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Controller performance analysis with LQG benchmark obtained under closed loop conditions

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Abstract

This paper proposes a new method for obtaining a linear quadratic Gaussian (LQG) benchmark in terms of the variances of process input and output from closed-loop data, for assessing the controller performance. LQG benchmark has been proposed in the literature to assess controller performance since the LQG tradeoff curve represents the limit of performance in terms of input and output variances. However, an explicit parametric model is required to calculate the LQG benchmark. In this work, we propose a data driven subspace approach to calculate the LQG benchmark under closed-loop conditions with certain external excitations. The optimal LQG-benchmark variances are obtained directly from the subspace matrices corresponding to the deterministic inputs and the stochastic inputs, which are identified using closed-loop data with setpoint excitation. These variances are used for assessing the controller performance. The method proposed in this paper is applicable to both univariate and multivariate systems. Profit analysis for the implementation of feedforward control to the existing feedback-only control system is also analyzed under the optimal LQG performance framework. The proposed method is illustrated through a simulation example and an application on a pilot scale process. © 2002 ISA—The Instrumentation, Systems, and Automation Society.

Keywords: Controller performance assessment; LQG benchmark; Subspace identification; Subspace matrices; Nonparametric models; State space model; Closed-loop identification

1. Introduction

A typical industrial plant can contain thousands of controllers ranging from PI/PID (proportional, integral, and derivative) controllers to the more advanced model predictive controllers like dynamic matrix control (DMC) [1,2], generalized predictive controller (GPC) [3,4], quadratic dynamic matrix controller (QDMC) [5], etc. With a goal towards optimal performance, energy conservation, and cost effectiveness of the process operations in the industry, controller performance assessment has been receiving attention both from the industry and from the academia since the notable work of Harris [6]. Periodic tuning of the controllers becomes an important task of control engineers for obtaining optimal performance from the control systems. Controller performance assessment techniques are used as a tool to check the optimality of the current controller tuning parameters settings. Several benchmarks such as minimum variance control (MVC) [6–21], linear quadratic Gaussian (LQG) control [13,15], and designed controller performance versus achieved controller performance [22–24], etc., have been proposed for assessing the controller performance. Among these approaches, MVC benchmark is one of the popular benchmarks due to its nonintrusive nature for the univariate case and routine closed-

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loop operating data can be used for the calculation of this benchmark. For the univariate application, only *a priori* knowledge of the process time delay is required for obtaining the MVC benchmark from routine operating data [7–9,15,12,25,6,17,] [20,21,26]. For the multivariate case, the calculations are more involved and require estimation of the unitary interactor matrix [13,10,11,12,14,16,] [27–29,19,20,30]. However, the MVC benchmark may not be a practical one for those control systems whose objective is not just minimizing process output variance but also keeping the input variablity (for example, valve movement) within some specified range to reduce upset to other processes, conserve energy, and lessen the equipment wear. The objective of such controllers may be expressed as minimizing a linear quadratic function of input and output variances. The LQG benchmark is a more appropriate benchmark for assessing the performance of such controllers. However, calculation of the LQG benchmark requires a complete knowledge of the process model [13,15], which is a demanding requirement or simply not possible in practice. An open-loop test for obtaining the process model may not always be feasible or may be expensive. The frequency domain approach was proposed by Kammer [31–33] for testing the LQ optimality for the performance assessment of a controller using closed-loop data with setpoint excitation. However, this approach does not give the quantitative values for the controller performance in terms of process input and output variances. In other words, it does not separate the nonoptimality/optimality with respect to process response (output) variance and process input variance. In this paper we propose subspace matrices based approach to obtain the LQG-benchmark variances of the process input and output to be used for the controller performance assessment. The required subspace matrices, those corresponding to the deterministic and stochastic inputs, are estimated from closed-loop data with setpoint excitation. The method proposed is applicable for both univariate and multivariate cases.

Subspace identification methods allow estimation of a state space model for the system directly from the process data. Certain subspace matrices, corresponding to the states, deterministic inputs, and stochastic inputs, are identified as an intermediate step in the subspace identification methods. Several approaches, such as N4SID (Numerical subspace state space identification), MOESP [MIMO (multi-input multi-output) output error state space model identification], and CVA (Canonical variate analysis), are popular for subspace identification using open-loop data. Subspace identification methods also exist for closed-loop data. Recently Van Overschee and De Moor [34] proposed a subspace identification method for the identification of the subspace matrices (all the three, corresponding to the states, deterministic input, and stochastic input) of the process using closed-loop data with the knowledge of the first N impulse response coefficients (Markov parameters for the multivariate systems) of the controller, where N is the maximum order of the state space model we want to identify. MOESP and CVA approaches were also used for the identification of a state space model using closed-loop data [35–37]. In addition to the setpoint excitation, the MOESP/CVA approach uses an external white-noise signal addition to the controller output to make it independent of the noise. The closed-loop state space model is first identified using the closed-loop data from which the open-loop state space matrices are retrieved. Ljung and McKelvey [38] presented a method for the identification of subspace matrices from closed-loop data using estimated predictors and state that their algorithm is an illustration of a "feasible" method rather than the "best way" of identifying systems operating in closed loop. The primary goal of all the above approaches is the identification of a state space model for the open-loop system.

Favoreel and co-workers [39–41] have recently proposed a method for the design of optimal LQG controllers directly from the subspace matrices, instead of using a state space model. Recent work by Kadali and Huang [42] allows identification of (only two of the subspace matrices, corresponding to) the deterministic subspace matrix and stochastic subspace matrix from closed-loop data without requiring any *a priori* knowledge of the controllers. This method requires set point excitation and is also extended to the case of measured disturbances [42]. These methods provide tools/means for the calculation of more practical controller performance benchmarks like LQG benchmark using closed-loop data. As will be shown later in this paper, the explicit process model is not required for obtaining the LQG benchmark.

The method for designing the optimal LQG controller directly from subspace matrices proposed in Refs. [39–41] is extended in this paper to the case of feedforward plus feedback control. If some of the disturbance variables are measurable, analysis of feedforward control performance is a worthwhile study. However, this analysis requires the subspace matrix corresponding to the measured disturbance variables. Using the subspace approach proposed in Ref. [42] the subspace matrix corresponding to the measured disturbance variables can also be estimated under closed-loop conditions, if the measured disturbances are assumed to be uncorrelated with the setpoint changes. This provides a means for the profit analysis of implementing feedforward control on the process.

The main contributions of this paper in the order of presentation are (i) derivation of the expressions for the calculation of the optimal LQG-benchmark variances of the process input and output directly from the subspace matrices, (ii) extension of the design of optimal LQG controllers using subspace matrices proposed in Refs. [39–41] to the feedforward plus feedback control case, (iii) extension of the analysis to the case of feedforward controller performance analysis, and (iv) illustration of the proposed method through an application on a pilot scale process.

This paper is arranged as follows. Section 2 explains the design of the LQG controller directly from the subspace matrices. Section 3 is the main section where the methodology of obtaining the LQG-benchmark variance for the process input and output from closed-loop data is presented. Incorporation of feedforward control in the optimal LQG control is discussed in Section 4. Controller performance analysis indices are defined and described in Section 5. A summary of the proposed method is presented in Section 6. Simulation results are presented in Section 7 followed by an application on a pilot scale process in Section 8. Conclusions are provided in Section 9. A brief review of the open-loop subspace identification methods is provided in Appendix A. Appendix C explains the closed-loop subspace identification method from Ref. [42].

