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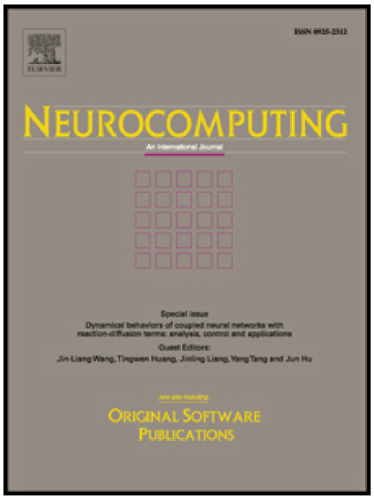

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Neural Network based Integral Sliding Mode Optimal Flight Control of Near Space Hypersonic Vehicle

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Abstract

In this paper, based on the integral sliding mode method and adaptive dynamic programming (ADP) algorithm, a robust optimal tracking control scheme is presented for near space hypersonic vehicle (NSHV) system in the presence of modeling uncertainty, external disturbance, and input saturation. Firstly, combining neural network, auxiliary system and integral sliding mode methods, an adaptive integral sliding mode control (AISMC) law is designed to guarantee system trajectories tend to a defined integral sliding surface and the effects of modeling uncertainty, external disturbance, and control input saturation are eliminated. Then, the robust optimal tracking control problem of original system is converted into the optimal control problem of a nominal system, and an ADP method with single critic network is utilized to acquire the corresponding optimal controller. Furthermore, Lyapunov analysis method shows that the overall control input which contains AISMC law and optimal controller can ensure all the signals in closed-loop system are stable in the sense of uniform ultimate boundedness (UUB). Finally, simulation results about attitude flight control of NSHV are given to verify the effectiveness of the proposed control scheme.

Keywords: Near space hypersonic vehicle, integral sliding mode control, optimal tracking control, adaptive dynamic programming, auxiliary system method, neural network

1. Introduction

It is well known that near space hypersonic vehicle (NSHV) has shown broad application prospects, especially in military field, due to its unique advantages such as large flight envelope, global rapid strike capability and multitasking ability [1, 2]. For some developed countries, the research of NSHV has attracted more and more attentions and many major strategic plans have been formulated and implemented since 21st century. The core research contents of NSHV is to design a reliable control scheme to guarantee the flight safety and quality. However, the NSHV system also possesses some characteristics of fast time varying, strong coupling and nonlinearity. And the effects of system uncertainty and external disturbance are inevitable during the flight process of NSHV for its complex flight environment and variable working modes. Meanwhile, the control input saturation problem should also be considered for the inherent physical property of actuator. Therefore, except for the basic requirements of stability and speedability, more properties of the flight control such as robustness, anti-windup and anti-disturbance abilities are equally important. In recent years, many advanced nonlinear control methods such as adaptive control [3, 4], backstepping control [5, 6] and sliding mode control [7, 8] have already been used for the flight control of NSHV. For instant, a novel adaptive control strategy was addressed for air-breathing hypersonic vehicles with uncertain aerodynamic parameters and actuator faults in [9]. In [10], for hypersonic reentry vehicle system with lumped uncertainties and input constraints, a neural network and backstepping based attitude tracking control scheme was proposed. In [11], a second-order dynamic sliding mode attitude control scheme was proposed for under-actuated hypersonic vehicles. And for air-breathing hypersonic vehicle system with aerodynamic parameter uncertainty, a finite time sliding mode control method was presented in [12]. In summary, the above advanced control methods have been successfully applied to the flight control of NSHV, and some good properties such as robustness, stability and anti-windup ability are guaranteed. However, to the best of our knowledge, there few studies focus on the optimal flight control of NSHV, which put forward a higher requirement (minimizing a predefined cost function) for the flight control process, especially when the effects of unknown modeling error, external disturbance and input saturation are considered under the same control framework.

As we know, the optimal tracking control problem (OTCP) of nonlinear system has become a hot issue in recent years. The core object of OTCP is not only stabilizing the error dynamic system, but also minimizing a predefined cost function composing of tracking error and control energy [13]-[15]. And the process of obtaining the optimal controller of nonlinear system usually needs to solve a Hamilton Jacobi Bellman (HJB) equation, which is a challenging or impossible task using analytic method [16]. As an advanced numerical approximation method, adaptive dynamic programming (ADP) method has been widely used to deal with this issue [17]. For instance, by constructing an augmented system with discounted cost function, an ADP based optimal tracking control scheme was proposed in [18]. And the same control scheme was applied for nonlinear system with unknown perturbation by introducing a modified cost function in [19]. In [20], by designing a feedforward control input, the OTCP can be transformed into a stabilization problem of an error system. Under the backstepping control frame, a novel optimal tracking control scheme was presented for strict-feedback nonlinear system [21]. And this optimal tracking control scheme was expanded to the uncertainty nonlinear system by means of neural network (NN) approximate technique [22]. From above analysis, the ADP algorithm can well handle the optimal control problem, even for uncertain system. However, for system with external time-varying disturbance, this control scheme cannot be directly used. In addition, for input saturation problem of optimal control, a generalized non-quadratic cost function has always been used [23]-[25]. Nevertheless, the non-quadratic cost function will further complicate the analytic process of optimal control, and this control scheme is also not suitable for optimal control problem with external disturbance.

For the purpose of attenuating or even eliminating the effects of external disturbances, many effective methods have been addressed. For instance, disturbance observer (DO) technique, which is deemed to be an effective way to estimate disturbance [26]-[29]. The DO-based control methods usually require the changing rate of disturbance to be bounded, which will limits the applied range of DO method. By contrast, sliding mode control (SMC) also can effectively handle the external disturbance, and the disturbance only require to be bounded [30]-[33]. Recently, several works have been carried out to deal with optimal control problem with external disturbance by combing the sliding mode control (SMC) method and ADP algorithm. In [34], a novel optimal guaranteed cost sliding mode control scheme was presented for nonlinear system with matched and unmatched disturbances and constrained input. And an adaptive sliding mode based optimal control scheme for nonlinear system with actuator faults and disturbances was proposed in [35]. In addition, for disposing input saturation problem, auxiliary system method can be a simple and effective option [36]. In [37], a backstepping based adaptive fuzzy neural controller was presented for unknown chaotic systems with input saturation, and a new system was constructed to deal with saturation. Similarly, constructed auxiliary system is applied to attenuate the effects of input and output saturation of a class of chaotic systems in [38].

Based on the above analysis, a robust optimal flight control strategy integrating radial basis function neural network (RBFNN) technique, auxiliary system method, integral SMC (ISMC) technique and optimal control theorem is proposed for NSHV attitude system with unknown modeling error, external disturbance and input saturation. And the proposed feedback controller not only handles the effects of input saturation, system unknown modeling error and external disturbance, but also a prescribed level of performance index is guaranteed. The remainder of this paper is organized as follows: Section 2 provides the problem descriptions of the robust optimal tracking control. Section 3 shows the processes of controller design and stability analysis of whole closed loop system. Section 4 presents the simulation results to verify the effectiveness of the proposed control scheme. Finally, in Section 5, the conclusion of this paper is given.

