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Stochastic Model Predictive Control for Economic/Environmental Operation Management of Microgrids

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***Abstract*— Microgrids are subsystems of the distribution grid which comprises generation capacities, storage devices and controllable loads, which can operate either connected or isolated from the utility grid. In this work, microgrid management system is developed in a stochastic framework. Uncertainties due to fluctuating demand and generation from renewable energy sources are taken into account and a *two-stage stochastic programming* approach is applied to efficiently optimize microgrid operations while satisfying a time-varying request and operation constraints. Mathematically, the stochastic optimization problem is stated as a mixed-integer linear programming problem, which can be solved in an efficient way by using commercial solvers. The stochastic problem is incorporated in a Model Predictive Control (MPC) scheme to further compensate the uncertainty though the feedback mechanism. Simulations show the effective performance of the proposed approach.**

I. Introduction

Microgrids are integrated energy systems, with the possibility of bidirectional power flows, consisting of interconnected loads and Distributed Energy Resources (DERs), which can operate in parallel with the grid or in an intentional island mode [1]. The growing need of reducing carbon emissions makes the concept of microgrid even more attractive.

In the *smart grid* scenario, new modeling requirements are needed, e.g. storage modeling must be incorporated into the operation planning problem in order to coordinate storage use with Renewable Energy Sources (RES) generation and energy prices, and address the complexity of the charging/discharging schedule [2]. It is important to notice that there are no current modeling tools including controllable loads and energy storage modeling in a smart grid environment [3].

Several models have been proposed for microgrids, some of them solving also the microgrid environmental/economic optimization problem (e.g., [4]–[6]). Recently, Model Predictive Control (MPC) has drawn the attention of the power system community [7] and several authors showed the advantages of applying MPC to dynamic economic dispatch (DED) in a stochastic or in an environmental framework (e.g., see [8], [9]).

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In most proposed solution methods, the optimization problem stays nonlinear and/or heuristics or decomposition techniques are applied; important microgrid features are in general neglected such as storage dynamics, grid interaction, generators operative constraints. An integrated optimization-based framework for microgrid operational planning has not yet been proposed, which considers the inherent uncertainty sources, all kinds of distributed energy resources technologies (DGs, RESs, energy storages and controllable loads), as well as detailed system features and operating constraints, from both an economic and environmental viewpoint.

A. Main assumptions and contributions

Since microgrid control requirements involve different control approaches and different time scales, a very reasonable approach is to develop a hierarchical control structure [10]. In a hierarchy of controllers, we aim at a high level optimization of microgrid operations; voltage stability, power quality, and frequency are supposed to be controlled at the lower control level. The local controllers have to guarantee that the system tracks the power reference values, while keeping voltage stability and power quality (see, for example, [11]). The high level controller deals with longer time scales and is very weakly dependent on the transient behavior of the fast dynamics. Then a steady state assumption for microgrid components can be safely made without much loss of accuracy. A further assumption of our approach is that the *Microgrid Operator* is the unique centralized entity in charge of microgrid management.

In this paper we propose an integrated optimization-based framework for microgrid operational planning in a stochastic framework. We propose the use of MPC in combination with Mixed Integer Linear Programming (MILP) [12] and two-stage stochastic programming [13]. Further, by taking the environmental concern into account, the microgrid operation optimization problem can be formulated as a multi-objective optimization (MOO). The *weighted min-max method* is applied to compute Pareto optimal points; this approach provides both necessary and sufficient conditions for Pareto optimality [14]. In our approach we strove to include as many details as possible and maintain the microgrid stochastic optimization problem solvable without resorting to any decomposition techniques or heuristics. Further, to guarantee a correct behavior for storage and grid interaction (e.g., non-simultaneous charging and discharging), we utilize the approach described in [15] and use the Mixed Logical Dynamical (MLD) framework.

B. Nomenclature

The parameters, the forecasts and the decision variables used in the proposed formulation are described respectively in Tables I, II and III, where, for simplicity, the subscript i when referring to the i^{th} unit is omitted. The fuel consumption cost for a DG unit is traditionally assumed to be a quadratic function of the generated power of the form $C^{DG}(P) = a_1P^2 + a_2P + a_3$.

