive SMC controllers were constructed to keep the system states onto the designed switching manifold within a limited time period. Ding *et al.* [35] considered the SMC problem of singular systems by designing a sliding mode controller such that the nonlinear singular

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system was exponentially stable and its trajectory can be forced into the prespecified sliding manifold in a limited time. It is noted that, however, these results are only applicable to descriptor linear models. From a practical point of view, it makes more sense to develop an appropriate SMC methodology for highly nonlinear descriptor systems, which can compensate the matched uncertainty effectively. Unfortunately, due to the difficulty in analysis, very few related results have been reported in this research area. As an alternative way, the development of SMC technique for fuzzy-model-based descriptor systems is feasible since the relatively mature theory for linear descriptor model can be used during sliding motion analysis. To date, a few results related to this topic have been reported in the literature. In [36], the fuzzy SMC design for a class of fuzzy-model-based descriptor models with state delay was investigated by resorting to an integral-type manifold function, and a suitable SMC controller was synthesized to force the states of the controlled system into the fuzzy manifold in a limited time. Based on the similar integral switching manifold, the fuzzy SMC design with dissipativity was further investigated in [37] for a type of time-delay T–S fuzzy singular systems, and a SMC law was proposed to guarantee the reachability of desired sliding motion.

It should be pointed out there is an obvious limitation to the existing results on SMC of fuzzy descriptor systems for the reason that they rely on an assumption that all linear local models share a common input matrix, which seriously restricts their practical applications. For example, the well-known inverted pendulum system cannot meet this requirement. As such, we are motivated to put forward a new fuzzy SMC strategy that is free of the restrictive requirement, which is the main purpose for this paper.

In this paper, we focus on the fuzzy SMC design for continuous-time fuzzy-model-based nonlinear descriptor systems with time-varying delay. To better adapt the model characteristics and circumvent the restrictive condition imposed on input matrices, an appropriate integral-type fuzzy switching manifold with time delay is designed, which is the crucial contribution of this paper. By resorting to Frobenius' theorem and double orthogonal complement, the existence condition of the fuzzy manifold is presented. New conditions to guarantee the asymptotic admissibility of corresponding sliding motion are further established. By designing a suitable fuzzy SMC controller, the reachability of the sliding motion is guaranteed in the presence of matched uncertainty and disturbance signal. An adaptive fuzzy SMC controller is also proposed to adapt the matched uncertainty with unknown upper bounds and input saturation. There are several important features of the developed result that are worth mentioning.

1) Input matrices via different subsystems are permitted to be diverse.
2) Matched uncertainty is fully rejected and the disturbance signal is not enlarged in sliding mode phase.
3) Input saturation is also considered since it often occurs in practical applications.

Finally, the inverted pendulum system is adopted to verify the applicability and virtue of our theoretical findings.

The remainder of this paper is structured as follows. Problems formulation and preliminaries are clarified in Section II. Details of fuzzy switching manifold and analysis of sliding motion are given in Section III. The problem of fuzzy SMC controller design is formulated in Section IV. Numerical example illustrating the applicability and virtue of the achieved fuzzy SMC methodology are offered in Section V. Section VI finally gives some concluding remarks.

Notations: $\mathbb{R}^{m\times n}$ stands for the set of all $m \times n$ real matrices. The terms induced by symmetry are denoted by "$*$." The inverse, transpose, and left inverse for a appropriated dimensioned matrix are indicated, respectively, by superscripts "-1" "T," and "$+$." I is adopted to denote the identity matrix. $B^{\perp}(x) \in \mathbb{R}^{n\times(n-m)}$ stands for the matrix whose columns are independent and span the null space of $B(x) \in \mathbb{R}^{n\times m}$. The block diagonal matrix is represented by diag$\{\cdots\}$.

II. Preliminaries

Consider a highly nonlinear uncertain descriptor system with time-varying delay, which can be exactly depicted by a T–S fuzzy model composed of the following fuzzy rules.

Model Rule i : **IF** $\vartheta_1(t)$ is π_{i1} and $\cdots\vartheta_p(t)$ is π_{ip}, **THEN**

$$\begin{cases} E\dot{x}(t) = A_i x(t) + A_{di}x(t-d(t)) \\ \qquad\quad + B_i(u(t) + f_m(x(t), x(t-d(t)))) + B_{wi}w(t) \\ y(t) = C_i x(t) + C_{di}x(t-d(t)) + D_{wi}w(t) \\ x(t) = \phi(t), t \in [-d_M, 0], i \in \mathcal{R} = \{1, 2, \dots, r\} \end{cases} \tag{1}$$

with $\pi_{i1}, \pi_{i2}, \dots, \pi_{ip}$ being fuzzy sets, r indicating the number of fuzzy rules, $x(t) \in \mathbb{R}^n$ being the system state, $\vartheta_1(t)$, $\vartheta_2(t), \dots, \vartheta_p(t)$ being the premise variables, and are functions of state $x(t)$, $u(t) \in \mathbb{R}^m$ being the input signal, $w(t) \in \mathbb{R}^q$ being the disturbance signal belonging to $L_2[0,\infty)$, $y(t) \in \mathbb{R}^p$ being the controlled output, $A_i \in \mathbb{R}^{n\times n}$, $A_{di} \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $B_{wi} \in \mathbb{R}^{n\times q}$, $C_i \in \mathbb{R}^{p\times n}$, and $D_{wi} \in \mathbb{R}^{p\times q}$ being known constant matrices. It is assumed that the singular matrix $E \in \mathbb{R}^{n\times n}$ satisfies rank$(E) = g \le n$. $d(t)$ represents a time-varying delay satisfying

$$0 \le d(t) \le d_M, 0 < \dot{d}(t) \le \mu < 1 \tag{2}$$

with d_M and μ being positive scalars. $\phi(t)$ denotes a compatible initial function on $[-d_M, 0]$. The nonlinear vector $f_m(x(t), x(t-d(t))) \in \mathbb{R}^m$ represents the matched uncertainty satisfying

$$\|f_m(x(t), x(t-d(t)))\| \le \kappa_1\|x(t)\| + \kappa_2\|x(t-d(t))\| \tag{3}$$

with $\kappa_1 > 0$ and $\kappa_2 > 0$ being known scalars.

Thus, based on the pair $(u(t), x(t))$, the whole fuzzy descriptor model with time-varying delay can be referred as

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{r}\mu_i(\vartheta(t))\big[A_i x(t) + A_{di}x(t-d(t)) \\ \qquad\quad + B_i(u(t) + f_m(x(t), x(t-d(t)))) + B_{wi}w(t)\big] \\ y(t) = \sum_{i=1}^{r}\mu_i(\vartheta(t))[C_i x(t) + C_{di}x(t-d(t)) + D_{wi}w(t)] \end{cases} \tag{4}$$

where

$$\mu_i(\vartheta(t)) = \frac{\prod_{i=1}^{p}\pi_{ij}\big(\vartheta_j(t)\big)}{\sum_{l=1}^{r}\prod_{j=1}^{p}\pi_{lj}\big(\vartheta_j(t)\big)} \ge 0, \sum_{i=1}^{r}\mu_i(\vartheta(t)) = 1$$

with $\pi_{ij}(\vartheta_j(t))$ denoting the grade of membership about $\vartheta_j(t)$ in π_{ij}. In this paper, the global input matrix for the fuzzy descriptor model $\mathcal{B}_x = \sum_{i=1}^{r} \mu_i(\vartheta(t))B_i$ is assumed to be column full rank.

We first present some definitions and lemmas to be used during sliding motion and controller synthesize. For this purpose, we consider the following unforced linear descriptor model with time-varying delay:

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_d x(t - d(t)) \\ x(t) = \phi(t), \quad t \in [-d_M, 0]. \end{cases} \tag{5}$$

Definition 1 [2]:

1) Descriptor model (5) is referred to be regular, if $\det(sE - A)$ is not identically zero.
2) Descriptor model (5) is referred to be impulse-free if $\deg(\det(sE - A)) = \mathrm{rank}(E)$.

Here, the requirements of regularity and absence of impulses for the system ensure the existence and uniqueness of impulse-free solution of (1) on $[0, \infty)$.

Definition 2: The descriptor model (5) is refereed to be asymptotically stable if, for any $\varepsilon > 0$, there exists a scalar $\delta(\varepsilon) > 0$ such that for any compatible initial condition $\phi(t)$ satisfying $\sup_{-d_M < t \le 0} \|\phi(t)\| < \delta(\varepsilon)$, the solution of (5) satisfies $\|x(t)\| < \varepsilon$ for $t \ge 0$. Furthermore, $\lim_{t\to\infty} \|x(t)\| = 0$.

Definition 3: The descriptor system (5) is said to be asymptotically admissible if it is regular, impulse-free, and asymptotically stable.