2. Designing LQG controller using subspace matrices

A linear time-invariant system can be described in a state space innovations form as

$$x_{k+1}=Ax_k+Bu_k+Ke_k, \tag{1}$$

$$y_k=Cx_k+Du_k+e_k, \tag{2}$$

where x_k, y_k, u_k, and e_k are the process states, outputs, deterministic inputs, and stochastic inputs, respectively. K is the Kalman filter gain and e_k is an unknown innovations sequence of white noise with the covariance matrix S. For an l-input and m-output system, A, B, C, D, K, and S are $(n\times n)$, $(n\times l)$, $(m\times n)$, $(m\times l)$, $(n\times m)$, and $(m\times m)$ matrices, respectively, where n is the state order.

The predictor equations (also called matrix input-output equations in subspace identification [43]) for the output of system in Eqs. (1) and (2) can be expressed as

$$y_f=\Gamma_N x_{t+1}+H_N u_f+H_N^s e_f, \tag{3}$$

$$=L_w w_p+L_u u_f+L_e e_f \tag{4}$$

where

$$y_f=\begin{bmatrix} y_{t+1}\\ \cdots \\ y_{t+N}\end{bmatrix};\quad u_f=\begin{bmatrix} u_{t+1}\\ \cdots \\ u_{t+N}\end{bmatrix};\quad w_p=\begin{bmatrix} y_p\\ u_p\end{bmatrix};$$

$$y_p=\begin{bmatrix} y_{t-N+1}\\ \cdots \\ y_t\end{bmatrix};\quad u_p=\begin{bmatrix} u_{t-N+1}\\ \cdots \\ u_t\end{bmatrix}$$

and Γ_N $(Nm\times n)$ is the extended observability matrix, H_N $(Nm\times Nl)$ and H_N^s $(Nm\times Nm)$ are the lower triangular Toeplitz matrices containing the impulse response coefficients (Markov parameters) corresponding to the deterministic input u_k and the unknown stochastic input e_k, respectively. p and f denote the past and the future, respectively. The subscript N follows from the number of steps ahead predictions represented in y_f. L_w $[Nm\times N(l+m)]$, L_u $(Nm\times Nl)$, and L_e $(Nm\times Nm)$ are the subspace matrices corresponding to the states, the deterministic inputs, and the stochastic inputs, respectively.

$$\Gamma_N=[C^T\ \ (CA)^T\ \ \cdots\ \ (CA^{N-1})^T]^T;$$

$$H_N=\begin{bmatrix} D & 0 & \cdots & 0\\ CB & D & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots\\ CA^{N-2}B & CA^{N-3}B & \cdots & \cdots\end{bmatrix};$$

$$H_N^s = \begin{bmatrix} I_m & 0 & \cdots & 0 \\ CK & I_m & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ CA^{N-2}K & CA^{N-3}K & \cdots & \cdots \end{bmatrix}.$$

Recent work by Refs. [40,39,41] shows the design of the optimal LQG controller by directly using the subspace matrices, instead of through a state space model, for the system in Eqs. (1) and (2). The linear quadratic Gaussian (LQG) controller is designed to minimize the following quadratic cost function J over the horizon N:

$$J = E\left\{ \sum_{k=1}^{N} [(y_{t+k} - r_{t+k})^T (y_{t+k} - r_{t+k}) + u_{t+k}^T (\lambda I_l) u_{t+k}] \right\} \quad (5)$$

$$= \sum_{k=1}^{N} [(\hat{y}_{t+k} - r_{t+k})^T (\hat{y}_{t+k} - r_{t+k}) + u_{t+k}^T (\lambda I_l) u_{t+k}], \quad (6)$$

where E is the expectancy operator, λ is the user defined non-negative input weighting parameter, and r_t is the reference for output trajectory. I_l is an l-order identity matrix. $\hat{y}_{t+k}$ is the k-step ahead predicted output given the past inputs and outputs and future inputs up to time t.

It should be noted that traditionally the following objective function is used for the design of the LQG controllers:

$$J = E\left\{ \sum_{k=1}^{N} [(y_{t+k} - r_{t+k})^T R (y_{t+k} - r_{t+k}) + u_{t+k}^T Q u_{t+k}] \right\}, \quad (7)$$

where R $(m \times m)$ and Q $(l \times l)$ are non-negative definite weighting matrices. To simplify the presentation the objective function in Eq. (5) is used throughout this paper. Eq. (5) basically means that all the inputs have the same weighting (or equal importance) in minimizing the objective function.

The optimal predictor equation from Eq. (4) is

$$\hat{y}_f = L_w w_p + L_u u_f. \quad (8)$$

The notation in the cost function can be simplified for regulatory control, by letting $r_{t+k} = 0$, as

$$J = \min_{u_f^2} [\hat{y}_f^T \hat{y}_f + u_f^T (\lambda I_{Nl}) u_f] \quad (9)$$

$$= (L_w w_p + L_u u_f)^T (L_w w_p + L_u u_f) + u_f^T (\lambda I_{Nl}) u_f. \quad (10)$$

Partial differentiation of J with respect to u_f and setting it to zero yields the LQG control law [40] as

$$u_f = -(\lambda I_{Nl} + L_u^T L_u)^{-1} L_u^T L_w w_p. \quad (11)$$

The above control law is the optimal LQG control law as $N \to \infty$ and is equivalent to an estimated state feedback control law,

$$u_f = -C \hat{x}_{t+1}, \quad (12)$$

where

$$C = (\lambda I_{Nl} + L_u^T L_u)^{-1} L_u^T \Gamma_N \quad (13)$$

and the relation $L_w w_p = \Gamma_N \hat{x}_{t+1}$ follows from Eqs. (3) and (4). Only the first control move is implemented and the calculation is repeated at each sampling interval.

3. Obtaining LQG benchmark from closed-loop data

To assess the controller performance, we compare the current controller performance, in terms of output (or input or both) variances, with the variances under the optimal control. This gives rise to the question of selection of the optimal benchmark controller. Though the primary objective of a control system is often to minimize the output variance, we may also want to limit input variance for reasons like energy conservation and equipment wear. In other words, a compromise between the process input variance and output variance is necessary. The optimal LQG control is one of such benchmarks that takes into account both input and output variances of the process and represents a limit of performance in terms of input and output variances [44,13,15].

To obtain the optimal LQG-benchmark variances we need to obtain the closed-loop expressions for process input and output in terms of the disturbances entering the process. From the predictor equation (4), we can write y_{t+1} in terms of the past inputs and past outputs as

$$y_{t+1}=l_{y_p}y_p+l_{u_p}u_p+g_0u_{t+1}+l_0e_{t+1}, \quad (14)$$

where

$$l_{y_p}=L_w(1:m,1:mN), \quad (15)$$

$$l_{u_p}=L_w[1:m,mN+1:(l+m)N], \quad (16)$$

and the notation $A(i:j,p:q)$ represents the rows i to j and columns p to q of the matrix A. Equation (14) can be transformed to alternatively express the process output in terms of past inputs and past noise with the process and noise model impulse response coefficients. Without giving a detailed mathematical derivation, the result is presented here as

$$y_{t+1}=[g_1\cdots g_N]\begin{bmatrix} u_t \\ \cdots \\ u_{t-N+1}\end{bmatrix} +[l_1\cdots l_N]\begin{bmatrix} e_t \\ \cdots \\ e_{t-N+2}\end{bmatrix}+g_0u_{t+1}+l_0e_{t+1}, \quad (17)$$