2. Problem Description and Transformation

2.1. Problem description

To deal with the attitude tracking control problem of NSHV, the following nonlinear MIMO system with input saturation, unknown modeling error and external disturbance is considered [39], [40].

$$\begin{cases} \dot{x} = f(x) + g(x)\mathrm{Sat}(u) + \phi(x) + d(t) \\ y = x \end{cases} \tag{1}$$

where $x \in \Re^n$ is a vector of system measurable state, $u \in \Re^n$ is control input vector and y denotes system output vector. $f(x) \in \Re^n$ and $g(x) \in \Re^{n\times n}$ are system state function and control gain matrix, respectively, which are known continuous smooth functions. $\phi(x) \in \Re^n$ represents the unknown modeling error which is a smooth function. $d(t) \in \Re^n$ denotes the external time-varying disturbance. $\mathrm{Sat}(u) = [\mathrm{sat}(u_1), \cdots, \mathrm{sat}(u_n)]^T \in \Re^n$ denotes a vector of saturation control input, and the saturation function $\mathrm{sat}(\cdot)$ is defined as [41]:

$$\mathrm{sat}(u_i) = \begin{cases} u_{\mathrm{max}i}, & u_i \geq u_{\mathrm{max}i} \\ u_i, & u_{\mathrm{min}i} < u_i < u_{\mathrm{max}i}; (i = 1, \cdots, n) \\ u_{\mathrm{min}i}, & u_i \leq u_{\mathrm{min}i} \end{cases} \tag{2}$$

where u_i is the i-th value of actual input signal calculated by control law, $u_{\mathrm{min}i}$ and $u_{\mathrm{max}i}$ are lower and upper bounds of u_i, respectively.

The core control objective for system (1) is to seek a robust optimal tracking control scheme such that the system output y can optimally track the desired signal x_r, not only eliminating the effects of input saturation, unknown modeling error and external disturbance, but also a prescribed level of performance index is guaranteed.

Firstly, some assumptions are shown as follows:

Assumption 1: [43] The control gain matrix $g(x)$ is assumed to be invertible and norm bounded, i.e., $\bar{g}_m \leq \|g(x)\| \leq \bar{g}_M$ for positive constants $\bar{g}_m$, $\bar{g}_M$.

Assumption 2: [11] The external disturbance $d(t)$ is assumed to be bounded such that $\|d(t)\| \leq \bar{d}_M$ for positive constant $\bar{d}_M$.

Assumption 3: [41] The reference signal x_r and its first-order derivative $\dot{x}_r$ are available signals and assumed to be bounded.

Assumption 4: [42] For system (1) subject to the input saturation (2) and desired signal x_r, there exists a feasible actual control input u such that the tracking objective can be achieved. Furthermore, the difference between the saturated control input and actual control input $\Delta u = \mathrm{Sat}(u) - u$ is assumed to be bounded such that $\|\Delta u\| \leq \bar{\Delta}_M$ for positive constant $\bar{\Delta}_M$.

2.2. Problem transformation

The unknown function $\phi(x)$ can be approximated using RBFNN as

$$\phi(x) = W_0^T \varphi_0(x) + \varepsilon_0(x) \tag{3}$$

where $W_0 \in \Re^{\iota \times n}$ and $\varphi_0(x) \in \Re^{\iota \times 1}$ are the ideal weight matrix and RBF basis function, respectively. $\varepsilon_0(x) \in \Re^{n \times 1}$ is the approximation error. It is well known that ε_0 can be small enough by suitably choosing the node number ι, hence we can assume $\|\varepsilon_0\| \leq \bar{\varepsilon}_0$ for positive constant $\bar{\varepsilon}_0$. Similarly, the ideal weight matrix W_0 and basis function φ_0 are also bounded, i.e., $\|W_0\| \leq \bar{W}_{0M}$, $\|\varphi_0\| \leq \bar{\varphi}_{0M}$ with positive constants $\bar{W}_{0M}$ and $\bar{\varphi}_{0M}$ [44].

Then, system (1) can be rewritten as

$$\dot{x} = f(x) + g(x)\mathrm{Sat}(u) + W_0^T \varphi_0(x) + D(t) \tag{4}$$

where $D = d + \varepsilon_0$ is the compound disturbance. Here, we can easily derive that D is bounded based on *Assumption 2* and boundedness of ε_0, and $\|D\| \leq \bar{D}_M$ for positive constant $\bar{D}_M$.

For the purpose of dealing with input saturation problem, the following auxiliary system is constructed.

$$\dot{\zeta} = -A\zeta + g(x)\Delta u \tag{5}$$

where $\zeta \in \Re^n$ denotes the state vector of auxiliary system, $A \in \Re^{n\times n}$ is a designed positive matrix, and Δu represents the input vector of auxiliary system which is defined in *Assumption 4* [37, 38].

Define the following variable:

$$z = x - x_r - \zeta \tag{6}$$

Taking the first derivative of z with respect to t, we have

$$\begin{aligned} \dot{z} &= \dot{x} - \dot{x}_r - \dot{\zeta} \\ &= f(x) + g(x)\mathrm{Sat}(u) + W_0^T\varphi_0(x) + D(t) - \dot{x}_r + A\zeta - g(x)\triangle u \\ &= f(x) + g(x)u + W_0^T\varphi_0(x) + D(t) - \dot{x}_r + A\zeta \end{aligned} \tag{7}$$

As the reference signal x_r is beforehand given, which can be regarded as an available signal, and let $F = f(x) - \dot{x}_r$, $G = g(x)$, then system (7) can be rewritten as

$$\dot{z} \quad = F + Gu + W_0^T\varphi_0 + A\zeta + D(t) \tag{8}$$

For system (8), a robust optimal tracking control scheme based on integral sliding mode control and ADP method will be designed in the next section. And the overall control input u consists of two parts:

$$u = u_o + u_s \tag{9}$$

where u_o is an optimal tracking control input for nominal system to guarantee a prescribed performance index requirement, and u_s is an adaptive integral sliding mode control (AISMC) law to eliminate the effects of input saturation, system uncertainty and external disturbance by driving the system trajectories toward the defined integral sliding manifold.

3. Controller Design and Stability Analysis

3.1. Adaptive integral sliding mode control

The integral sliding surface can be designed as follows:

$$S(z,t) = \Pi\left(z(t) - z(0) - \int_0^t F + Gu_o d\tau\right) \tag{10}$$

where $\Pi \in \Re^{n\times n}$ is a designed positive matrix which satisfies ΠG is invertible. Here, the optimal control input u_o is only an implicit express form, which will be designed in the next subsection.