Definition 4 [38]: The uncertain fuzzy descriptor system (4) is said to be strictly $(\mathcal{Z}, \mathcal{Y}, \mathcal{X})$-dissipative, for any $t > 0$ and initial state, if the following inequality holds:

$$\int_0^t \begin{bmatrix} y(t) \\ w(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathcal{Z} & \mathcal{Y} \\ * & \mathcal{X} \end{bmatrix} \begin{bmatrix} y(t) \\ w(t) \end{bmatrix} dt \ge \gamma \int_0^t w^{\mathrm{T}}(t)w(t)dt \tag{6}$$

where $\mathcal{Z} \le 0 \in \mathbb{R}^{p\times p}$, and $\mathcal{X} \in \mathbb{R}^{q\times q}$ are symmetric, and $\mathcal{Y} \in \mathbb{R}^{p\times q}$, $\gamma > 0$ is a positive scalar.

Lemma 1: For the given matrix $\mathcal{Q}_x \in \mathbb{R}^{n\times m}$ with $\mathrm{rank}(\mathcal{Q}_x) = l \le m$, suppose that $\bar{\mathcal{Q}}_x \in \mathbb{R}^{n\times l}$ is a full-column rank matrix formed by any l independent columns of $\mathcal{Q}_x$. If the distribution $\Delta_x = \mathrm{span}\{\bar{\mathcal{Q}}_{x,p}^{\perp}\}$, $p = 1, \ldots, n - l$, is involutive, that is,

$$\left[\bar{\mathcal{Q}}_{x,p}^{\perp}, \bar{\mathcal{Q}}_{x,q}^{\perp}\right] = \frac{\partial \bar{\mathcal{Q}}_{x,q}^{\perp}}{\partial x}\bar{\mathcal{Q}}_{x,p}^{\perp} - \frac{\partial \bar{\mathcal{Q}}_{x,p}^{\perp}}{\partial x}\bar{\mathcal{Q}}_{x,q}^{\perp} \in \Delta_x$$
$$\forall p, q = 1, \ldots, n - l$$

where $\bar{\mathcal{Q}}_p^{\perp}$ indicates the pth column about $\bar{\mathcal{Q}}^{\perp}$, $[\cdot, \cdot]$ represents the Lie bracket for any two vector fields. Then, the nonlinear vector $g_x = \left[\bar{g}_{x,1} \cdots \bar{g}_{x,l}\ 0_{1\times(m-l)}\right]^{\mathrm{T}} \in \mathbb{R}^{m\times 1}$ exists such that $(\partial g_x/\partial x) = \mathcal{T}_x \mathcal{Q}_x^{\mathrm{T}}$, with $\mathcal{T}_x \in \mathbb{R}^{m\times m}$ being a nonsingular matrix.

Proof: From Frobenius' theorem [39], since Δ_x is involutive, there exist l independent functions $\bar{g}_i(x)$ such that $[(\partial \bar{g}_{x,p})/\partial x]\bar{\mathcal{Q}}_{x,q}^{\perp} = 0$, $\forall 1 \le i \le l$. Let $[(\partial \bar{g}_x)/\partial x] \triangleq \bar{\mathcal{G}}_x$. Then, the equation $\mathrm{span}\{\bar{\mathcal{G}}_{x,p}^{\mathrm{T}}\} = (\mathrm{span}\{\bar{\mathcal{Q}}_{x,p}^{\perp}\})^{\perp}$ holds since the columns of $\bar{\mathcal{G}}_x^{\mathrm{T}}$ is full rank. The above equation is equivalent to $\mathrm{span}\{\bar{\mathcal{G}}_{x,p}^{\mathrm{T}}\} = \mathrm{span}\{\bar{\mathcal{Q}}_{x,p}\}$ by double orthogonal complement [40]. Therefore, the columns about matrices $\bar{\mathcal{G}}_x^{\mathrm{T}}$ and $\bar{\mathcal{Q}}_x$ constitute the bases for the same subspace, and $\bar{\mathcal{G}}_x = \bar{\mathcal{T}}_x \bar{\mathcal{Q}}_x^{\mathrm{T}}$, with $\bar{T}^{\mathrm{T}}(x) \in \mathbb{R}^{l\times l}$ being the their transformation matrix. Since the matrices $\mathcal{Q}_x$ and $\bar{\mathcal{Q}}_x$ are belong to the same subspace, there exists an invertable matrix $\mathcal{T}_x \in \mathbb{R}^{m\times m}$ such that

$$\mathcal{T}_x \mathcal{Q}_x^{\mathrm{T}} = \begin{bmatrix} \bar{\mathcal{T}}_x \bar{\mathcal{Q}}_x^{\mathrm{T}} \\ 0_{(m-l)\times n} \end{bmatrix} = \begin{bmatrix} \bar{\mathcal{G}}_x \\ 0_{(m-l)\times n} \end{bmatrix} = \frac{\partial g_x}{\partial x}.$$

■

Lemma 2: Given matrix $\mathcal{B}_x \in \mathbb{R}^{n\times m}$, $\bar{\mathcal{G}}_x = I - \mathcal{B}_x(\mathcal{G}_x\mathcal{B}_x)^{-1}\mathcal{G}_x$ with $\mathcal{G}_x = \mathcal{T}_x\mathcal{B}_x^{\mathrm{T}}$ has Euclidean norm one.

Proof: It is noted that

$$\begin{aligned} \bar{\mathcal{G}}_x &= I - \mathcal{B}_x\left(\mathcal{T}_x\mathcal{B}_x^{\mathrm{T}}\mathcal{B}_x\right)^{-1}\mathcal{T}_x\mathcal{B}_x^{\mathrm{T}} = I - \mathcal{B}_x\left(\mathcal{B}_x^{\mathrm{T}}\mathcal{B}_x\right)^{-1}\mathcal{B}_x^{\mathrm{T}} \\ &= I - \mathcal{B}_x\mathcal{B}_x^{+} = \bar{\mathcal{G}}_x^{\mathrm{T}} \end{aligned}$$

and

$$\begin{aligned} \bar{\mathcal{G}}_x^{\mathrm{T}}\bar{\mathcal{G}}_x &= \left[I - \mathcal{B}_x\mathcal{B}_x^{+}\right]\left[I - \mathcal{B}_x\mathcal{B}_x^{+}\right] = I - \mathcal{B}_x\mathcal{B}_x^{+} - \mathcal{B}_x\mathcal{B}_x^{+} \\ &\quad + \mathcal{B}_x\mathcal{B}_x^{+}\mathcal{B}_x\mathcal{B}_x^{+} = I - \mathcal{B}_x\mathcal{B}_x^{+} = \bar{\mathcal{G}}_x \end{aligned}$$

which clearly show that $\bar{G}(x)$ is idempotent and thus $\|\bar{G}(x)\| = 1$ or $\|\bar{G}(x)\| = 0$. Obviously, the rank of $I - \mathcal{B}_x\mathcal{B}_x^{+}$ is not zero due to fact that rank $(\mathcal{B}_x\mathcal{B}_x^{+}) < n$, and it is impossible that all its eigenvalues are zero, which proves that $\|\bar{G}(x)\| = 1$. ■

Lemma 3 [41]: Let Σ_1 and Σ_2 be two appropriated dimensioned real matrices. Thus, for any time varying matrix Δ satisfying $\Delta^{\mathrm{T}}\Delta \le I$, there exists a parameter $\varepsilon > 0$ satisfying

$$\Sigma_1\Delta\Sigma_2 + (\Sigma_1\Delta\Sigma_2)^{\mathrm{T}} \le \varepsilon^{-1}\Sigma_1\Sigma_1^{\mathrm{T}} + \varepsilon\Sigma_2^{\mathrm{T}}\Sigma_2.$$

Lemma 4 [42]: Let $P \in \mathbb{R}^{n\times n}$ be symmetric such that $E_L^{\mathrm{T}}PE_L > 0$ and $Q \in \mathbb{R}^{(n-g)\times(n-g)}$ is nonsingular. Then, $PE + R^{\mathrm{T}}QS^{\mathrm{T}}$ is nonsingular and its inverse is expressed as

$$\left(PE + R^{\mathrm{T}}QS^{\mathrm{T}}\right)^{-1} = \mathcal{P}E^{\mathrm{T}} + S\mathcal{Q}R$$

where $\mathcal{P} \in \mathbb{R}^{n\times n}$ is a symmetric matrix and $\mathcal{Q} \in \mathbb{R}^{(n-g)\times(n-g)}$ is a nonsingular matrix with

$$\mathcal{Q} = \left(S^{\mathrm{T}}S\right)^{-1}Q^{-1}\left(RR^{\mathrm{T}}\right)^{-1}, \; E_R^{\mathrm{T}}\mathcal{P}E_R = \left(E_L^{\mathrm{T}}PE_L\right)^{-1}$$

where R and S are matrices, which satisfy $RE = 0$ and $ES = 0$. E is decomposed as $E = E_LE_R^{\mathrm{T}}$ with $E_L \in \mathbb{R}^{n\times g}$ and $E_R \in \mathbb{R}^{n\times g}$ being of full column rank.

III. Switching Manifold Design and Sliding Motion Analysis

During this section, a new integral fuzzy switching manifold with time-varying delay will be designed and dissipativity-based admissibility conditions will be presented for sliding motion.