where g_i and l_i are the ith impulse response coefficients (Markov parameters for multivariate systems) of the process and noise models, respectively. In other words, we can express the past (state) contribution term $L_w w_p$ as

$$l_w w_p=[g_1\cdots g_N]\begin{bmatrix} u_t \\ \cdots \\ u_{t-N+1}\end{bmatrix}+[l_1\cdots l_N] \times\begin{bmatrix} e_t \\ \cdots \\ e_{t-N+2}\end{bmatrix}$$

$$\Rightarrow L_w w_p = \begin{bmatrix} g_1 & \cdots & g_{N-1} & g_N \\ g_2 & \cdots & g_N & 0 \\ \cdots & \cdots & \cdots & \cdots \\ g_N & 0 & \cdots & 0 \end{bmatrix}\begin{bmatrix} u_t \\ \cdots \\ u_{t-N+1}\end{bmatrix} + \begin{bmatrix} l_1 & \cdots & l_{N-1} & l_N \\ l_2 & \cdots & l_N & 0 \\ \cdots & \cdots & \cdots & \cdots \\ l_N & 0 & \cdots & 0 \end{bmatrix}\begin{bmatrix} e_t \\ \cdots \\ e_{t-N+1}\end{bmatrix}. \quad (18)$$

However, the controller output u_{t+1} is calculated using all the data available at time $t+1$, i.e., $\{u_t, y_{t+1}, u_{t-1}, y_t, \cdots\}$. Hence the original subspace predictor expression in Eq. (4) and the subspace based LQG-controller law in Eq. (11) have to be modified to obtain the closed-loop expressions for u_f and y_f. First, define

$$K=(\lambda I_{Nl}+L_u^T L_u)^{-1}L_u^T;$$

$$L_g=\begin{bmatrix} g_1 & g_2 & \cdots & g_{N-1} & g_N \\ g_2 & g_3 & \cdots & g_N & 0 \\ \cdots & \cdots & \cdots & \cdots & \\ g_N & 0 & 0 & \cdots & 0 \end{bmatrix};$$

$$L_h=\begin{bmatrix} l_0 & l_1 & \cdots & l_{N-1} & l_N \\ l_1 & l_2 & \cdots & l_N & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ l_{N-1} & 0 & 0 & \cdots & 0 \end{bmatrix};$$

$$\tilde{L}_e=\begin{bmatrix} 0 & 0 & \cdots & 0 \\ l_0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ l_N & l_{N-1} & \cdots & 0 \end{bmatrix};$$

$$\tilde{u}_p=\begin{bmatrix} u_t \\ u_{t-1} \\ \cdots \\ u_{t-N+1}\end{bmatrix};$$

$$\tilde{e}_p=\begin{bmatrix} e_{t+1} \\ e_t \\ \cdots \\ e_{t-N+1}\end{bmatrix}; \quad \tilde{e}_f=\begin{bmatrix} e_{t+2} \\ e_{t+3} \\ \cdots \\ e_{t+N+1}\end{bmatrix}.$$

Note that the matrices L_g and L_h contain the impulse response coefficients (Markov parameters) for the deterministic and stochastic inputs and can be formed using the subspace matrices L_u and L_e, respectively. From Eq. (11), we can write

$$u_f = -(\lambda I_{Nl} + L_u^T L_u)^{-1} L_u^T L_w w_p \tag{19}$$

$$= -K\{L_g \tilde{u}_p + L_h \tilde{e}_p\}. \tag{20}$$

Similarly substituting Eq. (20) in Eq. (4) we can write

$$y_f = L_g \tilde{u} + L_h \tilde{e} + L_u u_f + L_e e_f \tag{21}$$

$$= (I - L_u K) L_g \tilde{u}_p + (I - L_u K) L_h \tilde{e}_p + \tilde{L}_e \tilde{e}_f. \tag{22}$$

Now we have derived closed-loop expressions for both u and y; the next step is to calculate their variance expressions which are actually the H_2 norm of the closed-loop expressions weighted by the variance of e. A simple method to derive the variance expression is given below.

Let a disturbance enter the process at time $= t+1$, i.e.,

$$u_t = u_{t-1} = \cdots = u_{t-N+1} = 0,$$

$$e_t = e_{t-1} = \cdots = e_{t-N+1} = 0,$$

$$e_{t+2} = e_{t+3} = \cdots = e_{t+N} = 0.$$

Then according to Eqs. (20) and (22), we have

$$u_f = -K l_e e_{t+1} = \begin{bmatrix} \psi_0 \\ \psi_1 \\ \cdots \\ \psi_{N-1} \end{bmatrix} e_{t+1}, \tag{23}$$

$$y_f = (I - L_u K) l_e e_{t+1} = \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \cdots \\ \gamma_{N-1} \end{bmatrix} e_{t+1}, \tag{24}$$

where

$$l_e = \begin{bmatrix} l_0 \\ \cdots \\ l_{N-1} \end{bmatrix},$$

the vector of noise model impulse response coefficients/Markov parameters. From the above equations, we can calculate the LQG-benchmark variances of the process input and output as

$$\mathrm{Var}[u_t] = \sum_{i=0}^{N-1} \psi_i \mathrm{Var}[e_t] \psi_i^T, \tag{25}$$

$$\mathrm{Var}[y_t] = \sum_{i=0}^{N-1} \gamma_i \mathrm{Var}[e_t] \gamma_i^T. \tag{26}$$

As can be seen from the above equations only the subspace matrices L_u and L_e are required for obtaining the LQG-benchmark variances of the process input and output. The state subspace matrix, L_w, is not required. Therefore the closed-loop subspace identification method presented in Appendix C can be used for obtaining the optimal LQG control variances of process input and output.

For obtaining the LQG-benchmark limit curve, define

$$u_{lqg} = \mathrm{trace}\{\mathrm{Var}[u_t]\}, \tag{27}$$

$$y_{lqg} = \mathrm{trace}\{\mathrm{Var}[y_t]\}. \tag{28}$$

For different values of λ, the values for u_{lqg} and y_{lqg} are obtained. A plot of u_{lqg} vs y_{lqg} represents the optimal LQG performance limit curve that can be used for controller performance assessment.

4. Profit analysis of feedforward control

For the case of measured disturbance variables, obtaining optimal benchmark variances helps us in analyzing two things: (i) performance assessment of an existing feedforward plus feedback controller and (ii) profit analysis of implementing a feedforward controller on the process. This analysis in terms of process output variance using the MVC benchmark is provided in Refs. [13,28,29,19]. In this section we provide the analysis, in terms of both process output variance and process input variance, using the LQG benchmark.