Based on the defined sliding mode surface (10), the AISMC law u_s can be designed as

$$u_s = \quad -C(\Pi G)^{-1}\mathrm{Sgn}(S) - G^{-1}(\hat{W}_0^T\varphi_0 + A\zeta + K_s S) \tag{11}$$

where C and K_s are positive designed matrices with suitable dimension, $\hat{W}_0$ denotes the estimation value of W_0. And $\mathrm{Sgn}(S) = [\mathrm{sgn}(S_1), \cdots, \mathrm{sgn}(S_n)]^T$, where $\mathrm{sgn}(\cdot)$ is a sign function.

The adaptive law for $\hat{W}_0$ is designed as

$$\dot{\hat{W}}_0 = \Gamma(\varphi_0 S^T\Pi - \xi_0\hat{W}_0) \tag{12}$$

where $\Gamma \in \Re^{\iota\times\iota}$ is a positive definite matrix and ξ_0 is positive constant. Define $\tilde{W}_0 = \hat{W}_0 - W_0$ as the estimation error of W_0.

It is easy to see that the control input u_s is discontinuous due to the use of sign function, which may cause the chattering effect. In order to overcome this shortcoming, a hyperbolic tangent function $\varpi(S)$ is used here to replace the sign function $\mathrm{Sgn}(S)$, and defined as

$$\varpi(S) = \mathrm{Tanh}(\frac{S}{\epsilon}) \tag{13}$$

where $\mathrm{Tanh}(\frac{S}{\epsilon}) = [\tanh(S_1/\epsilon), \cdots, \tanh(S_n/\epsilon)]^T$ and ϵ is a small positive constant. Define the approximation error $\delta = \mathrm{Sgn}(S) - \varpi(S)$. It is easy to derive that δ is norm bounded such that $\|\delta\| \le \bar{\delta}_M$ for positive constant $\bar{\delta}_M$, and as ϵ decreases, the approximation error can be reduced.

Then, the actual control input of u_s is rewritten as

$$u_s = \quad -C(\Pi G)^{-1}\varpi(S) - G^{-1}(\hat{W}_0^T\varphi_0 + A\zeta + K_s S) \tag{14}$$

Next, *Theorem 1* will be given to show the stability of the sliding mode surface $S(t)$ with the AISMC law (14).

Theorem 1: Consider the system (8) with *Assumptions 1-4* hold. The integral sliding mode surface and AISMC law are designed in form of (10) and (14), respectively. And the updating law for $\hat{W}_0$ is given as (12). Then, the trajectory of sliding mode surface $S(t)$ and weight estimation error $\tilde{W}_0$ are UUB, and by suitably selecting the corresponding parameters Π, C, K_s, ξ_0, the ultimate convergence sets can be small enough.

Proof: Choose the Lyapunov candidate function L_1 as follows:

$$L_1 = \frac{1}{2}S^T S + \frac{1}{2}tr\left\{\tilde{W}_0^T \Gamma^{-1} \tilde{W}_0\right\} \tag{15}$$

Taking derivative of L_1 with respect to time t along the system dynamic (8), we have

$$\begin{aligned}
\dot{L}_1 &= S^T \dot{S} + tr\left\{\tilde{W}_0^T \Gamma^{-1} \dot{\tilde{W}}_0\right\} \\
&= S^T \Pi \left[G u_s + A\zeta + W_0^T \varphi_0 + D\right] + tr\left\{\tilde{W}_0^T \Gamma^{-1} \dot{\tilde{W}}_0\right\} \\
&= S^T \Pi[-CG(\Pi G)^{-1} \varpi(S) - \hat{W}_0^T \varphi_0 - K_s S \\
&\quad -A\zeta + D + W_0^T \varphi_0 + A\zeta] + tr\left\{\tilde{W}_0^T \Gamma^{-1} \dot{\tilde{W}}_0\right\} \\
&= -CS^T \mathrm{Sgn}(S) + CS^T \delta - S^T \Pi \tilde{W}_0^T \varphi_0 + S^T \Pi D - S^T \Pi K_s S \\
&\quad +tr\left\{\tilde{W}_0^T \varphi_0 S^T \Pi\right\} - \xi_0 tr\left\{\tilde{W}_0^T \hat{W}_0\right\}
\end{aligned} \tag{16}$$

Note that

$$S^T \Pi \tilde{W}_0^T \varphi_0 = tr\left\{S^T \Pi \tilde{W}_0^T \varphi_0\right\} = tr\left\{\tilde{W}_0^T \varphi_0 S^T \Pi\right\} \tag{17}$$

$$\xi_0 tr\left\{\tilde{W}_0^T \hat{W}_0\right\} \geq \frac{\xi_0}{2}\|\tilde{W}_0\|^2 - \frac{\xi_0}{2}\|W_0\|^2 \tag{18}$$

Then we have

$$\begin{aligned}
\dot{L}_1 &\leq -CS^T \mathrm{Sgn}(S) + CS^T \delta + S^T \Pi D - S^T \Pi K_s S + \frac{\xi_0}{2}\|W_0\|^2 - \frac{\xi_0}{2}\|\tilde{W}_0\|^2 \\
&\leq -\lambda_{min}(C)\|S\| + \|\Pi D\|\|S\| + \frac{\|C\|^2}{2}\|S\|^2 + \frac{\|\delta\|^2}{2} \\
&\quad -\lambda_{min}(\Pi K_s)\|S\|^2 - \frac{\xi_0}{2}\|\tilde{W}_0\|^2 + \frac{\xi_0}{2}\bar{W}_{0M}^2 \\
&\leq -\left(\lambda_{min}(C) - \lambda_{max}(\Pi)\bar{D}_M\right)\|S\| - \frac{\xi_0}{2}\|\tilde{W}_0\|^2
\end{aligned}$$

$$- \left(\lambda_{min}(\Pi K_s) - \frac{\|C\|^2}{2} \right) \|S\|^2 + \frac{\xi_0}{2} \bar{W}_{0M}^2 + \frac{\bar{\delta}_M^2}{2} \tag{19}$$

By suitably choosing the matrixes C, Π and K_s such that $\lambda_{min}(C) > \lambda_{max}(\Pi)\bar{D}_M$, $\lambda_{min}(\Pi K_s) > \frac{\|C\|^2}{2}$, then we have

$$\begin{aligned} \dot{L}_1 &\leq - \left(\lambda_{min}(\Pi K_s) - \frac{\|C\|^2}{2} \right) \|S\|^2 - \frac{\xi_0}{2} \|\tilde{W}_0\|^2 + \frac{\xi_0}{2} \bar{W}_{0M}^2 + \frac{\bar{\delta}_M^2}{2} \\ &\leq -\kappa L_1 + \varsigma \end{aligned} \tag{20}$$

where $\kappa = \min\{\lambda_{min}(\Pi K_s) - \frac{\|C\|^2}{2}, \frac{\xi_0}{2}\}$, $\varsigma = \frac{\xi_0}{2} \bar{W}_{0M}^2 + \frac{\bar{\delta}_M^2}{2}$.