For SMC design, the first important issue to be addressed is to propose a suitable switching manifold. For a descriptor system, the integral switching manifold containing the singular matrix is the most popular choice, which can avoid the difficulties yielded from singular matrix in deriving the sliding motion subsequently. In [36] and [37], the conventional integral switching manifolds proposed in [34] and [35] were extended to investigate the SMC design for fuzzy descriptor models. However, these results require that all the local input matrices B_i are the same, which seriously restrict their practical applications. To better adapt the fuzzy descriptor models and remove the restrictive requirement, in this paper, we propose a distinguished interval fuzzy switching manifold as

$$s(t) = \int_0^t \mathcal{G}_x E dx - \int_0^t \Bigg[\mathcal{G}_x \sum_{i=1}^r \mu_i(\vartheta(\alpha)) \sum_{j=1}^r \mu_j(\vartheta(\alpha)) \times (A_i x(\alpha) + B_i K_j x(\alpha) + A_{di} x(\alpha - d(\alpha))) \Bigg] d\alpha \quad (7)$$

where $\mathcal{G}_x = \mathcal{T}_x \mathcal{B}_x^{\mathrm{T}}$ with $\mathcal{T}_x \in \mathbb{R}^{m\times m}$ being nonsingular, $\sum_{j=1}^r \mu_j(\vartheta(t)) K_j x(t)$ is used to stabilize the sliding motion and $K_j \in \mathbb{R}^{m\times n}$ being the design parameters.

Based on $\dot{s}(t) = 0$, the corresponding fuzzy controller will be obtained as

$$u_{eq}(t) = -(\mathcal{G}_x \mathcal{B}_x)^{-1} \mathcal{G}_x \sum_{i=1}^r \mu_i(\vartheta(t)) B_{wi} w(t) + \sum_{j=1}^r \mu_j(\vartheta(t)) K_j x(t) - f_m(x(t), x(t - d(t))). \quad (8)$$

Combined (8) with (4), the following sliding motion is yielded:

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^r \mu_i(\vartheta(t)) \sum_{j=1}^r \mu_j(\vartheta(t)) \big[(A_i + B_i K_j) x(t) \\ \qquad + A_{di} x(t - d(t)) + \bar{\mathcal{G}}_x B_{wi} w(t)\big] \\ y(t) = \sum_{i=1}^r \mu_i(\vartheta(t)) [C_i x(t) + C_{di} x(t - d(t)) + D_{wi} w(t)] \end{cases} \quad (9)$$

with $\bar{\mathcal{G}}_x = I - \mathcal{B}_x(\mathcal{G}_x \mathcal{B}_x)^{-1} \mathcal{G}_x$ being the transition matrix for external disturbance signal.

Remark 1: To adapt the features of fuzzy descriptor models, a distinguished interval fuzzy switching manifold (7) is designed by involving the singular and projection matrices simultaneously. The precondition for the establishment of the fuzzy manifold lies in the existence of the nonlinear vector function $g_x \in \mathbb{R}^{m\times 1}$ satisfying $\partial g_x/\partial x = \mathcal{G}_x E$. Assume that $\mathrm{rank}(E^{\mathrm{T}} \mathcal{B}_x) = l$ and $\bar{\mathcal{B}}_x \in \mathbb{R}^{n\times l}$ is a full column rank matrix composed of any l independent columns of $E^{\mathrm{T}} B(x)$. Then, by resorting to Lemma 1, the requirement can be satisfied if the distribution span $\{\bar{B}_p^{\perp}\}$, $p = 1, 2, \ldots, n - l$ is involutive, with $\bar{B}_p^{\perp}$ representing the pth column about $\bar{B}^{\perp}$. In comparison with the previous manifolds adopted in [36] and [37], a significant advantage of our manifold is that the input matrices B_i via different subsystems are permitted to be diverse. Moreover, it can be seen from (9) and Lemma 2 that the matched uncertainty is fully rejected and the disturbance signal is not enlarged during corresponding sliding motion.

The following theorem is presented to ensure the asymptotic admissibility with $(\mathcal{Z}, \mathcal{Y}, \mathcal{X}) - \gamma$-dissipative performance for sliding motion (9).

Theorem 1: Given prespecified parameters $\mu > 0$ and $\gamma > 0$, symmetric matrices $0 \geq \mathcal{Z} \in \mathbb{R}^{p\times p}$ and $\mathcal{X} \in \mathbb{R}^{q\times q}$, and matrix $\mathcal{Y} \in \mathbb{R}^{p\times q}$, the asymptotic admissibility of sliding motion (9) with $(\mathcal{Z}, \mathcal{Y}, \mathcal{X}) - \gamma$-dissipative performance ensured, if there exist a nonsingular matrix Q, matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Z > 0$, scalars $\varepsilon_i > 0$, $i, j = 1, 2, \ldots, r$, such that LMIs (10) and (11) are feasible

$$\Lambda_{ii} < 0, i \in \mathcal{R} \quad (10)$$

$$\Lambda_{ij} + \Lambda_{ji} < 0, i < j, i, j \in \mathcal{R} \quad (11)$$

where

$$\Lambda_{ij} = \begin{bmatrix} \Xi_{11}^{ij} & \Xi_{12}^{i} & E^{\mathrm{T}}ZE & -C_i^{\mathrm{T}}\mathcal{Y} & \Xi_{15}^{ij} & C_i^{\mathrm{T}}\mathcal{Z} & 0 \\ * & \Xi_{22} & 0 & -C_{di}^{\mathrm{T}}\mathcal{Y} & d_M A_{di}^{\mathrm{T}} & C_{di}^{\mathrm{T}}\mathcal{Z} & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44}^{i} & 0 & D_{wi}^{\mathrm{T}}\mathcal{Z} & B_{wi}^{\mathrm{T}} \\ * & * & * & * & \Xi_{55}^{i} & 0 & 0 \\ * & * & * & * & * & \mathcal{Z} & 0 \\ * & * & * & * & * & * & -\varepsilon_i I \end{bmatrix}$$

$$\begin{aligned} \Xi_{11}^{ij} &= \big(E^{\mathrm{T}}P + SQ^{\mathrm{T}}R\big)A_i + A_i^{\mathrm{T}}\big(PE + R^{\mathrm{T}}QS^{\mathrm{T}}\big) \\ &\quad + \big(E^{\mathrm{T}}P + SQ^{\mathrm{T}}R\big)B_iK_j + K_j^{\mathrm{T}}B_i^{\mathrm{T}}\big(PE + R^{\mathrm{T}}QS^{\mathrm{T}}\big) \\ &\quad + Q_1 + Q_2 - E^{\mathrm{T}}ZE + \varepsilon_i\big(E^{\mathrm{T}}P + SQ^{\mathrm{T}}R\big)\big(PE + R^{\mathrm{T}}QS^{\mathrm{T}}\big) \\ \Xi_{12}^{i} &= \big(E^{\mathrm{T}}P + SQ^{\mathrm{T}}R\big)A_{di} \\ \Xi_{15}^{ij} &= d_M A_i^{\mathrm{T}} + d_M K_j^{\mathrm{T}} B_i^{\mathrm{T}} + \varepsilon_i d_M \big(E^{\mathrm{T}}P + SQ^{\mathrm{T}}R\big) \\ \Xi_{22} &= -(1 - \mu)Q_1,\ \Xi_{33} = -Q_2 - E^{\mathrm{T}}ZE \\ \Xi_{44}^{i} &= -D_{wi}^{\mathrm{T}}\mathcal{Y} - \mathcal{Y}^{\mathrm{T}}D_{wi} - \mathcal{X} + \gamma I \\ \Xi_{55}^{i} &= -Z^{-1} + \varepsilon_i d_M^2 I. \end{aligned}$$

R and S are matrices satisfying $RE = 0$ and $ES = 0$.

Proof: First, we prove the admissibility of the system (9). Since $\mathrm{rank}(E) = g \leq n$, there exist two nonsingular matrices H_1 and H_2 such that

$$H_1 E H_2 = \begin{bmatrix} I_g & 0 \\ 0 & 0 \end{bmatrix}. \quad (12)$$

Denote

$$H_1 A_i H_2 = \begin{bmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{bmatrix}, H_1^{-T} \bar{P} H_2 = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix} \quad (13)$$

where $\bar{P} = (PE + R^{\mathrm{T}}QS^{\mathrm{T}})$. It is obvious to see that $E^{\mathrm{T}}\bar{P} = \bar{P}^{\mathrm{T}}E$. By using the expressions in (12) and (13), we can obtain that $\bar{P}_{12} = 0$. Pre- and post-multiplying $\Xi_{11}^{ij} < 0$ by H_2^{T} and H_2, respectively, we have $A_{22i}^{\mathrm{T}}\bar{P}_{22} + \bar{P}_{22}^{\mathrm{T}}A_{22i} < 0$, which implies A_{22i} is nonsingular and thus the pairs (E, A_i) are regular and impulse free. Therefore, fuzzy system (9) is admissible by virtue of Definition 1.