Consider the case when measurements of some of the disturbance variables, v_t $(h \times 1)$, are available where v_t is assumed to be white noise; if this assumption is not true, then prewhitening is needed. The process state space representation (1) and (2) is modified to include measured disturbances as

$$x_{k+1} = A x_k + [B \ \ B_v] \begin{bmatrix} u_k \\ v_k \end{bmatrix} + K e_k, \tag{29}$$

$$y_k = Cx_k + [D \;\; D_v]\begin{bmatrix} u_k \\ v_k \end{bmatrix} + e_k. \tag{30}$$

Similarly, the predictor equations for the process output can be expressed as

$$y_f = \Gamma_N^b x_{t+1} + H_N u_f + H_N^v v_f + H_N^s e_f \tag{31}$$

$$= L_w^b w_p^b + L_u u_f + L_v v_f + L_e e_f, \tag{32}$$

where

$$v_p = \begin{bmatrix} v_{t-N+1} \\ \cdots \\ v_t \end{bmatrix}; \;\; v_f = \begin{bmatrix} v_{t+1} \\ \cdots \\ v_{t+N} \end{bmatrix}; \;\; W_p^b = \begin{bmatrix} Y_p \\ U_p \\ V_p \end{bmatrix}. \tag{33}$$

The optimal LQG control law, as $N \to \infty$, for feedback plus feedforward control, modifies to

$$u_f = -(\lambda I_{Nl} + L_u^T L_u)^{-1} L_u^T L_w^b w_p^b \tag{34}$$

$$= -C^b \hat{x}_{t+1}, \tag{35}$$

where

$$C^b = (\lambda I_{Nl} + L_u^T L_u)^{-1} L_u^T \Gamma_N^b. \tag{36}$$

The relation $L_w^b w_p^b = \Gamma_N^b \hat{x}_{t+1}$ follows from the comparison between subspace equations (31) and (32). Note that L_v is not required in the design of the controller and hence need not be calculated for implementing the controller. However, L_v is required for obtaining the LQG benchmark.

Similar to the previous section, define

$$L_{g^v} = \begin{bmatrix} g_0^v & g_1^v & \cdots & g_{N-1}^v & g_N^v \\ g_1^v & g_2^v & \cdots & g_N^v & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ g_{N-1}^v & 0 & 0 & \cdots & 0 \end{bmatrix};$$

$$\tilde{L}_e = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ g_0^v & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ g_N^v & g_{N-1}^v & \cdots & 0 \end{bmatrix};$$

$$\tilde{v}_p = \begin{bmatrix} v_{t+1} \\ v_t \\ \cdots \\ v_{t-N+1} \end{bmatrix}; \;\; \tilde{v}_f = \begin{bmatrix} v_{t+2} \\ v_{t+3} \\ \cdots \\ v_{t+N+1} \end{bmatrix},$$

where g_i^v is the ith impulse response coefficient (Markov parameter for multivariate systems) of the disturbance model corresponding to v_t. L_{g^v} can be formed from the subspace matrix L_v. Equations (34) and (32) can be written as

$$u_f = -K\{L_g u_p + L_{g^v} v_p + L_h e_p\}, \tag{37}$$

$$y_f = (I - L_u K) L_g \tilde{u}_p + (I - L_u K) L_{g^v} \tilde{v}_p + (I - L_u K) L_h \tilde{e}_p + \tilde{L}_v \tilde{v}_f + \tilde{L}_e \tilde{e}_f \tag{38}$$

Consider the measured and unmeasured disturbances enter the process at time $= t+1$, i.e.,

$$u_t = u_{t-1} = \cdots = u_{t-N+1} = 0,$$

$$v_t = v_{t-1} = \cdots = v_{t-N+1} = 0,$$

$$e_t = e_{t-1} = \cdots = e_{t-N+1} = 0,$$

$$v_{t+2} = v_{t+3} = \cdots = v_{t+N} = 0,$$

$$e_{t+2} = e_{t+3} = \cdots = e_{t+N} = 0.$$

Therefore

$$u_f = -K l_v v_t - K l_e e_t = \begin{bmatrix} \omega_0 \\ \omega_1 \\ \cdots \\ \omega_{N-1} \end{bmatrix} v_{t+1} + \begin{bmatrix} \psi_0 \\ \psi_1 \\ \cdots \\ \psi_{N-1} \end{bmatrix} e_{t+1}, \tag{39}$$

$$y_f = (I - L_u K) l_v v_t + (I - L_u K) l_e e_t = \begin{bmatrix} \Upsilon_0 \\ \Upsilon_1 \\ \cdots \\ \Upsilon_{N-1} \end{bmatrix} v_{t+1} + \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \cdots \\ \gamma_{N-1} \end{bmatrix} e_{t+1}, \tag{40}$$

where

$$l_v = \begin{bmatrix} g_0^v \\ \cdots \\ g_{N-1}^v \end{bmatrix} \text{ and } l_e = \begin{bmatrix} l_0 \\ \cdots \\ l_{N-1} \end{bmatrix},$$

the vectors of noise model impulse response coefficients/Markov parameters of measured and unmeasured disturbances, respectively. From the above equations, we can calculate the LQG-benchmark variances of the process input and output as

$$\mathrm{Var}[u_t]=\sum_{i=0}^{N-1}\omega_i \mathrm{Var}[v_t]\omega_i^T+\sum_{i=0}^{N-1}\psi_i \mathrm{Var}[e_t]\psi_i^T, \tag{41}$$

$$\mathrm{Var}[y_t]=\sum_{i=0}^{N-1}\Upsilon_i \mathrm{Var}[v_t]\Upsilon_i^T+\sum_{i=0}^{N-1}\gamma_i \mathrm{Var}[e_t]\gamma_i^T. \tag{42}$$

Hence only the subspace matrices L_u, L_v, and L_e are required for obtaining the LQG-benchmark variances for the process input and output. Now,

$$u_{lqg}=\mathrm{trace}\{\mathrm{Var}[u_t]\}, \tag{43}$$

$$y_{lqg}=\mathrm{trace}\{\mathrm{Var}[y_t]\} \tag{44}$$

By plotting u_{lqg} vs y_{lqg} for different values of λ, as explained in the previous section, an LQG feedforward plus feedback controller limit curve can be plotted. It can be compared with the feedback-only optimal LQG performance limit curve to analyze the benefits of implementing feedforward control. Further discussion is provided in the next section.

5. Controller performance analysis

One of the advantages of LQG benchmark is that the controller performance can be assessed in terms of both process response (output) variance and the process input variance. The LQG tradeoff curve in Fig. 1 represents the limit of controller performance, in terms of process input and output variances [44]. That is to say, all linear controllers (from PID, MPC, to any advanced control) can only operate in the region above the curve. Several useful performance indices in the analysis of the controller performance can be obtained from the LQG-benchmark curve.

5.1. Case 1: Feedback controller acting on the process and no measured disturbances

Consider the case when a feedback-only controller is acting on the process and the actual input and output variances are represented as $(V_u)^{fb}$ and $(V_y)^{fb}$, respectively. The closer $(V_u)^{fb}$ and $(V_y)^{fb}$ are to the limit curve, the closer is the controller performance to the optimal LQG controller. If the optimal output variance corresponding to $(V_u)^{fb}$ is $(V_y^o)^{fb}$ and the optimal input variance corresponding to $(V_y)^{fb}$ is $(V_u^o)^{fb}$, then the LQG performance indices can be defined in terms of process response variance, $(\eta)^{fb}$, and process input variance, $(E)^{fb}$, as

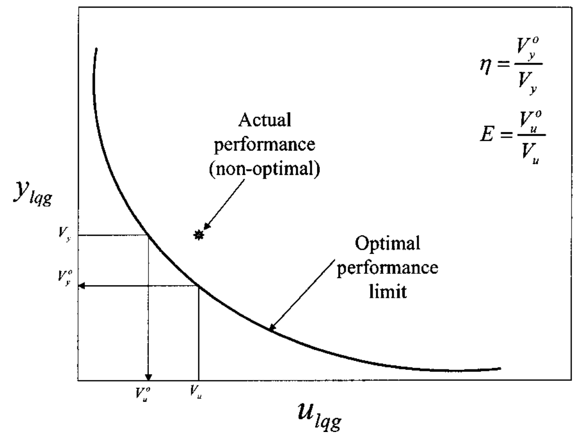

Fig. 1. Optimal LQG control performance limit curve. V_u and V_y represent the variances obtained from process data while V_u^o and V_y^o represent the optimal LQG-benchmark variances.

$$(\eta)^{fb}=\frac{(V_y^o)^{fb}}{(V_y)^{fb}}; \qquad (E)^{fb}=\frac{(V_u^o)^{fb}}{(V_u)^{fb}}. \tag{45}$$

$(\eta)^{fb}$ and $(E)^{fb}$ vary between 0 and 1. If $(\eta)^{fb}$ is equal to 1, for the given input variance, then the controller is giving optimal performance with respect to the process variance. If not, then the controller is nonoptimal and there is scope for improvement in terms of process response. Similarly if $(E)^{fb}$ is equal to 1, for the given output variance, then the controller is giving optimal performance with respect to the input variance. If not, then the controller is nonoptimal and there is scope to reduce input variance.