TABLE I
PARAMETERS

Parameters	Description
N_g, N_l, N_c	number respectively of DG units, critical loads and controllable loads
$C^{DG}(P)$	fuel consumption cost curve of a DG unit
$E(P)$	emission function of a DG unit
OM	operating and maintenance cost of a DG unit [€/kWh]
$R_{\max}$	ramp up limit of a DG unit [kW/h]
T	horizon of the optimization problem
T^{up}, T^{down}	minimum up and down time of a DG unit [h]
x^{sb}	storage 'physiological' energy loss [kWh]
$x^b_{\min}$, $x^b_{\max}$	minimum, maximum energy level of the storage unit [kWh]
$C^b_{\max}$	storage power limit [kW]
T^g	maximum interconnection power flow limit (at the point of common coupling) [kW]
$P_{\min}$, $P_{\max}$	minimum, maximum power level of a DG unit [kW]
η^c, η^d	storage charging and discharging efficiencies
$\beta_{\min}$, $\beta_{\max}$	minimum, maximum allowed curtailment of a controllable load
c^{SU}, c^{SD}	start-up, start-down costs of a DG unit [€]
D^c	power level of a controllable load [kW]
ρ_c	penalty weight on curtailments

TABLE II
FORECASTS

Forecasts	Description
P^{res}	sum of power production from RES [kW]
D	power level required from a critical load [kW]
c^P, c^S	purchasing, selling energy prices [€/kWh]

TABLE III
DECISION AND LOGICAL VARIABLES

Variables	Description
δ	off(0)/on(1) state of a DG unit
δ^b	discharging(0)/charging(1) mode of the storage unit
δ^g	exporting(0)/importing(1) mode to/from the utility grid
P	power level of a DG unit [kW]
P^b	power exchanged (positive for charging) with the storage unit [kW]
P^g	importing(positive)/exporting(negative) power level from/to the utility grid [kW]
x^b	stored energy level [kWh]
β	curtailed power percentage

II. SYSTEM DESCRIPTION AND MODELING

Here the key features of the microgrid architecture considered in this paper are briefly described. We point out that, although it is not illustrated for the sake of brevity, the proposed microgrid modeling includes the supply of thermal load with combined heat and power CHP capabilities, easily represented through the *power to heat ratio* [16]. Further details on microgrid modeling can be found in [17].

We remark that, due to constant sampling time $\Delta T = t_{k+1} - t_k$, there exists a constant ratio between energy and power at each interval.

A. Storage Dynamics

The following discrete time model of a storage unit is considered:

$$x^b(k+1) = x^b(k) + \eta P^b(k) - x^{sb}, \tag{1}$$

where $P^b(k)$ is the power exchanged with the storage at time k and

$$\eta = \begin{cases} \eta^c, & \text{if } P^b(k) > 0 \text{ (charging mode)}, \\ \eta^d, & \text{otherwise (discharging mode)}, \end{cases} \tag{2}$$

Typically $\eta^c < 1$ and $\eta^d = 1/\eta^c$ account for the losses.

By using the standard approach described in [15], a binary variable $\delta^b(k)$ and an auxiliary variable $z^b(k) = \delta^b(k)P^b(k)$ are introduced to model the logical conditions provided in (2); the storage dynamics and the corresponding constraints are rewritten in the following compact form:

$$\begin{aligned} &x^b(k+1) = x^b(k) - (\eta^d - \eta^c)z^b(k) + \eta^d P^b(k) - x^{sb}, \\ &\text{subject to} \quad E^b_1\delta^b(k) + E^b_2 z^b(k) \le E^b_3 P^b(k) + E^b_4, \end{aligned} \tag{3}$$

where the column vectors $E^b_1, E^b_2, E^b_3, E^b_4$ are provided in the Appendix.

B. Loads

We consider two types of loads: (*i*) *critical loads*, i.e. demand levels related to essential processes that must be always met; (*ii*) *controllable loads*, i.e. loads that can be reduced or shed during supply constraints or emergency situations. A continuous-valued variable, $0 \le \beta_h(k) \le 1$, associated to each controllable load h and to each sampling time k is defined. This variable represents the percentage of the given power level to be curtailed at time k.

C. Power balance

The balance between energy production and consumption which must be met at each time k; this is expressed as follows:

$$\begin{aligned} P^b(k) = &\textstyle\sum_{i=1}^{N_g} P_i(k) + P^{\text{res}}(k) + P^g(k) \\ &- \textstyle\sum_{j=1}^{N_l} D_j(k) - \sum_{h=1}^{N_c}[1 - \beta_h(k)]D^c_h(k). \end{aligned} \tag{4}$$

We denote by $u(k)$ the decision variables, which are the generators' power levels, the exchanged power with the utility grid and with the storage, the curtailments, the generators off/on states. We further denote by $w(k)$ the vector of random variables at time k, i.e. RES generation, demand, and the controllable power levels, which are known. Hence, the power balance can be rewritten from 4 as follows:

$$F_k^{'} u(k) + f^{'} w(k) = 0. \tag{5}$$

with F_k and f are provided in the Appendix.