In the following, we shall show system (9) is strictly $(\mathcal{Z}, \mathcal{Y}, \mathcal{X}) - \gamma$-dissipative. The following Lyapunov–Krasovskii functional is constructed:

$$\begin{aligned} V(t) &= x^{\mathrm{T}}(t)(E^{\mathrm{T}}P + SQ^{\mathrm{T}}R)Ex(t) \\ &\quad + \int_{t-d(t)}^{t} x^{\mathrm{T}}(\alpha)Q_1x(\alpha)d\alpha + \int_{t-d_M}^{t} x^{\mathrm{T}}(\alpha)Q_2x(\alpha)d\alpha \\ &\quad + d_M \int_{-d_M}^{0}\int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}ZE\dot{x}(\alpha)d\alpha d\beta \end{aligned} \quad (14)$$

where invertible matrix Q, and matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Z_1 > 0$, and $Z_2 > 0$ are to be designed.

Taking the derivative of $V(t)$ along the states of fuzzy descriptor system (9), considering dissipativity performance index, and by using Jensen inequality, we have

$$\begin{aligned}\dot{V}(t) \le\ & 2x^{\mathrm{T}}(t)(E^{\mathrm{T}}P + SQ^{\mathrm{T}}R)\sum_{i=1}^{r}\mu_i(\vartheta(t))\sum_{j=1}^{r}\mu_j(\vartheta(t))\\ &\times\left\{\big(A_i + B_iK_j\big)x(t) + A_{di}x(t-d(t)) + \bar{\mathcal{G}}_xB_{wi}w(t)\right\}\\ &+ x^{\mathrm{T}}(t)(Q_1+Q_2)x(t) - (1-\mu)x^{\mathrm{T}}(t-d(t))Q_1x(t-d(t))\\ &- x^{\mathrm{T}}(t-d_M)Q_2x(t-d_M) + d_M^2\dot{x}^{\mathrm{T}}(t)E^{\mathrm{T}}ZE\dot{x}(t)\\ &- [x(t)-x(t-d_M)]^{\mathrm{T}}E^{\mathrm{T}}ZE[x(t)-x(t-d_M)]. \end{aligned}\tag{15}$$

Considering dissipativity performance index, we have

$$\begin{aligned}&\dot{V}(t) - \Psi(t)\\ &\quad\le \sum_{i=1}^{r}\mu_i(\vartheta(t))\sum_{j=1}^{r}\mu_j(\vartheta(t))\xi^{\mathrm{T}}(t)\bar{\Lambda}_{ij}\xi(t)\\ &\quad= \sum_{i=1}^{r}\mu_i^2(\vartheta(t))\xi^{\mathrm{T}}(t)\bar{\Lambda}_{ii}\xi(t) + \sum_{i<j}^{r}\mu_i(\vartheta(t))\mu_j(\vartheta(t))\\ &\qquad\times\ \xi^{\mathrm{T}}(t)(\bar{\Lambda}_{ij}+\bar{\Lambda}_{ji})\xi(t)\end{aligned}\tag{16}$$

where

$$\begin{aligned}\Psi(t) &= y^{\mathrm{T}}(t)\mathcal{Z}y(t) + 2y^{\mathrm{T}}(t)\mathcal{Y}w(t) + w^{\mathrm{T}}(t)(\mathcal{X}-\gamma I)w(t)\\ \bar{\Lambda}_{ij} &= \begin{bmatrix}\bar{\Xi}_{11}^{ij} & \Xi_{12}^{i} & E^{\mathrm{T}}ZE & \bar{\Xi}_{14}^{i}\\ * & \Xi_{22} & 0 & -C_{di}^{\mathrm{T}}\mathcal{Y}\\ * & * & \Xi_{33} & 0\\ * & * & * & \Xi_{44}^{i}\end{bmatrix}\\ &\quad -\ \Gamma_1\mathcal{Z}\Gamma_1^{\mathrm{T}} + d_M^2\Gamma_2Z\Gamma_2^{\mathrm{T}}\\ \bar{\Xi}_{11}^{ij} &= \big(E^{\mathrm{T}}P + SQ^{\mathrm{T}}R\big)A_i + A_i^{\mathrm{T}}\big(PE + R^{\mathrm{T}}QS^{\mathrm{T}}\big)\\ &\quad+\ \big(E^{\mathrm{T}}P + SQ^{\mathrm{T}}R\big)B_iK_j\\ &\quad+\ K_j^TB_i^{\mathrm{T}}\big(PE + R^{\mathrm{T}}QS^{\mathrm{T}}\big) + Q_1 + Q_2 - E^{\mathrm{T}}ZE\\ \bar{\Xi}_{14}^{i} &= -C_i^{\mathrm{T}}\mathcal{Y} + \big(E^{\mathrm{T}}P + SQ^{\mathrm{T}}R\big)\bar{\mathcal{G}}_xB_{wi}\\ \Gamma_1 &= \begin{bmatrix}C_i & C_{di} & 0 & D_{wi}\end{bmatrix}^{\mathrm{T}}\\ \Gamma_2 &= \begin{bmatrix}A_i + B_iK_j & A_{di} & 0 & \bar{\mathcal{G}}_xB_{wi}\end{bmatrix}^{\mathrm{T}}\\ \xi(t) &= \begin{bmatrix}x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-d(t)) & x^{\mathrm{T}}(t-d_M) & w^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}}.\end{aligned}$$

On the other hand, in view of Lemma 2, we have

$$\bar{\mathcal{G}}_x\bar{\mathcal{G}}_x^{\mathrm{T}} \le I.\tag{17}$$

Thus, by Schur complement and Lemma 3, we can conclude from (10) and (11) that the following inequality holds:

$$\dot{V}(t) < \Psi(t).\tag{18}$$

Integrating both sides of (18) from 0 to t_p leads to $\int_0^t\Psi(\alpha)d\alpha \ge V(t) - V(0) \ge 0$. By resorting to Definition 4, the dissipativity performance of sliding motion (9) is ensured. By the similar algebraic manipulation, it can be derived that when $w(t) = 0$, there exists a sufficient small scalar $\lambda > 0$ such that $\dot{V}(t) < -\lambda\|x(t)\|^2$, for $x(t) \ne 0$. By Definition 3, we readily obtain that sliding motion (9) is asymptotically admissible. This completes the proof. ■

Based on Theorem 1, the following theorem is presented by which the involved variable matrices K_j in (7) will be solved.

Theorem 2: Given prespecified parameters $\mu > 0$ and $\gamma > 0$, symmetric matrices $0 \ge \mathcal{Z} \in \mathbb{R}^{p\times p}$ and $\mathcal{X} \in \mathbb{R}^{q\times q}$, and matrix $\mathcal{Y} \in \mathbb{R}^{p\times q}$, the asymptotic admissibility of sliding motion (9) with $(\mathcal{Z},\mathcal{Y},\mathcal{X}) - \gamma$-dissipative performance ensured, if there exist a nonsingular matrix $\mathcal{Q}$, matrices $\mathcal{P} > 0$, $\hat{Q}_1 > 0$, $\hat{Q}_2 > 0$, $\hat{Z} > 0$, J_{1j}, J_{2j}, scalars $\varepsilon_i > 0$, $i, j = 1, 2, \ldots, r$, such that the LMIs (19) and (20) are feasible

$$\hat{\Lambda}_{ii} < 0,\ i \in \mathcal{R}\tag{19}$$

$$\hat{\Lambda}_{ij} + \hat{\Lambda}_{ji} < 0,\ i < j,\ i, j \in \mathcal{R}\tag{20}$$

where

$$\hat{\Lambda}_{ij} = \begin{bmatrix}\hat{\Xi}_{11}^{ij} & A_{di}U & \hat{\Xi}_{13}^{ij} & \hat{\Xi}_{14}^{i} & \hat{\Xi}_{15}^{ij} & U^{\mathrm{T}}C_i^{\mathrm{T}}\mathcal{Z} & 0\\ * & \hat{\Xi}_{22} & 0 & \hat{\Xi}_{24}^{i} & \hat{\Xi}_{25}^{i} & U^{\mathrm{T}}C_{di}^{\mathrm{T}}\mathcal{Z} & 0\\ * & * & \hat{\Xi}_{33}^{ij} & 0 & 0 & 0 & 0\\ * & * & * & \hat{\Xi}_{44}^{i} & 0 & D_{wi}^{\mathrm{T}}\mathcal{Z} & B_{wi}^{\mathrm{T}}\\ * & * & * & * & \hat{\Xi}_{55}^{i} & 0 & 0\\ * & * & * & * & * & \mathcal{Z} & 0\\ * & * & * & * & * & * & -\varepsilon_iI\end{bmatrix}$$