Integration of (19) yields

$$0 \leq L_1 \leq \frac{\varsigma}{\kappa} + \left(L_1(0) - \frac{\varsigma}{\kappa} \right) \exp^{-\kappa t} \tag{21}$$

Therefore, the sliding mode surface $S(t)$ and weight estimation error $\tilde{W}_0$ are proved to be stable in the sense of UUB. Furthermore, by suitably selecting the parameters Π, C, K_s, ξ_0, the ultimate convergence sets of S and $\tilde{W}_0$ can be small enough. The proof is completed.

Remark 1: From *Theorem 1*, we can observe that by suitably designing the AISMC law (14), the trajectory of integral sliding mode surface $S(t)$ is UUB, which implies that the effects of input saturation, unknown modeling error and external disturbance are effectively suppressed. Then, the optimal controller u_o need to be designed for the inner dynamics system of sliding mode to guarantee the tracking performance. By this way, the robust optimal tracking control problem of system (8) is successfully converted to an optimal control problem of a nominal system. In the next subsections, the design process of u_o and stability of whole closed-loop system will be given.

3.2. ADP based Optimal Tracking Control Design

Considering the following nominal system:

$$\dot{z} = F + G u_o \tag{22}$$

The optimal tracking control input u_o contains two parts:

$$u_o = u_r + u_z \tag{23}$$

Firstly, u_r can be designed in form of

$$u_r = G^{-1}(\dot{x}_r - f(x_r) - K_z \nabla V_1(z)) \tag{24}$$

where $\nabla V_1(z) = \partial V_1(z)/\partial z$, and $V_1(z)$ is a defined Lyapunov candidate function with respect to z, K_z is a positive matrix. And u_z will be designed later.

Substituting (23) and (24) into (22), we have

$$\begin{aligned} \dot{z} &= f(x) - \dot{x}_r + Gu_z + \dot{x}_r - f(x_r) - K_z \nabla V_1 \\ &= f(x) - f(x_r) + Gu_z - K_z \nabla V_1 \end{aligned} \tag{25}$$

For convenience , let $F_z = f(x) - f(x_r)$, then (25) can be rewritten as

$$\dot{z} = F_z + Gu_z - K_z \nabla V_1 \tag{26}$$

Lemma 1: Considering the nominal system (22), the optimal tracking control input u_o consists of two parts, u_r and u_z. If the control policy u_r is designed in form of (24), and u_z is designed as an optimal controller of following system

$$\dot{z} = F_z + Gu_z. \tag{27}$$

Then, the optimal tracking control problem of (22) can be transformed into the optimal stabilization problem of system (27).

Proof: Choose the following Lyapunov candidate function

$$L_2(t) = V_1(z) \tag{28}$$

Taking derivative of L_2 with respect to system (22), we have

$$\begin{aligned} \dot{L}_2 &= \nabla V_1^T (F + Gu_z + Gu_r) \\ &= \nabla V_1^T (F_z + Gu_z) - \nabla V_1^T K_z \nabla V_1 \\ &\leq \nabla V_1^T (F_z + Gu_z) \end{aligned} \tag{29}$$

From (29), we can observe that if the control input u_z is designed as an optimal controller (denoted as u_z^*) for system (27) , then we can derive that $\nabla V_1^T(F_z + Gu_z^*) < 0$ [43], and we have $\dot{L}_2 < 0$. Then, the convergence of system (22) is guaranteed. Hence, the optimal tracking control problem of (22) can be transformed into the optimal stabilization problem of system (27). And in the next, the main mission is to design the the optimal controller u_z^* for system (27).

To deal with the optimal control problem, an infinite horizon cost function is defined as [46]

$$V(z(0)) = \int_0^\infty [z^T Q z + u_z^T R u_z] \mathrm{d}\tau \tag{30}$$

where $Q \in \Re^{n\times n}$ and $R \in \Re^{n\times n}$ are symmetric and positive definite matrices. Then, in reference to [45], the following definition about admissible control is given.

Definition 1: For system (27) with cost function (30), the control policy $u_z(t) \in \pi(\Phi)$ is called admissible control, if the continuous feedback control u_z not only stabilizes the system (26), but also guarantees the finiteness of the cost function $V(z)$ for $\forall z \in \Phi$, where Φ is a compact set.

The cost function at current time t is defined as [46]

$$V(z(t)) = \int_t^\infty [z^T Q z + u_z^T R u_z] \mathrm{d}\tau \tag{31}$$

Then we can define the following Hamiltonian function

$$H(z, u_z, \nabla V) = z^T Q z + u_z^T R u_z + \nabla V^T (F_z + G u_z) \tag{32}$$

where $\nabla V = \partial V / \partial z$.

The optimal cost function $V^*(z)$ is defined as [45]

$$V^*(z) = \min_{u_z \in \pi(\Phi)} \left(\int_t^\infty \left[z^T Q z + u_z^T R u_z \right] \mathrm{d}\tau \right) \tag{33}$$

Assume the minimum value on the right-hand side of (33) exists and is unique. Similar to [45], [46], the stationary condition $\partial H(z, u_z, \nabla V)/\partial u_z = 0$ is used and the optimal controller can be expressed as

$$u_z^* = -\frac{1}{2} R^{-1} G^T \nabla V^* \tag{34}$$

where $\nabla V^* = \partial V^*/\partial z$.

Substituting (34) into (32), we have following HJB equation:

$$z^T Qz + \nabla V^{*T} F_z - \frac{1}{4}\nabla V^{*T} GR^{-1}G^T \nabla V^* = 0 \tag{35}$$

In order to obtain the optimal controller u_z^*, we have to solve the HJB equation (35), which is extremely difficult using analytical method. Hence, a single network based ADP method is utilized to obtain its approximated solution. And the following two assumptions which are widely used in optimal control problem [46, 47, 48] are given.

Assumption 5: [46, 47] Considering system (27) with cost function (30), the optimal controller u_z^* is designed in form of (34). Assume $V_1(z)$ be a Lyapunov candidate function, then we have $\nabla V_1^T(F_z + Gu_z^*) < 0$, where $\nabla V_1 = \partial V_1(z)/\partial z$. Moreover, there always exsits a positive definite matrix $\Lambda(z)$ such that the following relationship $\nabla V_1^T(F_z + Gu_z^*) = -\nabla V_1^T \Lambda(z)\nabla V_1$ holds, where $\Lambda(z)$ is norm bounded and $\bar{\Lambda}_m \leq \|\Lambda(z)\| \leq \bar{\Lambda}_M$ for positive constants $\bar{\Lambda}_m$ and $\bar{\Lambda}_M$.

Assumption 6: [19, 21, 48] For system (27) with cost function (30) and the optimal controller (34), there exists a positive function $\eta(z)$, such that $\|F_z + Gu_z^*\| \leq \eta(z)$.

Based on neural network (NN) approximation technique, the optimal cost function $V^*(z)$ can be expressed as:

$$V^*(z) = W_c^T \varphi_c(z) + \varepsilon_c(z) \tag{36}$$

where $W_c \in \Re^h$ is the ideal weight vector, $\varphi_c(\cdot) \in \Re^h$ is the suitable activation function and $\varepsilon_c(z)$ denotes the approximation error. It is easy to derive that the approximation error ε_c can be small enough with the sufficient large number of neuron nodes h [44].