The maximum possible percent improvement in controller performance with respect to process response variance without increasing the input variance, by retuning the controller, can be calculated as

$$(I_\eta)^{fb}=\frac{(V_y)^{fb}-(V_y^o)^{fb}}{(V_y)^{fb}}100\%. \tag{46}$$

Similarly the maximum possible percent improvement in controller performance with respect to input variance without increasing the output variance, by retuning the controller, can be calculated as

$$(I_E)^{fb}=\frac{(V_u)^{fb}-(V_u^o)^{fb}}{(V_u)^{fb}}100\%. \qquad (47)$$

5.2. Case 2: Feedforward plus feedback controller acting on the process

For the case of measured disturbances, and a feedforward (FF) plus feedback (FB) controller acting on the process, let the actual input and output variances be denoted by $(V_u)^{ff\&fb}$ and $(V_y)^{ff\&fb}$, respectively. Then the LQG curve is plotted using $\{L_u,\ L_e,$ and $L_v\}$,[1] and represents the limit of performance in terms of process input and output variances for a feedforward plus feedback controller. Let the optimal output variance corresponding to $(V_u)^{ff\&fb}$ be $(V_y^o)^{ff\&fb}$ and the optimal input variance corresponding to $(V_y)^{ff\&fb}$ be $(V_u^o)^{ff\&fb}$, then the optimal FF-FB LQG performance indices can be defined in terms of process response variance, $(\eta)^{ff\&fb}$, and process input variance, $(E)^{ff\&fb}$, as

$$(\eta)^{ff\&fb}=\frac{(V_y^o)^{ff\&fb}}{(V_y)^{ff\&fb}};$$

$$(E)^{ff\&fb}=\frac{(V_u^o)^{ff\&fb}}{(V_u)^{ff\&fb}}. \qquad (48)$$

$(\eta)^{ff\&fb}$ and $(E)^{ff\&fb}$ vary between 0 and 1. If $(\eta)^{ff\&fb}$ is equal to 1, then the controller is giving optimal feedforward plus feedback controller performance, for the given input variance. If not, then the controller is nonoptimal and has potential for improvement by retuning. Similarly, if $(E)^{ff\&fb}$ is equal to 1, then the controller is giving optimal feedforward plus feedback controller performance, for the given output variance.

The maximum possible percent improvement in the controller performance, with respect to process response variance without increasing the input variance, by retuning the controller, is calculated for the feedforward plus feedback control case as

$$(I_\eta)^{ff\&fb}=\frac{(V_y)^{ff\&fb}-(V_y^o)^{ff\&fb}}{(V_y)^{ff\&fb}}100\%. \qquad (49)$$

Similarly, we can define in terms of the input variance,

$$(I_E)^{ff\&fb}=\frac{(V_u)^{ff\&fb}-(V_u^o)^{ff\&fb}}{(V_u)^{ff\&fb}}100\%. \qquad (50)$$

5.3. Case 3: Feedback controller acting on the process and measured disturbances being available

Consider the case where a feedback-only controller is acting on the process and measured disturbance variables are available. We want to know how much improvement in the controller performance is possible by implementing a feedforward control in addition to the existing feedback-only controller. By implementing optimal feedforward control on the system the process response variance will decrease. The same may not hold for the input variance. The process input variance may increase or decrease by the implementation of feedforward control with the measured disturbances. The following analysis helps in determining the incentive for the implementation of a feedforward controller on the process.

We can obtain $(V_u)^{fb}$ and $(V_y)^{fb}$ from process data. We construct two LQG limit curves. (i) Identify $\{L_u,\ L_v,$ and $L_e\}$ and construct the FF and FB LQG controller limit curve to obtain $(V_y^o)^{ff\&fb}$ and $(V_u^o)^{ff\&fb}$. (ii) Treat measured and unmeasured disturbances as a lumped set of $(m\times 1)$ unmeasured disturbances and identify $\{L_u$ and $L_e\}$ and construct the FB-only LQG controller limit curve to obtain $(V_y^o)^{fb}$ and $(V_u^o)^{fb}$.

The maximum possible improvement in the optimal controller performance with the implementation of an optimal feedforward controller is obtained in terms of the process response variance and process input variance as

$$\frac{(V_y^o)^{fb}-(V_y^o)^{ff\&fb}}{(V_y)^{fb}}100\% \qquad (51)$$

and

[1]It should be noted that the subspace matrix corresponding to the measured disturbances L_v cannot be identified when a feedforward plus feedback controller is acting on the process [42]. A feedback-only controller should be acting on the process for identifying L_v.

$$\frac{(V_u^o)^{fb}-(V_u^o)^{ff\&fb}}{(V_u)^{fb}}100\%, \qquad (52)$$

respectively.

The performance analysis indices presented for three different cases above can be used in analyzing the incentives, in terms of decreasing both process response variance and process input variance, for retuning the controller.

6. Summary of the subspace matrices approach to the calculation of the LQG benchmark

Controller performance analysis using the LQG benchmark involves comparing the current process input and output variances with the variances if an LQG controller were implemented on the process. The method proposed in this paper allows the calculation of the LQG-benchmark variances directly from the deterministic and stochastic process subspace matrices, thus not requiring a parametric model, and principally consists of the following steps:

1. Estimation for the deterministic and stochastic process subspace matrices from the process data. The subspace matrices can be identified by either

(a) Using the process open-loop data [45,43] as shown in Appendix A.

(b) Using the process closed-loop data with setpoint excitation [42] as shown in Appendix C.

2. Estimation of the process stochastic noise and obtaining the variance Var$[e_t]$. Also estimate Var$[v_t]$ if any measured disturbances are available; if the measured disturbance is not white noise, then prewhitening is necessary. Routine operating data can be used for this purposes.

3. For different values of λ calculate the LQG-benchmark variances u_{lqg} and y_{lqg}. Plot u_{lqg} vs y_{lqg} to obtain the optimal LQG performance limit curve.