D. Interaction with the utility grid

By following the same procedure outlined above, a binary variable $\delta^g(k)$ and an auxiliary variable $C^g(k)$ are introduced to model the possibility either to purchase or to sell energy from/to the utility grid; $C^{\rm g}(k)$ is the cost of the power exchanged with the utility grid, which can be negative if energy is sold to the utility grid.

Then, the purchasing/selling microgrid behavior can be compactly expressed by the following mixed integer linear inequalities:

$$E_1^g \delta^g(k) + E_2^g C^g(k) \leq E_3^g(k) P^g(k) + E_4^g. \qquad (6)$$

The column vectors $E_1^g, E_2^g, E_3^g(k), E_4^g$ are provided in the Appendix. The matrix $E_3^g(k)$ is generally time-varying due to the time varying energy prices.

E. Generator operating conditions

The operating constraints, at each sampling time k, on the minimum amount of time for which a controllable generation unit must be kept on/off (minimum up/down times) can be expressed by the following mixed integer linear inequalities without resorting to any additional variable:

$$\begin{aligned} &\delta_i(k) - \delta_i(k-1) \leq \delta_i(\tau^{\rm up}), \quad \text{(off-on switch)},\\ &\delta_i(k-1) - \delta_i(k) \leq 1 - \delta_i(\tau^{\rm down}), \ \text{(on-off switch)}, \end{aligned} \qquad (7)$$

with $i = 1, \ldots, N_g$, $\tau^{\rm up} = k+1, \ldots, \min(k + T_i^{\rm up} - 1, T)$ and $\tau^{\rm down} = k{+}1, \ldots, \min(k{+}T_i^{\rm down}{-}1, T)$ otherwise. The DG unit start up and shut down behavior are also modeled in order to account for the corresponding costs. For this reason, two auxiliary variables, $SU_i(k)$ and $SD_i(k)$, are introduced, representing respectively the start up cost and the shut down cost for the $i^{\rm th}$ DG unit at time k. These auxiliary variables must satisfy the following mixed integer linear constraints:

$$\begin{aligned} &SU_i(k) \geq c_i^{SU}(k)[\delta_i(k) - \delta_i(k-1)],\\ &SD_i(k) \geq c_i^{SD}(k)[\delta_i(k-1) - \delta_i(k)],\\ &SU_i(k) \geq 0,\\ &SD_i(k) \geq 0, \end{aligned} \qquad (8)$$

with $i = 1, \ldots, N_g$ (see [18] and references therein).

III. Problem Description

In this section, the MOO problem is formulated. The objectives considered in the MOO problem formulation are microgrid operating costs and emissions.

A. Cost Functions

Microgrid operating costs include costs associated with energy production and start-up and shut-down decisions, along with possible earnings and curtailment penalties.

1) Microgrid Operating Costs: The cost $V_r(u(k))$ at time step k is given by

$$\begin{aligned} &V_r(u(k)) := \textstyle\sum_{i=1}^{N_g} [C_i^{DG}(P_i(k)) + OM_i\, P_i(k) + SU_i(k)\\ &+SD_i(k)] + C^{\rm g}(k) + OM^b\, |P^b(k)| + \rho_c \textstyle\sum_{h=1}^{N_c} \beta_h(k) D_h^c(k). \end{aligned}$$

Depending on the DG unit, maintenance costs can be also expressed by the term $OM_i\, \delta_i(k)$. The absolute value, $|P^b(k)|$, can be easily expressed as a linear function of $P^b(k)$ and $z^b(k)$ [19]. We approximate each quadratic term $C_i^{DG}(P_i)$ with a convex piecewise affine function, which provides very similar results, but can be entered into a mixed integer linear program.