Taking the partial derivative of $V^*(z)$ with respect to z. We have

$$\nabla V^* = \nabla \varphi_c^T W_c + \nabla \varepsilon_c \tag{37}$$

Assumption 7: [44] The ideal weight vector W_c is assumed to be bounded, $\|W_c\| \leq \bar{W}_{cM}$ for constant $\bar{W}_{cM}$. The reconstruction error ε_c and its gradient are assumed to be norm bounded, $\|\varepsilon_c\| \leq \bar{\varepsilon}_{cM}$, $\|\nabla\varepsilon_c\| \leq \bar{\varepsilon}'_{cM}$ for constants $\bar{\varepsilon}_{cM}$ and $\bar{\varepsilon}'_{cM}$. The gradient form of activation function φ_c is assumed to be norm bounded such that $\bar{\varphi}'_{cm} \leq \|\nabla\varphi_c\| \leq \bar{\varphi}'_{cM}$ for positive constants $\bar{\varphi}'_{cm}, \bar{\varphi}'_{cM}$.

Using (34) and (37), the optimal control u_z^* can be

$$u_z^* = -\frac{1}{2}R^{-1}G^T(\nabla\varphi_c^T W_c + \nabla\varepsilon_c) \tag{38}$$

And the HJB equation (35) can be rewritten as

$$H(z, W_c) = z^T Q z + W_c^T \nabla\varphi_c F_z - \frac{1}{4}W_c^T \nabla\varphi_c \Xi \nabla\varphi_c^T W_c + \varepsilon_H = 0 \tag{39}$$

where $\Xi = GR^{-1}G^T$. Based on *Assumption 1*, we can derive that $\bar{\Xi}_m \leq \|\Xi\| \leq \bar{\Xi}_M$ for positive constants $\bar{\Xi}_m$, $\bar{\Xi}_M$.

And ε_H is the function reconstruction error in form of

$$\begin{aligned} \varepsilon_H &= \nabla\varepsilon_c^T F_z - \frac{1}{2}W_c^T \nabla\varphi_c \Xi \nabla\varepsilon_c - \frac{1}{4}\nabla\varepsilon_c^T \Xi \nabla\varepsilon_c \\ &= \nabla\varepsilon_c^T (F_z + G u_z^*) + \frac{1}{4}\nabla\varepsilon_c^T \Xi \nabla\varepsilon_c \end{aligned} \tag{40}$$

Based on *Assumption 6* and *7*, ε_H can assume to be bounded, i.e., $|\varepsilon_H| \leq \bar{\varepsilon}'_{cM}\eta(z) + \bar{\varepsilon}'^2_{cM}\bar{\Xi}_M$.

Define $\hat{W}_c$ as the estimation value of W_c, then the approximate control $\hat{u}_z$ can be described as

$$\hat{u}_z = -\frac{1}{2}R^{-1}G^T \nabla\varphi_c^T \hat{W}_c \tag{41}$$

Then the nonlinear HJB equation (39) can be approximated as

$$\hat{H}(z, \hat{W}_c) = z^T Q z + \hat{W}_c^T \nabla\varphi_c F_z - \frac{1}{4}\hat{W}^T \nabla\varphi_c \Xi \nabla\varphi_c^T \hat{W}_c = e_c \tag{42}$$

where e_c is the residual error.

In the next step, the updating law will be designed for the estimated NN weight value $\hat{W}_c$. Before this, we define the following objective function to be minimized:

$$E = \frac{1}{2}e_c^T e_c \tag{43}$$

Reference to the design process of [45], the update law for NN weight is designed as

$$\begin{aligned}\dot{\hat{W}}_c &= -\xi_1\left(\frac{\partial E}{\partial \hat{W}_c}\right)+\frac{\xi_2}{2}\Sigma(z,\hat{u}_z)\nabla\varphi_c\Xi\nabla V_1(z)\\ &= -\xi_1\sigma\left(z^TQz+\hat{W}_c^T\nabla\varphi_cF_z-\frac{1}{4}\hat{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^T\hat{W}_c\right)\\ &\quad+\frac{\xi_2}{2}\Sigma(z,\hat{u}_z)\nabla\varphi_c\Xi\nabla V_1(z)\end{aligned}\tag{44}$$

where $\sigma=\nabla\varphi_cF_z-\nabla\varphi_c\Xi\nabla\varphi_c^T\hat{W}_c/2$, $\xi_1>0$ and $\xi_2>0$ are design constants, $V_1(z)$ is a Lyapunov function described in *Assumption 5*. And the last term $\Sigma(z,\hat{u}_z)$ is defined as following form [45]:

$$\Sigma(z,\hat{u}_z)=\begin{cases}0, & \text{if } \nabla V_1^{\mathrm{T}}(F_z+G\hat{u}_z)<0\\ 1, & \text{otherwise}\end{cases}\tag{45}$$

Observing that

$$\begin{aligned}\sigma &= \nabla\varphi_cF_z-\nabla\varphi_c\Xi\nabla\varphi_c^T\hat{W}_c/2\\ &= \nabla\varphi_c(\dot{z}^*+\Xi\nabla\varepsilon_c/2)+\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c/2\end{aligned}\tag{46}$$

where $\dot{z}^*=F_z+Gu_z^*$ and $\tilde{W}_c=W_c-\hat{W}_c$ denotes the estimation error.

And

$$\begin{aligned}e_c &= z^TQz+\hat{W}_c^T\nabla\varphi_cF_z-\frac{1}{4}\hat{W}^T\nabla\varphi_c\Xi\nabla\varphi_c^T\hat{W}_c\\ &= \frac{1}{2}\tilde{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^TW_c-\tilde{W}_c^T\nabla\varphi_cF_z-\varepsilon_H-\frac{1}{4}\tilde{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c\\ &= -\tilde{W}_c^T\nabla\varphi_c(\dot{z}^*+\frac{\Xi\nabla\varepsilon_c}{2})-\varepsilon_H-\frac{1}{4}\tilde{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c\end{aligned}\tag{47}$$

Then the error dynamics of $\tilde{W}_c$ can be written as

$$\begin{aligned}\dot{\tilde{W}}_c &= -\dot{\hat{W}}_c\\ &= -\xi_1\left(\nabla\varphi_c(\dot{z}^*+\frac{\Xi\nabla\varepsilon_c}{2})+\frac{1}{2}\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c\right)\\ &\quad\times\left(\tilde{W}_c^T\nabla\varphi_c(\dot{z}^*+\frac{\Xi\nabla\varepsilon_c}{2})+\frac{1}{4}\tilde{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c+\varepsilon_H\right)\\ &\quad-\frac{\xi_2}{2}\Sigma(z,\hat{u}_z)\nabla\varphi_c\Xi\nabla V_1\end{aligned}\tag{48}$$