4. For the current process input and output variances, V_u and V_y, respectively, obtain the optimal variance values V_u^o and V_y^o, for both feedback-only and feedforward plus feedback control cases. Calculate the controller performance analysis indices,

$$(\eta)^{fb}, \qquad (E)^{fb},$$

$$(I_\eta)^{fb}, \qquad (I_E)^{fb},$$

$$(\eta)^{ff\&fb}, \qquad (E)^{ff\&fb},$$

$$(I_\eta)^{ff\&fb}, \qquad (I_E)^{ff\&fb}.$$

7. Simulations

Consider the following state space system (modified from the example in Ref. [43]):

$$x_{k+1}=\begin{bmatrix}0.6 & 0.6 & 0\\ -0.6 & 0.6 & 0\\ 0 & 0 & 0.7\end{bmatrix}x_k+\begin{bmatrix}1.6161\\ -0.3481\\ 2.6319\end{bmatrix}u_k$$

$$+\begin{bmatrix}0.5\\ -0.5\\ 0.4\end{bmatrix}v_k+\begin{bmatrix}-1.1472\\ -1.5204\\ -3.1993\end{bmatrix}e_k,$$

$$y_k=[-0.4373 \;\; -0.5046 \;\; 0.0936]x_k$$

$$+[-0.7759]u_k+[-0.5]v_k+e_k.$$

A process time delay of three samples is introduced for the above system in MATLAB-*Simulink*. A PID controller, $[0.1+0.05/s+0.05s]$, is tuned for the above system. We assume the controller knowledge is not known.

Closed-loop input/output data is obtained by exciting the system using a designed random binary signal (RBS) signal (with *idinput* function in MATLAB), with bandpass limits [0 0.06] and magnitude 1, for the setpoint. The measured disturbance and the unmeasured disturbance are random white noise with standard deviations 0.2 and 0.1, respectively. Note that although the measured and unmeasured disturbances can have the same standard deviation, different *seeds* have to be used for generating them in MATLAB. Using closed-loop subspace identification [42], with $i=30$ (rows) and $j=3000$ (columns) in the data Hankel matrices, the subspace matrices L_u (30×30) and L_e (30×30) are identified. Due to the presence of noise, the upper nondiagonal elements in L_u and L_e will not be exactly zero but negligibly small numbers (they approach to zero as $N\rightarrow\infty$). An optimal feedback-only LQG controller is considered as a benchmark for controller performance assessment. For a range of values of λ (1–30), the LQG-benchmark variances of the process input and output are obtained for both feedback-only control case and feedforward plus feedback control case

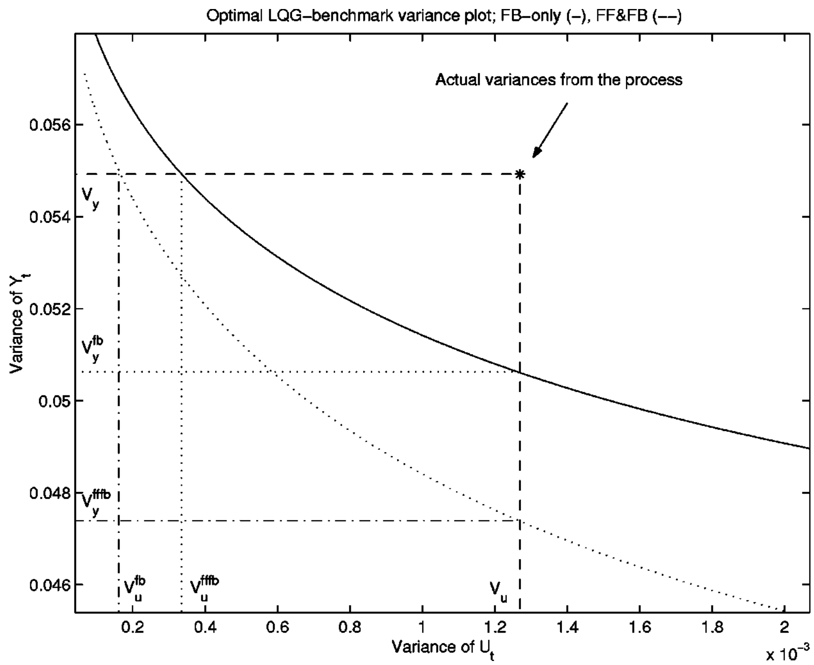

Fig. 2. Optimal LQG control performance limit curve.

and plotted in Fig. 2. The controller performance analysis parameters are obtained from Fig. 2 (see Table 1).

We make the following observations from Table 1.

From feedback-only LQG limit curve

From the FB-only optimal LQG-benchmark variances we see that although the controller performance is close to optimal with respect to the process output variance (92.17% of the optimal), the performance index with respect to the input variability is only 13.85% (see Fig. 3). Hence there is a maximum possible scope of 86.15% to reduce the input variance without increasing the output variance (see Fig. 4).

Table 1
Controller performance analysis parameters.

Parameter		Value	Parameter		Value
$(V_y)^{fb}$	.....	0.0549			
$(V_u)^{fb}$	.....	13×10^{-4}			
$(V_y^o)^{fb}$	.....	0.0506	$(V_y^o)^{ff\&fb}$	.....	0.0474
$(V_u^o)^{fb}$	.....	1.8×10^{-4}	$(V_u^o)^{ff\&fb}$	.....	8.2×10^{-4}
$(\eta)^{fb}$	.....	0.9217	$(\eta)^{ff\&fb}$	.....	0.8634
$(I_\eta)^{fb}$	.....	07.83%	$(I_\eta)^{ff\&fb}$	.....	13.66%
$(E)^{fb}$	.....	0.1385	$(E)^{ff\&fb}$	.....	0.6308
$(I_E)^{fb}$	.....	86.15%	$(I_\eta)^{ff\&fb}$	.....	36.92%
			$\frac{(V_y^o)^{fb}-(V_y^o)^{ff\&fb}}{(V_y)^{fb}}100\%$	.....	05.83%
			$\frac{(V_u^o)^{fb}-(V_u^o)^{ff\&fb}}{(V_u)^{fb}}100\%$	.....	-49.23%

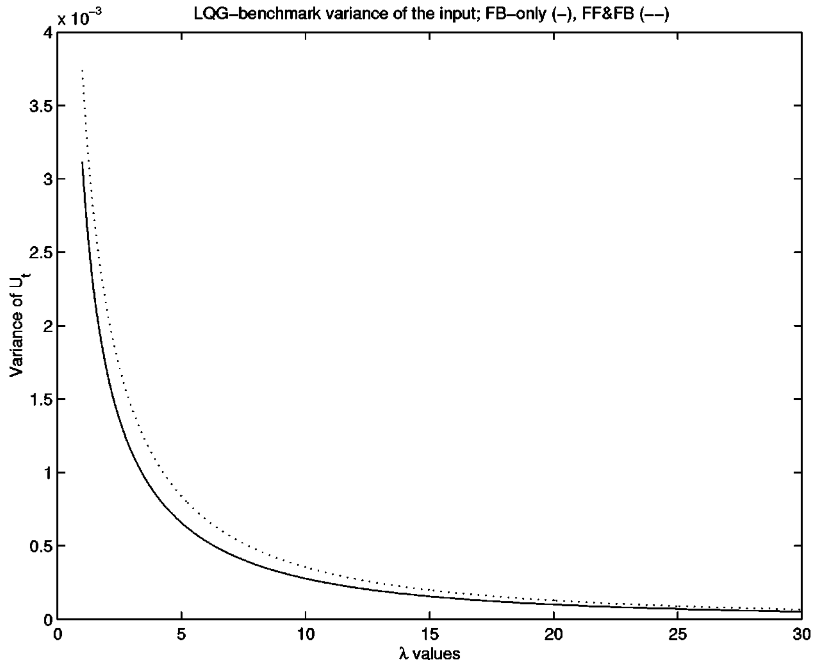

Fig. 3. LQG-benchmark variances of the input.