2) Microgrid Emissions: The total emission of the $i^{\rm th}$ unit can be generally expressed as [20]:

$$E_i(P) = \alpha_i + \beta_i P + \gamma_i P^2 + \zeta_i e^{\xi_i P} \qquad (9)$$

where α_i, β_i, γ_i, ζ_i and ξ_i are nonnegative coefficients of the emission characteristics. The cost related to microgrid emissions at time step k is

$$V_e(u(k)) := \sum_{i=1}^{N_g} E_i(P_i(k)).$$

Each function $E_i(P)$ can be approximated with a piecewise affine function in order to keep the problem linear [18]. We assume that the emissions generated by the utility energy mix is negligible; however, if the corresponding CO_2 signal is known, it can be easily added to (9).

B. Capacity and Terminal Constraints

To pose the final MILP optimization problem, additional operational constraints must be met:

$$x_{\min}^b \leq x^b(k) \leq x_{\max}^b \qquad (10a)$$

$$P_{i,\min}\, \delta_i(k) \leq P_i(k) \leq P_{i,\max}\, \delta_i(k) \qquad (10b)$$

$$|P_i(k+1) - P_i(k)| \leq R_{i,\max} \qquad (10c)$$

$$\beta_{h,\min} \leq \beta_h(k) \leq \beta_{h,\max} \qquad (10d)$$

with $i = 1, \ldots, N_g$ and $h = 1, \ldots, N_c$. The constraints above model the physical bounds on the storage device (inequality (10a)), the power flow limits of the DG units (inequality (10b)) and their ramp up and ramp down rates (inequality (10c)), the bounds on controllable loads curtailments (inequality (10d)).

C. Uncertainty Modeling and Scenario Generation

In order to make the proposed control action effective, the stochastic nature of RES and demand is considered. The uncertainty during the k^{th} sampling time, ω_k, is decomposed as $\omega_k = \bar{\omega}_k + \tilde{\omega}_k$ where $\bar{\omega}_k$ is the forecast and $\tilde{\omega}_k$ is the forecast error at time k. In this work, historical data are employed to train a hidden Markov model (HMM) for generating the finite number of paths, i.e. scenarios, and the corresponding probability of occurrence [21], [22]; the resulting set of paths is called *fan*. To properly decrease the scenario number and make the stochastic problem tractable, scenario reduction algorithms can be applied. The *Backward Reduction Algorithm* has been proved to provide very good performances in the two-stage mixed integer stochastic programming framework [23], [24].

D. A Stochastic Programming Approach with Simple Recourse

In this work, we apply a *stochastic programming with recourse*, also known as *two-stage stochastic programming*, approach to the microgrid operation management under uncertainty.

In the two-stage stochastic programming approach, the decision variables are partitioned into two sets. The first stage variables are those that have to be decided before the actual realization of the uncertainty becomes available; once the random events occur, the values of the second stage or recourse variables can be decided. These recourse variables are also interpreted as *correction actions* as they are used to compensate any infeasibility from the first-stage decisions; thus, violations are accepted, but their costs affect the choice of the first stage variables. The objective is to choose the first stage variables in order to minimize the sum of first stage costs and the expected value of the random second stage or recourse costs [13].

A two-stage linear stochastic program with simple continuous recourse can be stated as follows:

$$\begin{aligned}&\min c^{'}x + \mathcal{Q}(x)\\ &s.t.\\ &\quad Ax = b\\ &\quad x \in \mathcal{X},\end{aligned}$$

where A and b are given matrices of appropriate size, $\omega \in \mathbb{R}^r$ is the random vector with probability distribution P_ω (being independent from x), x is the first stage decision vector and $\mathcal{Q}(x) = \mathbf{E}_\omega[Q(x,\omega)]$, is the *expected recourse*, where $Q(x,\omega)$ is the *second-stage function*:

$$\begin{aligned}&Q(x,\omega) = \min\left(qp^{'}\xi^{+} + qm^{'}\xi^{-}\right)\\ &s.t.\\ &\quad \xi^{+} \geq Hx - \omega\\ &\quad \xi^{-} \geq -(Hx - \omega)\\ &\quad \xi^{+}, \xi^{-} \geq 0,\end{aligned}$$

where qp and qm are penalty coefficient vectors of appropriate size and ξ^{+} and ξ^{-} are the recourse vectors. The random constraints are denoted by the compact form $Hx - \omega = 0$. By considering P_ω as a finite discrete probability distribution with the marginal distribution of ω_i given by $p_{ij} = P_\omega(\omega_i = \hat{\omega}_{ij})$, $j = 1, \dots S_i$, $\mathcal{Q}(x) = \sum_{i=1}^{r} E_{\omega_i} Q(x_i, \omega_i)$ and the problem (11) is equivalent to the following equivalent deterministic linear problem:

$$\begin{aligned}&\min c^{'}x + \textstyle\sum_{i=1}^{r}\sum_{j=1}^{S_i} p_{ij}\left(qp_i^{'}{\xi^{+}}_{ij} + qm_i^{'}{\xi^{-}}_{ij}\right)\\ &s.t.\\ &\quad A\mathbf{x} = b\\ &\quad {\xi^{+}}_{ij} \geq Hx_i - \hat{\omega}_{ij}\\ &\quad {\xi^{-}}_{ij} \geq -(Hx_i - \hat{\omega}_{ij})\\ &\quad {\xi^{+}}_{ij}, {\xi^{-}}_{ij} \geq 0\\ &\quad x \in \mathcal{X}.\end{aligned}$$

In the microgrid scenario, recourse actions are needed when imbalances between energy supply and demand occur: higher demand leads to energy *shortage*, while lower demand results in unexpected energy *surplus*. An example of recourse action in case of energy deficit, is to purchased the needed amount of energy from the utility grid. Hence, we focus on the power balance constraint (5) at each time k, which contains the random variables; all the other constraints can be considered as *first stage constraints*.

Thus, at time k, the second-stage function for the microgrid problem can be expressed as follows:

$$\begin{aligned}&\min \textstyle\sum_{j=1}^{S} p_j\left(qp^{'}\xi_j^{+}(k) + qm^{'}\xi_j^{-}(k)\right)\\ &s.t.\\ &\quad \xi_j^{+}(k) \geq F_k^{'}u(k) - \tilde{\omega}_j(k)\\ &\quad \xi_j^{-}(k) \geq -(F_k^{'}u(k) - \tilde{\omega}_j(k))\\ &\quad \xi_j^{+}(k), \xi_j^{+}(k) \in \mathbb{R}_{+},\end{aligned}$$

where S is the number of scenarios and $\tilde{\omega}(k) = -f^{'}w(k)$.

IV. Stochastic Model Predictive Control problem

In this section the stochastic MPC optimization problem is formulated. We denote by $x^b(k+j|k)$, with $j \geq 0$, the state at time step $k+j$ predicted at time k employing the storage model (3). The initial storage level at time k is denoted by x_k^b and the vectorized input sequence $\left(u^{'}(k)\dots u^{'}(k+T-1)\right)^{'}$ is denoted by $\mathbf{u}_k^{T-1}$, given an initial time step k and a time duration T.

At each time step k, given an initial storage state x_k^b and a time duration T, the MPC scheme computes the optimal control sequence $\mathbf{u}_k^{T-1}$: then, only the first sample of the input sequence is implemented, and subsequently the horizon is shifted. At the next sampling time, the new state of the system is measured or estimated, and a new optimization problem is solved using this new information.

In order to formulate the MPC problem at each time step k, each single term $V_i(u(k))$, with $i \in \{r, e\}$, has to be appropriately normalized [14]. We denote by $\tilde{V}_i(u(k))$ the normalized objective function i. Further, we denote by $\Omega_{j,k}^{T-1}$ the vector containing all the random variables from k to $k+T-1$, for scenario j. We assume a discrete distribution of random variables and a finite number of S scenarios, $\Omega_1, \dots, \Omega_S$, with corresponding probability $p_1, \dots, p_S$. When applying the weighted min-max method, the two stage stochastic problem for microgrid management can be formulated as the following mixed integer linear program:

$$\begin{aligned}&J(x_k^b) = \min\left(\mu + \textstyle\sum_{j=1}^{S} p_j\left(qp^{'}{\xi^{+}}_{j,k}^{T-1} + qm^{'}{\xi^{-}}_{j,k}^{T-1}\right)\right)\\ &\text{s.t.}\\ &\quad \alpha_i \textstyle\sum_{\tau=0}^{T-1}\left(\tilde{V}_i(u(k+\tau)) + \rho\sum_{j\in\{r,e\}}\tilde{V}_j(u(k+\tau))\right) \leq\\ &\quad \leq \mu\\ &\quad \text{storage model (3);}\\ &\quad \text{constraints (6), (7), (8);}\\ &\quad \text{constraints (10);}\\ &\quad {\xi^{+}}_{j,k}^{T-1} \geq Fu_k^{T-1} - \Omega_{j,k}^{T-1}\\ &\quad {\xi^{-}}_{j,k}^{T-1} \geq -(Fu_k^{T-1} - \Omega_{j,k}^{T-1})\\ &\quad {\xi^{+}}_{j,k}^{T-1}, {\xi^{-}}_{j,k}^{T-1} \in {\mathbb{R}_{+}}^{T},\\ &\quad x^b(k|k) = x^b(k),\end{aligned}$$