$$\begin{aligned}\hat{\Xi}_{11}^{ij} &= A_iU + U^{\mathrm{T}}A_i^{\mathrm{T}} + B_i\big(J_{1j}E^{\mathrm{T}} + J_{2j}R\big) + \Big(EJ_{1j}^{\mathrm{T}} + R^{\mathrm{T}}J_{2j}^{\mathrm{T}}\Big)B_i^{\mathrm{T}}\\ &\quad+\ \hat{Q}_1 + \hat{Q}_2 - EU - U^{\mathrm{T}}E^{\mathrm{T}} + \hat{Z} + \varepsilon_iI\\ \hat{\Xi}_{13} &= EU + U^{\mathrm{T}}E^{\mathrm{T}} - \hat{Z},\ \hat{\Xi}_{14}^{i} = -U^{\mathrm{T}}C_i^{\mathrm{T}}\mathcal{Y}\\ \hat{\Xi}_{15}^{ij} &= d_MU^{\mathrm{T}}A_i^{\mathrm{T}} + d_M\Big(EJ_{1j}^{\mathrm{T}} + R^{\mathrm{T}}J_{2j}^{\mathrm{T}}\Big)B_i^{\mathrm{T}} + \varepsilon_id_MI\\ \hat{\Xi}_{22} &= -(1-\mu)\hat{Q}_1,\ \hat{\Xi}_{24}^{i} = -U^{\mathrm{T}}C_{di}^{\mathrm{T}}\mathcal{Y},\ \hat{\Xi}_{25}^{i} = d_MU^{\mathrm{T}}A_{di}^{\mathrm{T}}\\ \hat{\Xi}_{33} &= -\hat{Q}_2 - EU - U^{\mathrm{T}}E^{\mathrm{T}} + \hat{Z}\\ \hat{\Xi}_{44}^{i} &= -D_{wi}^{\mathrm{T}}\mathcal{Y} - \mathcal{Y}^{\mathrm{T}}D_{wi} - \mathcal{X} + \gamma I\\ \hat{\Xi}_{55}^{i} &= -\hat{Z} + \varepsilon_id_M^2I,\ U = \mathcal{P}E^{\mathrm{T}} + S\mathcal{Q}R.\end{aligned}$$

R and S are matrices satisfying $RE = 0$ and $ES = 0$.

Furthermore, the parameter matrices K_j involved in fuzzy manifold (7) are computed by

$$K_j = (J_{1j}E^{\mathrm{T}} + J_{2j}R)U^{-1}.\tag{21}$$

Proof: According to Lemma 4, we know that $PE + R^{\mathrm{T}}QS^{\mathrm{T}}$ is nonsingular and its inverse is $U = \mathcal{P}E^{\mathrm{T}} + S\mathcal{Q}R$. Based on the fact $(U_i^{\mathrm{T}}E^{\mathrm{T}} - Z^{-1})Z(U_i^{\mathrm{T}}E^{\mathrm{T}} - Z^{-1})^{\mathrm{T}} \ge 0$, the following inequality holds:

$$\begin{aligned}&\begin{bmatrix}-U^{\mathrm{T}}E^{\mathrm{T}}ZEU & U^{\mathrm{T}}E^{\mathrm{T}}ZEU\\ U^{\mathrm{T}}E^{\mathrm{T}}ZEU & -U^{\mathrm{T}}E^{\mathrm{T}}ZEU\end{bmatrix} = \begin{bmatrix}I\\ -I\end{bmatrix}\big(-U^{\mathrm{T}}E^{\mathrm{T}}ZEU\big)\begin{bmatrix}I\\ -I\end{bmatrix}^{\mathrm{T}}\\ &\quad\le [I\ -I]^{\mathrm{T}}(-EU - U^{\mathrm{T}}E^{\mathrm{T}} + \hat{Z})[I\ -I].\end{aligned}\tag{22}$$

Noting above inequality, pre- and post-multiplying (10) and (11) by $\mathrm{diag}\{U^{\mathrm{T}}, U^{\mathrm{T}}, U^{\mathrm{T}}, I, I, I, I\}$ and its transpose, respectively, and let $J_{1j} = K_j\mathcal{P}$, $J_{2j} = K_jS\mathcal{Q}$, $\hat{Q}_1 = U^{\mathrm{T}}Q_1U$, $\hat{Q}_2 = U^{\mathrm{T}}Q_2U$, $\hat{Z} = Z^{-1}$, we can obtain (19) and (20). This completes the proof. ■

IV. Fuzzy SMC Controller Synthesis

During this section, we shall construct a PDC-based fuzzy SMC controller to force the states of fuzzy descriptor system (4) onto the proposed fuzzy switching manifold (7) in the presence of matched uncertainty and disturbance signal. Moreover, an adaptive SMC controller is also provided to deal with the input saturation and uncertainty with unknown upper bounds.

For fuzzy descriptor system (4), the fuzzy SMC controller for rule j is designed as

Model Rule j: **IF** $\vartheta_1(t)$ is π_{j1} and $\cdots \vartheta_p(t)$ is π_{jp}, **THEN**

$$u(t) = K_j x(t) - \rho(t)(\mathcal{G}_x\mathcal{B}_x)^{-1}\frac{s(t)}{\|s(t)\|}. \tag{23}$$

Thus, the overall fuzzy SMC controller is referred as

$$u(t) = \sum_{j=1}^{r}\mu_j(\vartheta(t))K_j x(t) - \rho(t)(\mathcal{G}_x\mathcal{B}_x)^{-1}\frac{s(t)}{\|s(t)\|} \tag{24}$$

where

$$\rho(t) = \delta + \kappa_1\|\mathcal{G}_x\mathcal{B}_x\|\|x(t)\| + \kappa_2\|\mathcal{G}_x\mathcal{B}_x\|\|x(t-d(t))\| + \|\mathcal{G}_x B_w(x)\|\|w(t)\|$$

were $B_w(x) = \sum_{i=1}^{r}\mu_i(\vartheta(t))B_{wi}$, and $\delta > 0$ is a small scalar.

Theorem 3: Consider the uncertain fuzzy descriptor system (4). Assume that the fuzzy switching manifold is given by (7) and conditions in Theorem 2 are feasible. Then, under the fuzzy SMC controller (24), the states of closed-loop system (4) can be forced onto the proposed fuzzy switching manifold $s(t) = 0$ in spite of matched uncertainty and disturbance signal.

Proof: Define a regular Lyapunov function as

$$V(t) = \frac{1}{2}s^{\mathrm{T}}(t)s(t). \tag{25}$$

Taking the derivative of switching manifold (7) yields

$$\dot{s}(t) = \mathcal{G}_x\sum_{i=1}^{r}\mu_i(\vartheta(t))\sum_{j=1}^{r}\mu_j(\vartheta(t)) \times \big[-B_iK_jx(t) + B_i(u(t) + f_m(x(t), x(t-d(t)))) + B_{wi}w(t)\big]. \tag{26}$$

Considering the above equation, we have

$$\begin{aligned}\dot{V}(t) &= s^{\mathrm{T}}(t)\dot{s}(t)\\ &= s^{\mathrm{T}}(t)\mathcal{G}_x\sum_{i=1}^{r}\mu_i(\vartheta(t))\sum_{j=1}^{r}\mu_j(\vartheta(t))\\ &\quad\times\big[-B_iK_jx(t) + B_i(u(t) + f_m(x(t), x(t-d(t)))) + B_{wi}w(t)\big]\\ &\le \kappa_1\|s(t)\|\|\mathcal{G}_x\mathcal{B}_x\|\|x(t)\| + \kappa_2\|s(t)\|\\ &\quad\times\|\mathcal{G}_x\mathcal{B}_x\|\|x(t-d(t))\| + \sum_{i=1}^{r}\mu_i(\vartheta(t))\|s(t)\|\\ &\quad\times\|\mathcal{G}_xB_{wi}\|\|w(t)\| + s^{\mathrm{T}}(t)\mathcal{G}_x\mathcal{B}_xu(t)\\ &\quad- s^{\mathrm{T}}(t)\mathcal{G}_x\mathcal{B}_x\sum_{j=1}^{r}\mu_j(\vartheta(t))K_jx(t).\end{aligned} \tag{27}$$

Substituting (24) into (27), the following inequality holds:

$$\dot{V}(t) \le -\delta\|s(t)\| \le 0 \tag{28}$$

which implies that $\dot{V}(t) < 0$ for all $s(t) \ne 0$. By resorting to Barbalat's lemma, the states of closed-loop system (4) will be forced onto the proposed fuzzy manifold $s(t) = 0$ in presence of matched uncertainty and disturbance signal. The proof is completed. ■

Remark 2: By analyzing the reachability of sliding motion, a fuzzy SMC controller is developed in (24). Since it is discontinuous due to the existence of term $s(t)/\|s(t)\|$, solutions of the fuzzy descriptor system are understood in a Filippov [43] sense. Compared with previous results [36], [37], the key advantage of the developed fuzzy SMC controller is that the very restrictive requirement imposed on local input matrices has been removed, which is benefited from the new fuzzy manifold (7). Thus, the applicability has been significantly improved. Moreover, it is worth mentioning that the achieved fuzzy SMC strategy can be further extended to descriptor systems with state dependent uncertainties [44] and fuzzy Markovian jump descriptor models with incomplete transition rates [45], which is a topic worthy of future research.

The unknown vector $f_m(x(t), x(t-d(t)))$ may represent the unmodeled dynamics and increased bias force of a physical system, whose upper bounds are often unavailable. In addition, input saturation, which arises as a manifestation of the physical limitations of the control capacity, is unavoidable in most actuators and in enforced at all time. In the next, an adaptive SMC controller will be provided to ensure the reaching condition of sliding motion in the presence of input saturation and unknown upper bounds of uncertainty.