Based on the results of (24) and (41), the optimal tracking control input u_o can be designed as

$$u_o = G^{-1}(\dot{x}_r - f(x_r) - K_z \nabla V_1(z)) - \frac{1}{2} R^{-1} G^T \nabla \varphi_c^T \hat{W}_c \tag{49}$$

3.3. Stability Analysis

Based on the results of (11) and (49), the overall control input u can be

$$\begin{aligned} u &= u_s + u_o \\ &= -C(\Pi G)^{-1}\varpi(S) - G^{-1}(\hat{W}_0^T \varphi_0 + A\zeta + K_s S) \\ &\quad + G^{-1}(\dot{x}_r - f(x_r) - K_z \nabla V_1(z)) - \frac{1}{2} R^{-1} G^T \nabla \varphi_c^T \hat{W}_c \end{aligned} \tag{50}$$

The aforementioned design and analysis procedures can be summarised as following theorem:

***Theorem 2*:** Consider the system (8) with *Assumption 1-7* hold. The integral sliding mode surface is designed in form of (10). The overall control input for closed-loop is design as (50), which contains the AISMC law (14) and optimal control input (49). And the updating laws for NN weights $\tilde{W}_0$ and $\tilde{W}_c$ are designed as (12) and (44), respectively. Then, the trajectory of sliding mode surface $S(t)$, the estimation value of NN weights $\tilde{W}_0, \tilde{W}_c$ and the defined function ∇V_1 are UUB, and the upper bound for each signal can be described as

$$\|S\| \leq \max \left\{ \sqrt[2]{\frac{\varsigma_1}{\kappa_{s1}}}, \sqrt[2]{\frac{\varsigma_2}{\kappa_{s2}} + \frac{\xi_1^2 K^{*2} \psi_2^2}{\kappa_{s2}\kappa_{z2}} + \frac{\xi_2^2 \bar{\Xi}_M^2 \bar{\varepsilon}_{cM}'^2}{4\kappa_{s2}\kappa_{z2}}} \right\} \tag{51}$$

$$\|\tilde{W}_0\| \leq \max \left\{ \sqrt[2]{\frac{\varsigma_1}{\kappa_{w1}}}, \sqrt[2]{\frac{\varsigma_2}{\kappa_{w2}} + \frac{\xi_1^2 K^{*2} \psi_2^2}{\kappa_{w2}\kappa_{z2}} + \frac{\xi_2^2 \bar{\Xi}_M^2 \bar{\varepsilon}_{cM}'^2}{4\kappa_{w2}\kappa_{z2}}} \right\} \tag{52}$$

$$\|\tilde{W}_c\| \leq \max \left\{ \sqrt[4]{\frac{\varsigma_1}{\xi_1 \psi_1}}, \sqrt[4]{\frac{\varsigma_2}{\xi_1 \psi_1} + \frac{\xi_1 K^{*2} \psi_2^2}{\psi_1 \kappa_{z2}} + \frac{\xi_2^2 \bar{\Xi}_M^2 \bar{\varepsilon}_{cM}'^2}{4\xi_1 \psi_1 \kappa_{z2}}} \right\} \tag{53}$$

$$\|\nabla V_1\| \leq \max \left\{ \sqrt[2]{\frac{\varsigma_1}{\kappa_{z1}}}, \sqrt[2]{\frac{2\varsigma_2}{\kappa_{z2}} + \frac{2\xi_1^2 K^{*2} \psi_2^2}{\kappa_{z2}^2} + \frac{\xi_2^2 \bar{\Xi}_M^2 \bar{\varepsilon}_{cM}'^2}{2\kappa_{z2}^2}} \right\} \tag{54}$$

where $\kappa_{s1} = \lambda_{min}(\Pi K_s) - \frac{1}{2}\|K_s\|^2 - \frac{\|C\|^2}{2}$, $\kappa_{z1} = \xi_2\lambda_{min}(K_z) - \frac{\xi_2^2}{2} - \frac{\xi_2^2}{2\lambda_2} - \frac{3}{2}$, $\kappa_{w1} = \frac{\xi_0}{2} - \frac{\lambda_2}{2}\bar{\varphi}_{0M}^2$, $\varsigma_1 = \xi_1\varrho(\varepsilon) + \frac{\bar{\delta}_M^2}{2} + \frac{\xi_2^2}{2}\bar{D}_M^2 + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2} + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2}\bar{\delta}_M^2 + \frac{\xi_0}{2}\bar{W}_{0M}^2$. and $\kappa_{s2} = \lambda_{min}(\Pi K_s) - \frac{\|K_s\|^2}{2} - \frac{\|C\|^2}{2}$, $\kappa_{z2} = \xi_2\lambda_{min}(K_z) + \xi_2\bar{\Lambda}_m - \frac{1}{2}\xi_2^2 - \frac{\xi_2^2}{2\lambda_2} - \frac{3}{2}$, $\kappa_{w2} = \frac{\xi_0}{2} - \frac{\lambda_2}{2}\bar{\varphi}_{0M}^2$, $\varsigma_2 = \xi_1\varrho(\varepsilon) + \frac{\bar{\delta}_M^2}{2} + \frac{\xi_2^2}{2}\bar{D}_M^2 + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2} + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2}\bar{\delta}_M^2 + \frac{\xi_0}{2}\bar{W}_{0M}^2$. In addition, by suitably selecting the relative parameters, the ultimate convergence set for each signal can be small enough.

***Proof*:** Choosing the following Lyapunov candidate function:

$$L(t) = L_1(t) + \xi_2 L_2(t) + L_3(t) \tag{55}$$

where $L_1(t) = \frac{1}{2}S^T S + \frac{1}{2}tr\left\{\tilde{W}_0^T\Gamma^{-1}\tilde{W}_0\right\}$, $L_2(t) = \nabla V_1(z)$, $L_3(t) = \frac{1}{2}\tilde{W}_c^T\tilde{W}_c$.

Firstly, based on the analysis results of *Theorem 1*, the first derivative of $L_1(t)$ can be

$$\dot{L}_1 \leq -CS^T\text{sgn}(S) + CS^T\delta + S^T\Pi D - S^T\Pi K_s S + \frac{\xi_0}{2}\|W_0\|^2 - \frac{\xi_0}{2}\|\tilde{W}_0\|^2 \tag{56}$$

Taking the derivative of $\xi_2 L_2(t)$ respect to system (8), we have

$$\begin{aligned}\xi_2\dot{L}_2 &= \xi_2\nabla V_1\dot{z}\\ &= \xi_2\nabla V_1^T[F_z + G\hat{u}_z - K_z\nabla V_1 + W_0^T\varphi_0 + A\zeta + D - C\Pi^{-1}\varpi(S)\\ &\quad -\hat{W}_0^T\varphi_0 - A\zeta - K_s S]\\ &= \xi_2\nabla V_1^T(F_z + G\hat{u}_z) - \xi_2\nabla V_1^T\tilde{W}_0^T\varphi_0 + \xi_2\nabla V_1^T D - \xi_2\nabla V_1^T C\Pi^{-1}\text{Sgn}(S)\\ &\quad +\xi_2\nabla V_1^T C\Pi^{-1}\delta - \xi_2\nabla V_1^T K_s S - \xi_2\nabla V_1^T K_z\nabla V_1\end{aligned} \tag{57}$$