where μ is a scalar auxiliary variable and $i \in \{r, e\}$, $\alpha_r = \alpha$, $\alpha_e = 1 - \alpha$ and ρ is a sufficiently small positive scalar whose values should be between 0.0001 and 0.01 [14]. If the DGs' fuel consumption and emission functions are nonlinear, continuous variables and linear constraints derived from piecewise affine approximation must be added. The matrix $\mathbf{F}'$ is given by $\mathrm{diag}(\mathrm{F}'_1, \dots, \mathrm{F}'_\mathrm{T})$. The vectors ${\xi^+}_{j,k}^{T-1}$, ${\xi^-}_{j,k}^{T-1}$, $\mathbf{q}^+$ and $\mathbf{q}^-$ contain the recourse variables and their costs over the whole prediction horizon.

V. Simulation results

We had in mind to compare the proposed approach with other approaches proposed in the literature so far. We selected the work [6] for several reasons: (*i*) the authors propose a stochastic approach to microgrid environmental/economic operation optimization; (*ii*) it provides a comparison with related studies, showing that the proposed approach outperforms them; (*iii*) it is possible to retrieve an almost complete set of data. However, there are significant differences, such as, several DG operational constraints are neglected, storage dynamics and grid interaction are described by different constraints, curtailments are not considered, nor a MPC scheme is applied. The system under study is a grid-connected microgrid with four DG units, PV panels and a NiMH battery. Storage efficiencies are not reported in [6], so we consider the typical charging/discharging efficiency of NiMH batteries, which is 0.66. The planning horizon is 24 hours and the sampling time is 1 hours. We compare the results for the case in [6] with S3 and 30 scenarios. Results are reported in Table IV and in Figure 2; they show that our approach yields a better value of the objective functions thanks to a more efficient storage usage and grid interaction.

We want now to investigate our approach on a grid-connected microgrid comprising 10 units, considering all operational constraints described in Section II. The 10 DG units parameters are evaluated using the data available in [25]. The microgrid comprises PV panels and an energy storage, bounded between 50 kWh and 500 kWh and with maximal charge and discharge rates respectively 100 kW and −100 kW. The charge and discharge efficiencies are both equal to 0.9. Bounds on the power exchanged with the utility grid are set to 100 kW.

We compare the following strategies for the microgrid optimization problem:

- Deterministic MPC (D-MPC): the 24h horizon plan obtained by solving the optimization problem stated in Section IV based on demand and PV power generation forecasts computed through a Least-Square Support Vector regressor [26];
- Stochastic MPC (S-MPC): it is the daily microgrid operation plan obtained by solving the stochastic optimization problem problem described in Section IV;
- Benchmark: the 24h horizon plan obtained by solving the optimization problem assuming that the scheduler knows all the future realizations of random variables. The benchmark can be never achieved but it provides an ideal value to evaluate a strategy against.

Demand and energy price data, for the 3 weeks of October 2012, are obtained from the New York ISO (http://www.nyiso.com), while solar generation data are provided by the Elia (http://www.elia.be/en/grid-data). Data for the first 20 days are used for generating load and solar generation scenarios, while the microgrid under three different controllers is simulated for 24 of October. The planning horizon is 24 hours, the sampling period is 15 minutes and the prediction horizon is 96 sampling periods. In order to evaluate the suitability of the scenario generation method described in Section III-C, we performed the stability tests as described in [24]; the results showed that a fan of 50 scenarios can guarantee good performance.

Figure 1 shows the Pareto curve for the three strategies obtained by varying the weights on the objective functions from 0 to 1 with a sampling of 0.1. It can be seen that the S-MPC is significantly closer to the benchmark.

The formulation presented in the previous section was implemented using Matlab. We used ILOG's CPLEX 12.0 to solve the MILP optimizations, which is an efficient solver based on the branch-and-bound algorithm; when it terminates, the solutions are known to be globally optimal. We solve optimization problems of 12672 variables and 155298 constraints. We point out that the average computational time for each time step is 1.7 minutes and in this study the optimal solution is always achieved.