Consider the fuzzy descriptor system (4) with input saturation

$$E\dot{x}(t) = \sum_{i=1}^{r}\mu_i(\vartheta(t))\big[A_ix(t) + A_{di}x(t-d(t)) + B_i(\mathrm{sat}(u) + f_m(x(t), x(t-d(t)))) + B_{wi}w(t)\big] \tag{29}$$

where $\mathrm{sat}(u) = \big[\,\mathrm{sat}(u_1)\ \mathrm{sat}(u_2)\ \cdots\ \mathrm{sat}(u_m)\,\big]^{\mathrm{T}}$ is the actual control input. As in [46], the saturation function $\mathrm{sat}(u)$ can be expressed as

$$\mathrm{sat}(u) = \chi(u(t))\cdot u(t) \tag{30}$$

where $\chi(u(t)) = \mathrm{diag}\{\chi_1(u(t)), \ldots, \chi_m(u(t))\}$ with $\chi_l(u(t)) \in [0, 1]$ being the indicator for the saturation degree of the l th entry of the control vector. According to density property of a real number, there exists a scalar σ satisfying

$$0 < \sigma \le \min(\chi_l(u(t))) \le 1. \tag{31}$$

Theorem 4: Consider the uncertain fuzzy descriptor system (29) with input constraint (30), and assume that the upper bounds κ_1 and κ_2 for $f_m(x(t), x(t-d(t)))$ are unavailable. Assume that the fuzzy switching manifold is given by (7) and conditions in Theorem 2 are feasible. Then, the states of closed-loop system (4) can be forced onto the proposed fuzzy switching manifold $s(t) = 0$ in spite of uncertainty and external disturbance signal by the following adaptive fuzzy SMC controller:

$$u(t) = -\rho(t)(\mathcal{G}_x\mathcal{B}_x)^{-1}\frac{s(t)}{\|s(t)\|} \tag{32}$$

where

$$\rho(t) = \delta + \nu\hat{\sigma}\big(\hat{\kappa}_1\|x(t)\| + \hat{\kappa}_2\|x(t-d(t))\|\big)\|\mathcal{G}_x\mathcal{B}_x\| + \nu\hat{\sigma}(\|\mathcal{G}_xB_w(x)\|\|w(t)\| + \|\mathcal{G}_x\mathcal{B}_x\mathcal{K}_xx(t)\|)$$

and $\mathcal{B}_{w,x} = \sum_{i=1}^{r}\mu_i(\vartheta(t))B_{wi}$, $\mathcal{K}_x = \sum_{j=1}^{r}\mu_j(\vartheta(t))K_j$, and $\delta > 0$ is a constant. The adaptation laws for estimated

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parameters $\hat{\kappa}_1$, $\hat{\kappa}_2$ and $\hat{\sigma}$ are chosen as

$$\dot{\hat{\kappa}}_1 = p_1\|x(t)\|\|\mathcal{G}_x\mathcal{B}_x\|\|s(t)\|, \hat{\kappa}_1(0) > 0 \tag{33}$$

$$\dot{\hat{\kappa}}_2 = p_2\|x(t-d(t))\|\|\mathcal{G}_x\mathcal{B}_x\|\|s(t)\|, \hat{\kappa}_2(0) > 0 \tag{34}$$

$$\begin{aligned}\dot{\hat{\sigma}} &= \nu\hat{\sigma}^3\big(\hat{\kappa}_1\|x(t)\| + \hat{\kappa}_2\|x(t-d(t))\|\big)\|\mathcal{G}_x\mathcal{B}_x\|\|s(t)\| \\ &\quad + \nu\hat{\sigma}^3\big(\big\|\mathcal{G}_x\mathcal{B}_{w,x}\big\|\ \|w(t)\| + \|\mathcal{G}_x\mathcal{B}_x\mathcal{K}_x x(t)\|\big)\|s(t)\|, \hat{\sigma}(0) > 0\end{aligned} \tag{35}$$

where $\nu > 1$ and p_1 and p_2 are design parameters.

Proof: Choose the following Lyapunov function:

$$V(t) = \frac{1}{2}s^{\mathrm{T}}(t)s(t) + \frac{1}{2p_1}\tilde{\kappa}_1^2 + \frac{1}{2p_2}\tilde{\kappa}_2^2 + \frac{1}{2}\tilde{\sigma}^2 \tag{36}$$

where $\tilde{\sigma} = \sigma - \hat{\sigma}^{-1}$, $\tilde{\kappa}_1 = \kappa_1 - \hat{\kappa}_1$, and $\tilde{\kappa}_2 = \kappa_2 - \hat{\kappa}_2$. Then, from (27), the time derivative of $V(t)$ can be expressed as

$$\begin{aligned}\dot{V}(t) &\le \kappa_1\|s(t)\|\|\mathcal{G}_x\mathcal{B}_x\|\|x(t)\| + \kappa_2\|s(t)\|\|\mathcal{G}_x\mathcal{B}_x\| \\ &\quad \times \|x(t-d(t))\| + \|s(t)\|\big\|\mathcal{G}_x\mathcal{B}_{w,x}\big\|\|w(t)\| \\ &\quad + \|s(t)\|\|\mathcal{G}_x\mathcal{B}_x K(x)x(t)\| + s^{\mathrm{T}}(t)\mathcal{G}_x\mathcal{B}_x\chi(u(t))u(t) \\ &\quad - \frac{1}{p_1}\tilde{\kappa}_1\dot{\hat{\kappa}}_1 - \frac{1}{p_2}\tilde{\kappa}_2\dot{\hat{\kappa}}_2 + \tilde{\sigma}\hat{\sigma}^{-2}\dot{\hat{\sigma}}.\end{aligned} \tag{37}$$

Then, noticing (31), with control law (32) and adaptation laws (33)–(35), we have

$$\begin{aligned}\dot{V}(t) &\le \big(\hat{\kappa}_1\|x(t)\| + \hat{\kappa}_2\|x(t-d(t))\|\big)\|\mathcal{G}_x\mathcal{B}_x\|\|s(t)\| \\ &\quad + \big(\big\|\mathcal{G}_x\mathcal{B}_{w,x}\big\|\|w(t)\| + \|\mathcal{G}_x\mathcal{B}_x\mathcal{K}_x x(t)\|\big)\|s(t)\| - \sigma\delta\|s(t)\| \\ &\quad - \sigma\nu\hat{\sigma}\big(\hat{\kappa}_1\|x(t)\| + \hat{\kappa}_2\|x(t-d(t))\|\big)\|\mathcal{G}_x\mathcal{B}_x\|\|s(t)\| \\ &\quad - \sigma\nu\hat{\sigma}\big(\big\|\mathcal{G}_x\mathcal{B}_{w,x}\big\|\|w(t)\| + \|\mathcal{G}_x\mathcal{B}_x\mathcal{K}_x x(t)\|\big)\|s(t)\| \\ &\quad + \tilde{\sigma}\nu\hat{\sigma}\big(\hat{\kappa}_1\|x(t)\| + \hat{\kappa}_2\|x(t-d(t))\|\big)\|\mathcal{G}_x\mathcal{B}_x\|\|s(t)\| \\ &\quad + \tilde{\sigma}\nu\hat{\sigma}\big(\big\|\mathcal{G}_x\mathcal{B}_{w,x}\big\|\|w(t)\| + \|\mathcal{G}_x\mathcal{B}_x\mathcal{K}_x x(t)\|\big)\|s(t)\| \\ &= -\sigma\delta\|s(t)\| + (1-\nu)\big(\hat{\kappa}_1\|x(t)\| + \hat{\kappa}_2\|x(t-d(t))\|\big) \\ &\quad \times \|\mathcal{G}_x\mathcal{B}_x\|\|s(t)\| + (1-\nu)\big(\big\|\mathcal{G}_x\mathcal{B}_{w,x}\big\|\|w(t)\| \\ &\qquad + \|\mathcal{G}_x\mathcal{B}_x\mathcal{K}_x x(t)\|\big)\|s(t)\| \\ &\le 0\end{aligned} \tag{38}$$

which implies the system states can be forced onto the prespecified manifold $s(t) = 0$ in the presence of matched uncertainty and disturbance signal. This completes the proof. ■

Remark 3: An adaptive fuzzy SMC controller is further developed in (32), where the update laws are used to deal with input saturation and unknown upper bounds of matched uncertainty. It is worth mentioning that very few literatures have been published on SMC controller design for descriptor systems in the presence of input saturation. According to (38), all of the signals are bounded, and thus, $u(t)$ is bounded, which implies the existence of σ since $\chi_l(u(t))$ will not converge to zero. Therefore, assumption (31) is reasonable. Also, it is clear from (30), (31), and (38) that the larger the u_{Ml} is, the bigger the σ becomes, and thus, the higher the converge rate of the state variables is.