Taking the time derivative of $L_3(t)$ with respect to t, we have

$$\begin{aligned}\dot{L}_3 &= \tilde{W}_c^T\dot{\tilde{W}}_c\\ &= -\xi_1\left(\tilde{W}_c^T\nabla\varphi_c(\dot{z}^* + \frac{\Xi\nabla\varepsilon_c}{2})\right)^2 - \xi_1\tilde{W}_c^T\nabla\varphi_c(\dot{z}^* + \frac{\Xi\nabla\varepsilon_c}{2})\varepsilon_H\\ &\quad -\frac{\xi_1}{8}(\tilde{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c)^2 - \frac{\xi_1}{2}\tilde{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c\varepsilon_H\\ &\quad -\frac{3\xi_1}{4}\tilde{W}_c^T\nabla\varphi_c(\dot{z}^* + \frac{\Xi\nabla\varepsilon_c}{2})\times\tilde{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c\\ &\quad -\frac{\xi_2}{2}\Sigma(z,\hat{u}_z)\tilde{W}_c^T\nabla\varphi_c\Xi\nabla V_1\end{aligned} \tag{58}$$

Recombining the equality (58), we have

$$\begin{aligned}\dot{L}_3 &= -\frac{\xi_1}{2}\left(\tilde{W}_c^T\nabla\varphi_c(\dot{z}^*+\frac{\Xi\nabla\varepsilon_c}{2})\right)^2-\frac{\xi_1}{16}(\tilde{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c)^2+\frac{3\xi_1}{2}\varepsilon_H^2\\&-\frac{3\xi_1}{4}\left(\tilde{W}_c^T\nabla\varphi_c(\dot{z}^*+\frac{\Xi\nabla\varepsilon_c}{2})\right)\times\left(\tilde{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c\right)\\&-\frac{\xi_1}{2}\left(\tilde{W}_c^T\nabla\varphi_c(\dot{z}^*+\frac{\Xi\nabla\varepsilon_c}{2})+\varepsilon_H\right)^2-\frac{\xi_1}{16}(\tilde{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c+4\varepsilon_H)^2\\&-\frac{\xi_2}{2}\Sigma(z,\hat{u}_z)\tilde{W}_c^T\nabla\varphi_c\Xi\nabla V_1 \qquad (59)\end{aligned}$$

Then, we have

$$\begin{aligned}\dot{L}_3 &\le 4\xi_1\left(\tilde{W}_c^T\nabla\varphi_c(\dot{z}^*+\frac{\Xi\nabla\varepsilon_c}{2})\right)^2+\frac{3\xi_1}{2}\varepsilon_H^2-\frac{\xi_1}{32}(\tilde{W}_c^T\nabla\varphi_c\Xi\nabla\varphi_c^T\tilde{W}_c)^2\\&-\frac{\xi_2}{2}\Sigma(z,\hat{u}_z)\tilde{W}_c^T\nabla\varphi_c\Xi\nabla V_1 \qquad (60)\end{aligned}$$

Taking norm bounds on (60), and completing the square with respect to $\|\tilde{W}_c^T\nabla\varphi_c\|^2$ and $\|\dot{z}^*+\frac{\Xi\nabla\varepsilon_c}{2}\|$, we have

$$\begin{aligned}\dot{L}_3 &\le \frac{3\xi_1}{2}\varepsilon_H^2-\frac{\xi_1}{64}\|\tilde{W}_c^T\nabla\varphi_c\|^4\bar{\Xi}_m^2+\frac{256\xi_1}{\bar{\Xi}_m^2}\|\dot{z}^*+\frac{\Xi\nabla\varepsilon_c}{2})\|^4\\&-\xi_1\bar{\Xi}_m^2(\frac{\|\tilde{W}_c^T\nabla\varphi_c\|^2}{8}-\frac{16}{\bar{\Xi}_m^2}\|\dot{z}^*+\frac{\Xi\nabla\varepsilon_c}{2}\|^2)^2\\&-\frac{1}{2}\xi_2\Sigma(z,\hat{u}_z)\tilde{W}_c^T\nabla\varphi\Xi\nabla V_1\\&\le -\frac{\xi_1}{64}\|\tilde{W}_c^T\nabla\varphi_c\|^4\bar{\Xi}_m^2+\frac{256\xi_1}{\bar{\Xi}_m^2}\|\dot{z}^*+\frac{\Xi\nabla\varepsilon_c}{2}\|^4+\frac{3\xi_1}{2}\varepsilon_H^2\\&-\frac{1}{2}\xi_2\Sigma(z,\hat{u}_z)\tilde{W}_c^T\nabla\varphi_c\Xi\nabla V_1 \qquad (61)\end{aligned}$$

Using the Pythagorean Theorem and applying the relationship $|\varepsilon_H|\le\bar{\varepsilon}'_{cM}\eta(z)+\bar{\varepsilon}'^2_{cM}\bar{\Xi}_M$, we have

$$\begin{aligned}\dot{L}_3 &\le \frac{3\xi_1}{2}(\bar{\varepsilon}'^4_{cM}+\eta^4(z)+2\bar{\varepsilon}'^4_{cM}\bar{\Xi}_M^2)-\frac{\xi_1}{64}\|\tilde{W}_c^T\nabla\varphi_c\|^4\bar{\Xi}_m^2\\&+\frac{2048\xi_1}{\bar{\Xi}_m^2}(\|\dot{z}^*\|^4+\|\frac{\Xi\nabla\varepsilon_c}{2}\|^4)-\frac{1}{2}\xi_2\Sigma(z,\hat{u}_z)\tilde{W}_c^T\nabla\varphi_c\Xi\nabla V_1 \qquad (62)\end{aligned}$$

According to *Assumption 6*, we have $\|\dot{z}^*\|\le\eta(z)$ and with $\bar{\varphi}'_{cm}\le\|\nabla_e\varphi_c\|\le\bar{\varphi}'_{cM}$, inequality (62) can be written as

$$\dot{L}_3 \le -\frac{\xi_1}{64}\bar{\varphi}'^4_{cm}\bar{\Xi}_m^2\|\tilde{W}_c\|^4+\frac{3\xi_1}{2}(\bar{\varepsilon}'^4_{cM}+2\bar{\varepsilon}'^4_{cM}\bar{\Xi}_M^2)+\frac{128\xi_1\bar{\Xi}_M^4\bar{\varepsilon}'^4_{cM}}{\bar{\Xi}_m^2}$$

$$+\frac{3\xi_1}{2}\eta^4(z)+\frac{2048\xi_1}{\bar{\Xi}_m^2}\eta^4(z)-\frac{1}{2}\xi_2\Sigma(z,\hat{u}_z)\tilde{W}_c^T\nabla\varphi_c\Xi\nabla V_1 \tag{63}$$