From feedforward and feedback LQG limit curve
From the FF and FB optimal LQG-benchmark variances we see that the controller performance is still close to optimal with respect to the process response variance (86.34% of the optimal), whereas the performance index with respect to the input variability is better than with a feedback-only controller 63.08%. Hence there is a maxi-

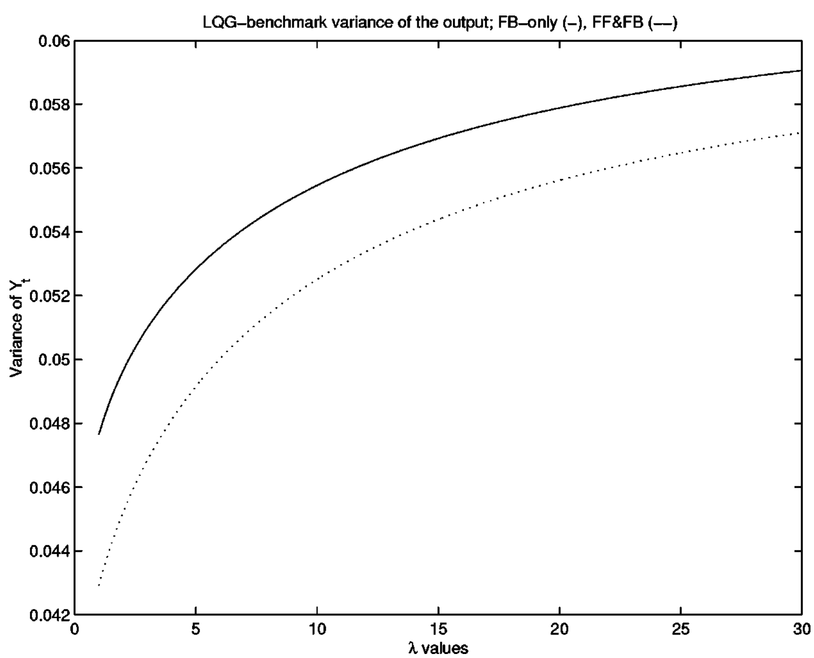

Fig. 4. LQG-benchmark variances of the output.

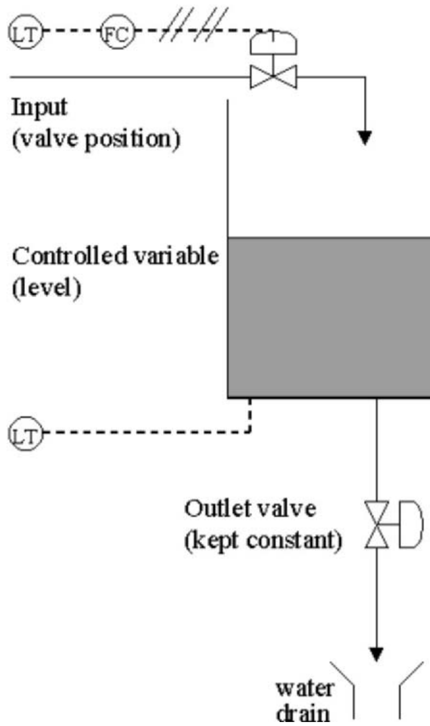

Fig. 5. Experimental setup.

mum possible scope of 36.92% to reduce the input variance without increasing the output variance.

For the profit analysis of implementation of feedforward controller on the process, we see that there is an incentive of only 5.83% reduction in the process response variance possible by the implementation of an optimal feedforward controller on the process. However, there is 49.23% maximum possible scope for increase in the process input variance. Hence it can be concluded that there is not much incentive from the implementation of a feedforward controller in this case.

8. Application on a pilot scale process

The proposed method of controller performance analysis using optimal LQG benchmark is tested on a pilot scale system. The system considered is shown in Fig. 5. The input (u) is the valve position of the input water flow valve and the process variable to be controlled (y) is the level of water in the tank. The tank outlet flow valve is kept at a constant position. The head of the water in the inlet pipe can be considered as (an unmeasured) disturbance. The tank level is controlled by a PID controller, $5+0.05/s+0.05s$. The controller sampling rate is 5 s. An RBS signal of series of

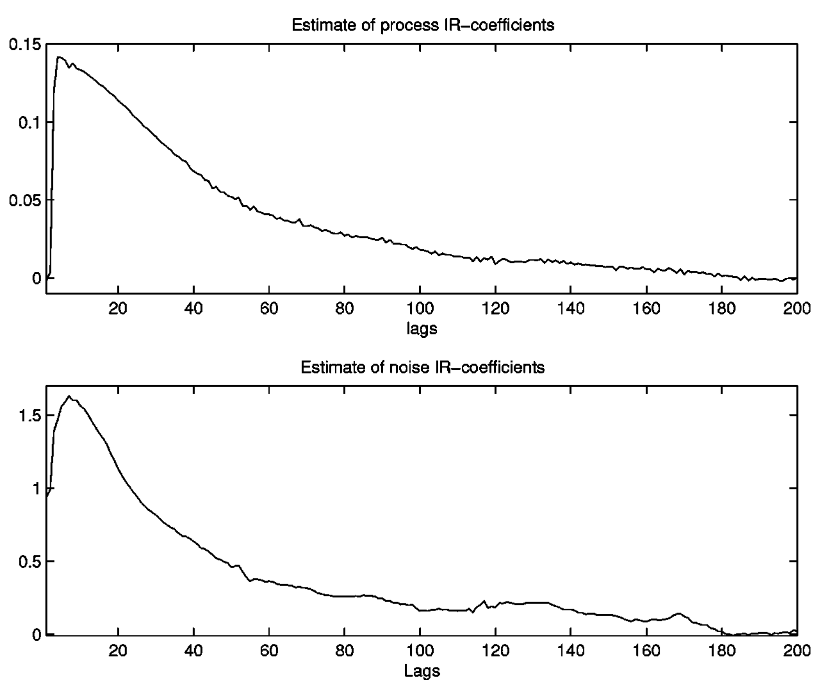

Fig. 6. Impulse response models for the process and noise identified using closed-loop data.

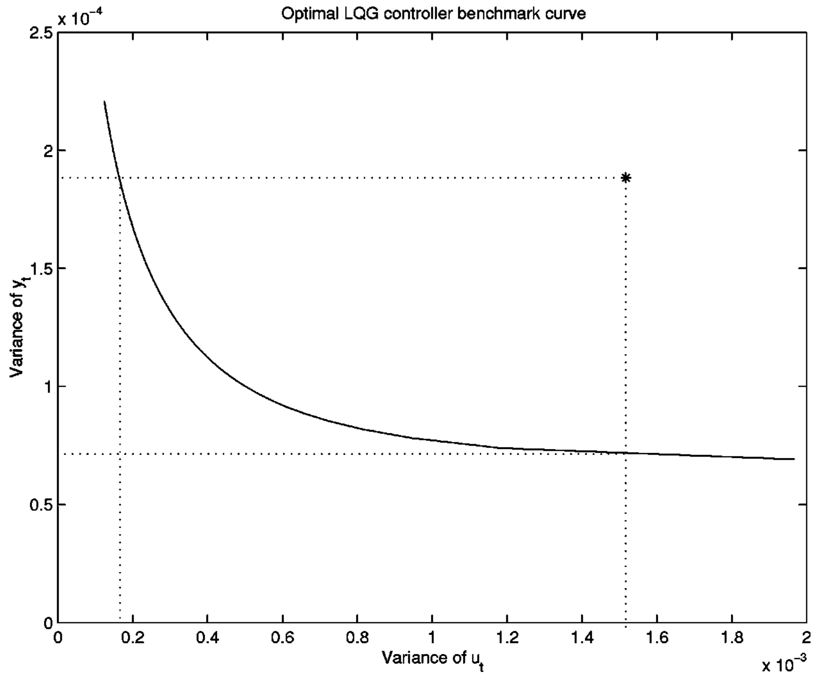

Fig. 7. Optimal LQG-benchmark curve for the CSTH.

setpoint changes to the level is designed in MATLAB. Closed-loop data of the process input and output are collected at a sampling rate of 5 s. The subspace matrices L_u and L_e of dimension 200 are identified using the closed-loop subspace identification method from Ref. [42]. The impulse response coefficients models for the process and noise are plotted in Fig. 6. The optimal LQG-benchmark curve is plotted for a range of values of λ, as shown in Fig. 7. The actual process input and output variances are compared with the optimal variances for the controller performance analysis as shown in Table 2.