Remark 4: As in the existing results about SMC, the main drawback of the sliding mode controllers proposed in (24) and (32) is that they are discontinuous when crossing the fuzzy sliding manifold, which will cause control chattering involving high frequency dynamics. One common way to alleviate this issue is to replace the discontinuous term $s(t)/\|s(t)\|$ by smooth term $s(t)/(\|s(t)\| + \bar{\alpha})$, where $\bar{\alpha}$ is a small positive parameter. As a result, the fuzzy manifold will not converge to zero accurately but vary within a small range.

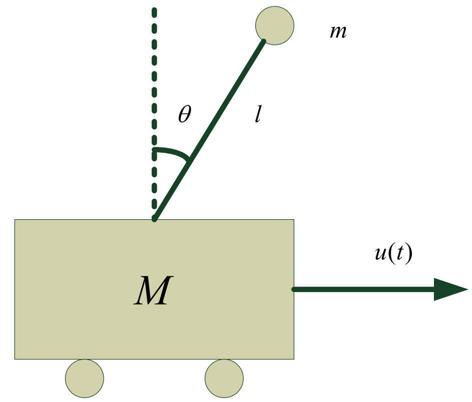

Fig. 1. Schematic of the inverted pendulum system.

V. Simulation Example

During this section, an inverted pendulum system will be adopted to demonstrate the effectiveness and virtue the of proposed fuzzy SMC scheme (see Fig. 1). The whole physical system is given by

$$\begin{cases}\dot{x}_1(t) = x_2(t) \\ \big[(M+m)\big(J+ml^2\big) - m^2l^2\cos^2(x_1)\big]\dot{x}_2(t) = (M+m) \\ \qquad \times\ mgx_3(t) - m^2lx_2^2(t)x_3(t)\cos(x_1) - ml\cos(x_1)u(t) \\ 0 = l\sin(x_1) - x_3(t)\end{cases} \tag{39}$$

where $x_1(t) \in [-\pi/2, \pi/2]$ denotes the angle for the pendulum, $x_2(t)$ represents the angular velocity, and $x_3(t)$ denotes the relative horizontal distance. $u(t)$ represents the control input, $g = 9.8\ \mathrm{m/s^2}$ indicates the acceleration of gravity. $l = 0.304$ m represents the length of the pendulum. $m = 0.22$ kg and $M = 1.3282$ kg represents the mass for pendulum and the cart, respectively. $J = ml^2/3$ denotes moment of inertia.

It is assumed there are perturbed time delay terms in the system along values of the scalar $\lambda \in [0, 1]$. By taking the matched uncertainty and disturbance signal into account, system (39) can be further represented by an uncertain time-delay T–S fuzzy descriptor model, which is given by

$$\begin{cases}E\dot{x}(t) = \sum_{i=1}^{2}\mu_i(x_1(t))[\lambda A_i x(t) + (1-\lambda)A_i x(t-d(t)) \\ \qquad + B_i(u(t) + f_m(x(t), x(t-d(t)))) + B_{wi}w(t)\big] \\ y(t) = \sum_{i=1}^{2}\mu_i(x_1(t))[C_i x(t) + C_{di}x(t-d(t)) + D_{wi}w(t)]\end{cases} \tag{40}$$

where

$$E = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}, A_1 = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & a_{123}\\ a_{131} & 0 & -1\end{bmatrix}$$

$$A_2 = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & a_{223}\\ a_{231} & 0 & -1\end{bmatrix}$$

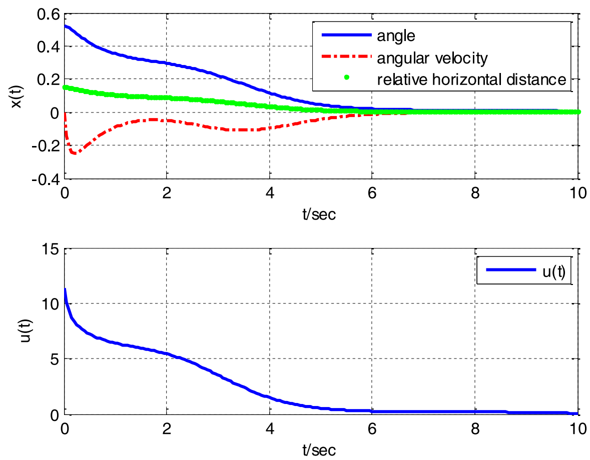

Fig. 2. Responses of system with small matched uncertainty under fuzzy controller (43).

$$
\begin{aligned}
B_1 &= \begin{bmatrix} 0 & b_{12} & 0 \end{bmatrix}^{\mathrm{T}}, B_2 = \begin{bmatrix} 0 & b_{22} & 0 \end{bmatrix}^{\mathrm{T}} \\
D_{w1} &= D_{w2} = 0.1 \\
B_{w1} &= B_{w2} = 0.1 * \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}}, a_1 = (M+m)\left(J+ml^2\right) \\
&\quad - m^2 l^2 \\
C_1 &= C_2 = C_{d1} = C_{d2} = 0.01 * \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \\
w(t) &= 0.9/\left(1+t^2\right) \\
a_2 &= (M+m)\left(J+ml^2\right) - m^2 l^2 \cos^2\theta_0 \\
a_{123} &= (M+m)mg/a_1, a_{223} = (M+m)mg/a_2 \\
a_{231} &= l\sin\theta_0/\theta_0, b_{22} = -ml\cos\theta_0/a_2 \\
b_{12} &= -ml/a_1, a_{131} = l \\
d(t) &= 0.2 + 0.2\cos(1.5t), \theta_0 = \pi/3
\end{aligned}
$$

$f_m(x(t), x(t-d(t))) = -m_u \sin\ (t)(x_1(t) + x_1(t-d(t)))$ with m_u denoting the amplitude of matched uncertainty.

The membership functions are given by

$$
\begin{aligned}
\mu_1(x_1(t)) &= \left(\sin^2(\theta_0) - \sin^2(x_1(t))\right)/\sin^2(\theta_0) \\
\mu_2(x_1(t)) &= 1 - \mu_1(x_1(t)).
\end{aligned}
$$

Remark 5: It is noted that the local control input matrices of obtained fuzzy descriptor model are not the same. Thus, the fuzzy SMC approaches developed in [36] and [37] cannot be applicable in this situation. By contrast, benefited from the novel fuzzy manifold (7), the developed fuzzy SMC approach in this paper is not limited by this restriction.

In this paper, we choose $\lambda = 0.9$, $\mathcal{Z} = -1.5$, $\mathcal{Y} = 1.5$, $\mathcal{X} = 1.5$, $\mu = 1.0$, and $\gamma = 0.5$. By solving the conditions in Theorem 2, and by (22), we obtain

$$
\begin{aligned}
K_1 &= \begin{bmatrix} 5.6176 & 7.2192 & 42.8551 \end{bmatrix} \\
K_2 &= \begin{bmatrix} 8.6762 & 10.22646 & 69.4804 \end{bmatrix}.
\end{aligned}
$$

Define

$$
\mathcal{T}_x = \left(m^2 l^2 \cos^2(x_1) - (M+m)(J+ml^2)\right)/(ml\cos(x_1)).
$$

Thus, the fuzzy manifold proposed in (7) is written as

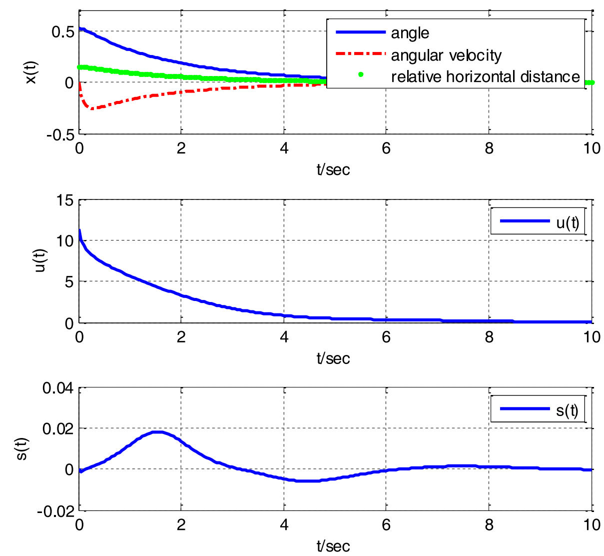

Fig. 3. Responses of system with small matched uncertainty under fuzzy SMC controller (42).