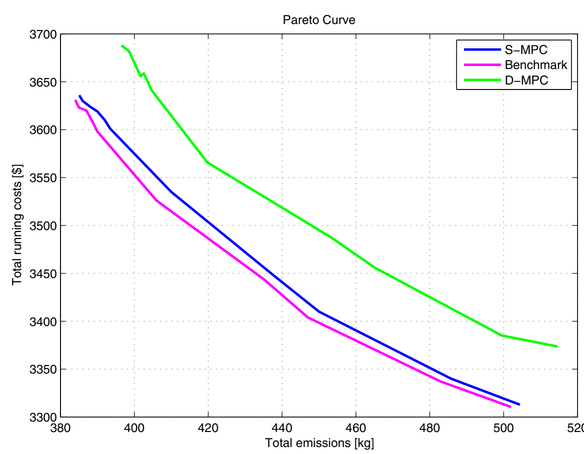

Fig. 1. Pareto curve for the three strategies.

TABLE IV

Comparison results

α_r	α_e	Total running costs *vs* [cent] Total running costs [6]	Total emissions *vs* [kg] Total emissions [6]
1	0	302.65 *vs* 308.58	701.7 *vs* 735.317
0.5	0.5	702.31 *vs* 713.22	520.79 *vs* 564.64
0	1	1290.8 *vs* 1319.99	407.77 *vs* 440.1177

VI. Conclusions and Future Studies

In this paper we present a control-oriented stochastic approach to microgrid modeling and high level economic/environmental optimization. Our main contributions

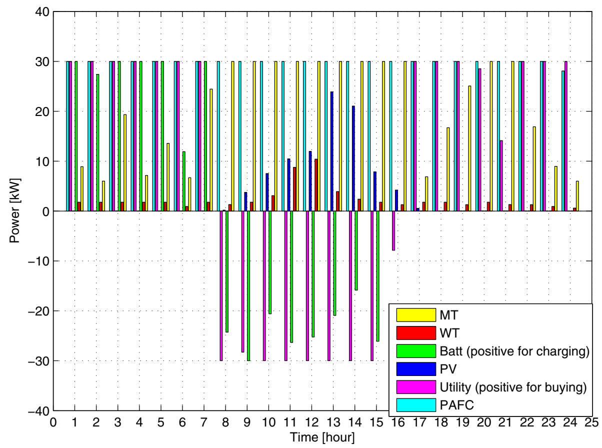

Fig. 2. DG power schedule.

are: (*i*) the storage and grid interaction modeling by using a MLD framework; (*ii*) the development of a an integrated optimization-based framework for microgrid operational planning; (*iii*) the formulation of a microgrid control strategy based on feedback and stochastic optimization.

Currently stability and feasibility properties are under studies though extensive numerical computations have given reassuring results.

Appendix I
Matrices

$$
\begin{aligned}
&E_1^b = \left[C^b \;\; -(C^b+\varepsilon) \;\; C^b \;\; C^b \;\; -C^b \;\; -C^b\right]' \\
&E_2^b = E_2^g = [0 \;\; 0 \;\; 1 \;\; -1 \;\; 1 \;\; -1]' \\
&E_3^b = [1 \;\; -1 \;\; 1 \;\; -1 \;\; 0 \;\; 0]' \\
&E_4^b = \left[C^b \;\; -\varepsilon \;\; C^b \;\; C^b \;\; 0 \;\; 0\right]' \\
&E_1^g = [T^g \;\; -(T^g+\varepsilon) \;\; M^g \;\; M^g \;\; -M^g \;\; -M^g]' \\
&E_3^g(k) = \left[1 \;\; -1 \;\; c^P(k) \;\; -c^P(k) \;\; c^S(k) \;\; -c^S(k)\right]' \\
&E_4^g = [T^g \;\; -\varepsilon \;\; M^g \;\; M^g \;\; 0 \;\; 0]' \\
&F(k) = [\underbrace{1\ldots1}_{Ng} \;\; 1 \;\; \underbrace{\ldots D_i^c(k)\ldots}_{Nc} \;\; \underbrace{0\ldots0}_{Ng}]' \\
&f = [1 \;\; \underbrace{-1\cdots-1}_{Nl} \;\; \underbrace{-1\cdots-1}_{Nc}]',
\end{aligned}
$$

where $M^g = \max_k(c^P(k), c^S(k)) \cdot T^g$, ε is a small tolerance (typically the machine precision).

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