Table 2
Controller performance analysis parameters: experiment.

Parameter		Value
$(V_y)^{fb}$	·····	1.885×10^{-4}
$(V_u)^{fb}$	·····	1.52×10^{-3}
$(V_y^o)^{fb}$	·····	0.71×10^{-4}
$(V_u^o)^{fb}$	·····	1.66×10^{-4}
$(\eta)^{fb}$	·····	0.378
$(I_\eta)^{fb}$	·····	62.30%
$(E)^{fb}$	·····	0.11
$(I_\eta)^{fb}$	·····	89.00%

From Table 2 we can see that the controller performance is nonoptimal with respect to both input and output variances. There is a maximum possible scope of 62.30% to reduce the process output variance without increasing the input variance and 89.00% to reduce the process input variance without increasing the output variance.

9. Conclusions

A subspace identification based approach is proposed in this paper for obtaining the optimal LQG benchmark from closed-loop data, for controller performance assessment. It has been shown that instead of explicit process models, only the subspace matrices corresponding to the deterministic and stochastic inputs, L_u and L_e, are required to obtain the LQG benchmark. The closed-loop subspace identification method is used to obtain L_u and L_e which are subsequently used to obtain the LQG benchmark. The optimal LQG benchmark method is extended to the case of feedforward plus feedback control. Profit analysis for the implementation of feedforward control under optimal LQG control performance framework is also derived and explained in this paper. The results of the paper are illustrated through a simulation example and a pilot-scale experiment.

Acknowledgments

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Appendix A: Review of open-loop subspace identification methods

For the system described in Eqs. (1) and (2), suppose we have the measurements of the inputs and the outputs u_k, y_k for $k \in \{0,1,\dots,(2i+j-2)\}$. The data block Hankel matrices with i-block rows and j-block columns are defined for u_k as

$$U_p = \begin{bmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \cdots & \cdots & \cdots & \cdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \end{bmatrix};$$

$$U_f = \begin{bmatrix} u_i & u_{i+1} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \cdots & u_{i+j} \\ \cdots & \cdots & \cdots & \cdots \\ u_{2i-1} & u_{2i} & \cdots & u_{2i+j-2} \end{bmatrix}. \tag{A1}$$

Similarly we can form the data block Hankel matrices for y_k. Then if (i) the deterministic input u_k is uncorrelated with e_k, (ii) u_k is persistently exciting of the order $2i$, and (iii) the measurements go to infinity, $j \rightarrow \infty$, the open-loop models can be consistently identified and identification involves finding the prediction of the future outputs Y_f using a linear predictor. The least-squares prediction $\hat{Y}_f$ can be found by solving the least-squares problem:

$$\min_{L_w, L_u} \left\| Y_f - (L_w \;\; L_u) \begin{pmatrix} W_p \\ U_f \end{pmatrix} \right\|_F^2. \tag{A2}$$

$\hat{Y}_f$ is found by the orthogonal projection of the row space of Y_f into the row space spanned by W_p and U_f defined as [43]

$$\hat{Y}_f = Y_f \Big/ \begin{bmatrix} W_p \\ U_f \end{bmatrix},$$

$$[L_w \;\; L_u] = Y_f \begin{bmatrix} W_p \\ U_f \end{bmatrix}^{\dagger} = Y_f [W_p^T \;\; U_f^T] \left(\begin{bmatrix} W_p \\ U_f \end{bmatrix} [W_p^T \;\; U_f^T] \right)^{-1}. \tag{A3}$$

This projection can be implemented in a numerically robust way with a QR (orthogonal-triangular) decomposition [45,46,43,47,48] or using PLS [49].

Appendix B: Stochastic subspace identification

For the case of no deterministic input to the system, i.e., for a white-noise driven process (for example, a closed-loop system with no setpoint changes, driven by an external disturbance), the past data Hankel matrix is taken as $W_p = y_p$. Therefore the subspace expression becomes

$$\hat{Y}_f = L_w y_p. \tag{B1}$$

The first row of $\hat{Y}_f$ represents the one-step-ahead predictions of the output. Therefore the white-noise disturbance sequence entering the process can be estimated as

$$e_f = [e_i \;\; e_{i+1} \cdots e_{i+j-1}]^T = Y_f(1,:) - \hat{Y}_f(1,:). \tag{B2}$$

Appendix C: Subspace matrices identification using closed-loop data [42]

Consider the case when this system is operating under closed loop with a linear time-invariant controller Q, expressed in transfer function form as

$$u_k = Q(r_k - y_k), \tag{C1}$$

where r_k is the setpoint for the process output at sampling instant k and $(r_k - y_k)$ is the setpoint deviation. Assume that the controller does not cancel the process dynamics. The control system can be expressed in state space representation as

$$x_{(k+1)}^c = A_c x_k^c + B_c (r_k - y_k), \tag{C2}$$

$$u_k = C_c x_k^c + D_c (r_k - y_k). \tag{C3}$$

The block Hankel representation for the control system is

$$U_f=\Gamma_i^c X_f^c+H_i^c(R_f-Y_f) \tag{C4}$$

$$=L_m M_p+L_c R_f-L_c Y_f, \tag{C5}$$

where $M_p=[U_p^T\ R_p^T\ Y_p^T]^T$. L_m and L_c are the subspace matrices of the control system. Using Eq. (4) and the above expressions we obtain

$$Y_f=L^O M_p+L_c^O R_f+L_e^O E_f, \tag{C6}$$

$$U_f=L^I M_p+L_c^I R_f+L_e^I E_f, \tag{C7}$$

where L^I is the combination of $(I+L_cL_u)^{-1}L_m$ and $(I+L_cL_u)^{-1}L_cL_w$; L^O is the combination of $(I+L_uL_c)^{-1}L_uL_m$ and $(I+L_uL_c)^{-1}L_w$; and

$$L_c^I=(I+L_cL_u)^{-1}L_c,$$

$$L_e^I=-(I+L_cL_u)^{-1}L_cL_e=-L_c^I L_e,$$

$$L_c^O=(I+L_uL_c)^{-1}L_uL_c,$$

$$L_e^O=-(I+L_uL_c)^{-1}L_e.$$

With setpoint excitation, estimation of the closed-loop subspace matrices L_c^I, L_e^I, L_c^O, and L_e^O becomes an open-loop identification problem. Therefore the subspace matrix corresponding to the deterministic and stochastic inputs can be extracted as

$$L_u=L_c^O(L_c^I)^{-1}, \tag{C8}$$

$$L_e=-(L_c^I)^{-1}L_e^I. \tag{C9}$$

Notice that we can identify only L_u and L_e using the above closed-loop subspace identification method. These two subspace matrices are sufficient to obtain the LQG-benchmark variance.

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