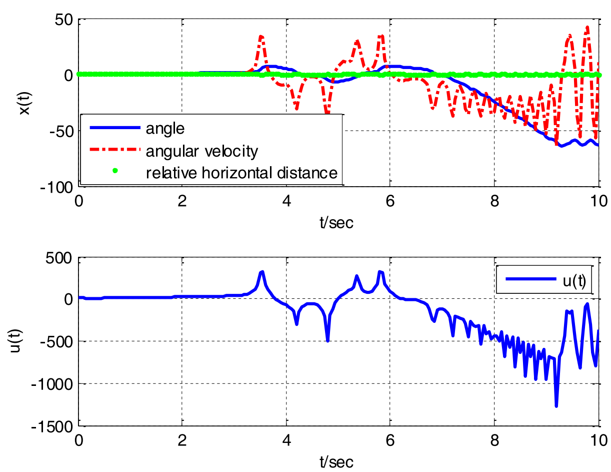

Fig. 4. Responses of system with big matched uncertainty under fuzzy controller (43).

$$
\begin{aligned}
s(t) &= x_2(t) - x_2(0) \\
&\quad - \int_0^t \left\{ \mathcal{G}_x \sum_{i=1}^{2} \mu_i(x_1(\alpha)) \sum_{j=1}^{2} \mu_j(x_1(\alpha)) \right. \\
&\qquad \left. \times \ \left(A_i x(\alpha) + B_i K_j x(\alpha) + A_{di} x(\alpha - d(\alpha))\right) \right\} d\alpha
\end{aligned} \tag{41}
$$

with $\mathcal{G}_x = \mathcal{T}_x \mathcal{B}_x^{\mathrm{T}} = \begin{bmatrix} 0 \ 1 \ 0 \end{bmatrix}$

Moreover, the fuzzy SMC controller in (24) is given by

$$
u(t) = \sum_{j=1}^{2} \mu_j(x_1(t)) K_j x(t) - \rho(t)(\mathcal{G}_x \mathcal{B}_x)^{-1} \frac{s(t)}{\|s(t)\| + \bar{\alpha}} \tag{42}
$$

where

$$
\begin{aligned}
\rho(t) &= \delta + m_u \|\mathcal{G}_x \mathcal{B}_x\| \|x(t)\| + m_u \|\mathcal{G}_x \mathcal{B}_x\| \|x(t-d(t))\| \\
&\quad + 0.9 \left\|\mathcal{G}_x \mathcal{B}_{w,x}\right\|.
\end{aligned}
$$

First, we make a comparison of the proposed SMC methodology with the conventional PDC-based fuzzy controller $u_n(t)$,

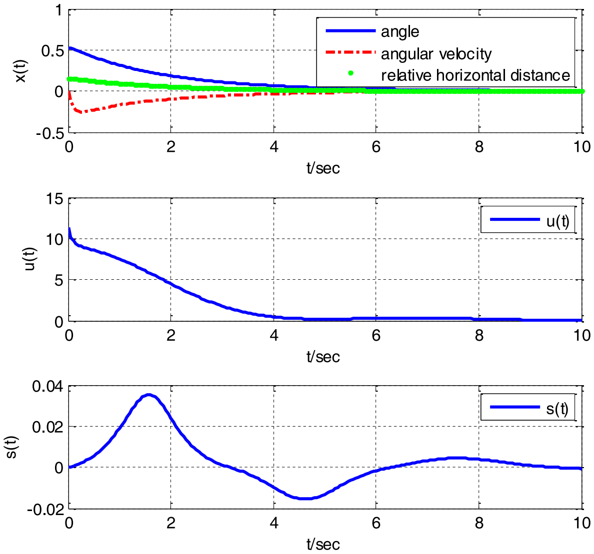

Fig. 5. Responses of system with big matched uncertainty under fuzzy SMC controller (42).

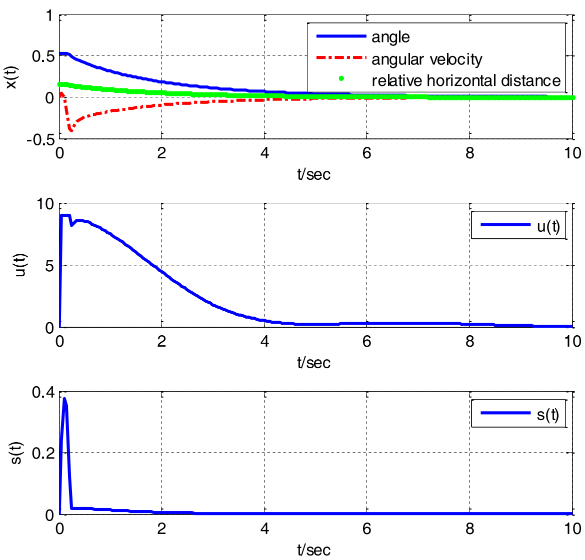

Fig. 6. System responses under adaptive SMC law (44).

which is formulated as

$$u_n(t) = \sum_{j=1}^{2} \mu_j(x_1(t))K_j x(t). \tag{43}$$

Suppose the initial state $x(0) \in \left[0.5236\ 0\ l/2\right]$, and the tuning scalars $\delta = 0.4$ and $\bar{\alpha} = 0.01$. First, we assume that the matched uncertainty is small ($m_u = 3$). In this case, the simulation results under the controllers formulated in (43) and (42) are depicted in Figs. 2 and 3, respectively. From the figures, we can observe that both conventional fuzzy controller (43) and fuzzy SMC controller (42) can tolerate the small matched uncertainty. Second, we consider the case of the big matched uncertainty ($m_u = 10$). Figs. 4 and 5 illustrate the simulation results for system (40) under controllers in (43) and (42), respectively. We can see from Fig. 4 that, despite the huge control efforts, the conventional fuzzy controller (43) cannot

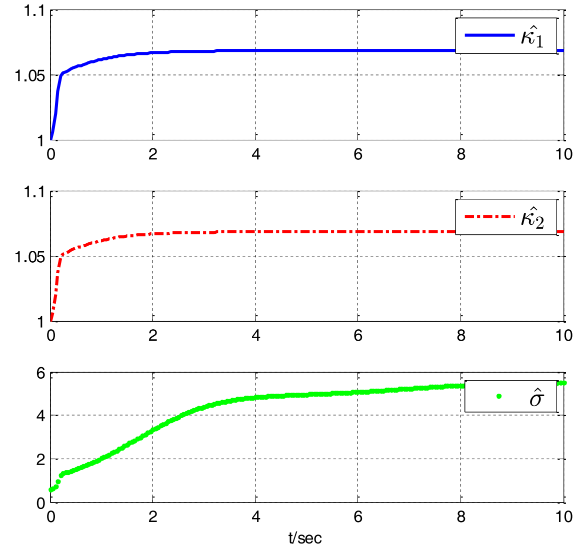

Fig. 7. Estimated parameters $\hat{\kappa}_1$, $\hat{\kappa}_2$, and $\hat{\sigma}$.

stabilize the closed-loop fuzzy descriptor system (40) in the presence of big matched uncertainty. While applying the fuzzy SMC controller (42), the system can be stabilized successfully, which verifies the advantage of the proposed control strategy. The reason lies in that the discontinuous part of fuzzy SMC law can drive the whole system toward a sliding motion that the matched uncertainty is completely rejected. Moreover, benefited from the introduction of the small scalar $\bar{\alpha}$ into (42), the chattering is very small during the period of response.

Finally, we shall further show the effectiveness of proposed adaptive fuzzy SMC law in the presence of unknown upper bounds of matched uncertainty and input saturation. In this example, the adaptive SMC law (32) is given as

$$u(t) = -\rho(t)(\mathcal{G}_x\mathcal{B}_x)^{-1}\frac{s(t)}{\|s(t)\| + \bar{\alpha}} \tag{44}$$

where

$$\begin{aligned}\rho(t) &= \delta + \nu\hat{\sigma}\left(\hat{\kappa}_1\|x(t)\| + \hat{\kappa}_2\|x(t-d(t))\|\right)\|\mathcal{G}_x\mathcal{B}_x\| \\ &\quad + \nu\hat{\sigma}\left(0.9\left\|\mathcal{G}_x\mathcal{B}_{w,x}\right\| + \|\mathcal{G}_x\mathcal{B}_x\mathcal{K}_x x(t)\|\right).\end{aligned}$$

It is assumed that the upper bounds of $f_m(x(t), x(t-d(t)))$ are unknown and the input are constrained by $|u(t)| \leq 9N$. Given the design parameters $p_1 = p_2 = 1.1$, $\nu = 1.2$, and the initial set $\hat{\kappa}_1(0) = \hat{\kappa}_2(0) = 1.0$, and $\hat{\sigma}(0) = 0.5$, the simulation results under adaptive fuzzy SMC controller (44) are provided in Figs. 6 and 7, which show that the designed adaptive SMC controller can achieve asymptotic stability of the system despite unknown upper bounds of matched uncertainty and input saturation.

VI. Conclusion

In this paper, the fuzzy SMC design problem has been addressed for a type of uncertain fuzzy-model-based nonlinear descriptor systems with time-varying delay. A suitable integral fuzzy switching manifold with time delay has been proposed to adapt the system features. Criteria have been provided to ensure the asymptotic admissibility of sliding motion satisfying dissipative performance. A fuzzy SMC controller has been constructed to assure reaching condition. A modified adaptive fuzzy SMC controller has also been developed to adapt input saturation and matched uncertainty with unknown upper bounds. The applicability and virtue of our theoretical findings have been illustrated by numerical simulations for an inverted pendulum system.

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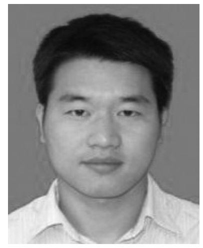

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