Let $\psi_1=\bar{\varphi}_{cm}'^4\bar{\Xi}_m^2/64$, $\psi_2=(2048/\bar{\Xi}_m^2)+3/2$, and $\varrho(\varepsilon)=128\bar{\varepsilon}_{cM}'^4\bar{\Xi}_M^4/\bar{\Xi}_m^2+(3/2)(\bar{\varepsilon}_{cM}'^4+2\bar{\varepsilon}_{cM}'^4\bar{\Xi}_M^2)$. Then, we have

$$\dot{L}_3\leq\xi_1\varrho(\varepsilon)-\xi_1\psi_1\|\tilde{W}_c\|^4+\xi_1\psi_2\eta^4(z)-\frac{1}{2}\xi_2\Sigma(z,\hat{u}_z)\tilde{W}_c^T\nabla\varphi_c\Xi\nabla V_1 \tag{64}$$

Based on the results of (56), (57) and (64), $\dot{L}$ can be rewritten as:

$$\begin{aligned}\dot{L}\leq\;&-CS^T\text{sgn}(S)+CS^T\delta+S^T\Pi D-S^T\Pi K_sS-\frac{\xi_0}{2}\|\tilde{W}_0\|^2+\frac{\xi_0}{2}\|W_0\|^2\\&+\xi_2\nabla V_1^T(F_z+G\hat{u}_z)-\xi_2\nabla V_1^TK_z\nabla V_1+\xi_2\nabla V_1^TD-\xi_2\nabla V_1^TC\Pi^{-1}\text{sgn}(S)\\&+\xi_2\nabla V_1^TC\Pi^{-1}\delta-\xi_2\nabla V_1^TK_sS-\xi_2\nabla V_1^T\tilde{W}_0^T\varphi_0+\xi_1\varrho(\varepsilon)-\xi_1\psi_1\|\tilde{W}_c\|^4\\&+\xi_1\psi_2\eta^4(z)-\frac{1}{2}\xi_2\Sigma(z,\hat{u}_z)\tilde{W}_c^T\nabla\varphi_c\Xi\nabla V_1\end{aligned} \tag{65}$$

Case 1: In this case, $\Sigma(z,\hat{u}_z)=0$. Since $\nabla V_1^T(F_z+G\hat{u}_z)<0$, we have $-\nabla V_1^T(F_z+G\hat{u}_z)>0$. There exists a constant $\vartheta>0$ such that $0<\|\nabla V_1\|\vartheta\leq-\nabla V_1^T\dot{z}$ holds for all $z\in\Phi$. And here we chose the positive function $\eta(z)=\sqrt[4]{K^*\|\nabla V_1\|}$ for positive constant K^*. Then, we have

$$\begin{aligned}\dot{L}\leq\;&-CS^T\text{sgn}(S)+CS^T\delta+S^T\Pi D-S^T\Pi K_sS-\frac{\xi_0}{2}\|\tilde{W}_0\|^2+\frac{\xi_0}{2}\|W_0\|^2\\&-\xi_2\nabla V_1^T\tilde{W}_0^T\varphi_0+\xi_1\varrho(\varepsilon)+\xi_2\nabla V_1^TD-\xi_2\nabla V_1^TC\Pi^{-1}\text{sgn}(S)\\&+\xi_2\nabla V_1^TC\Pi^{-1}\delta-\xi_2\nabla V_1^TK_sS-\xi_2\nabla V_1^TK_z\nabla V_1-\xi_1\psi_1\|\tilde{W}_c\|^4\\&+\xi_1K^*\psi_2\|\nabla V_1\|-\xi_2\vartheta\|\nabla V_1\|\\\leq\;&-\left(\lambda_{min}(C)-\lambda_{max}(\Pi)\bar{D}_M\right)\|S\|+\frac{\|C\|^2}{2}\|S\|^2+\frac{\bar{\delta}_M^2}{2}-\lambda_{min}(\Pi K_s)\|S\|^2\\&-\frac{\xi_0}{2}\|\tilde{W}_0\|^2+\frac{\xi_0}{2}\bar{W}_{0M}^2+\frac{1}{2}\|\nabla V_1\|^2+\frac{\xi_2^2}{2}\bar{D}_M^2+\frac{1}{2}\|\nabla V_1\|^2+\frac{\xi_2^2}{2}\|C\Pi^{-1}\|^2\\&+\frac{1}{2}\|\nabla V_1\|^2+\frac{\xi_2^2}{2}\|C\Pi^{-1}\|^2\bar{\delta}_M^2-\xi_2\lambda_{min}(K_z)\|\nabla V_1\|^2+\frac{\xi_2^2}{2}\|\nabla V_1\|^2\\&+\frac{\|K_s\|^2}{2}\|S\|^2+\frac{\xi_2^2}{2\lambda_2}\|\nabla V_1\|^2+\frac{\lambda_2}{2}\bar{\varphi}_{0M}^2\|\tilde{W}_0\|^2+\xi_1\varrho(\varepsilon)-\xi_1\psi_1\|\tilde{W}_c\|^4\\&-(\xi_2\vartheta-\xi_1K^*\psi_2)\|\nabla V_1\|\end{aligned} \tag{66}$$

where λ_2 is a positive constant.

By suitable selecting the parameters ξ_1,ξ_2, K^*, Π and C such that $\frac{\xi_2}{\xi_1}>$

$\frac{K^*\psi_2}{\vartheta}$, $\lambda_{min}(C) > \lambda_{max}(\Pi)\bar{D}_M$, then we have

$$\begin{aligned}\dot{L} \quad &\leq \quad -\left(\lambda_{min}(\Pi K_s) - \frac{\|K_s\|^2}{2} - \frac{\|C\|^2}{2}\right)\|S\|^2 \\ &\quad -\left(\xi_2\lambda_{min}(K_z) - \frac{\xi_2^2}{2} - \frac{\xi_2^2}{2\lambda_2} - \frac{3}{2}\right)\|\nabla V_1\|^2 \\ &\quad -\left(\frac{\xi_0}{2} - \frac{\lambda_2}{2}\bar{\varphi}_{0M}^2\right)\|\tilde{W}_0\|^2 - \xi_1\psi_1\|\tilde{W}_c\|^4 + \frac{\xi_0}{2}\bar{W}_{0M}^2 \\ &\quad +\xi_1\varrho(\varepsilon) + \frac{\delta_M^2}{2} + \frac{\xi_2^2}{2}\bar{D}_M^2 + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2} + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2}\delta_M^2 \end{aligned} \tag{67}$$

Let $\kappa_{s1} = \lambda_{min}(\Pi K_s) - \frac{1}{2}\|K_s\|^2 - \frac{\|C\|^2}{2}$, $\kappa_{z1} = \xi_2\lambda_{min}(K_z) - \frac{\xi_2^2}{2} - \frac{\xi_2^2}{2\lambda_2} - \frac{3}{2}$, $\kappa_{w1} = \frac{\xi_0}{2} - \frac{\lambda_2}{2}\bar{\varphi}_{0M}^2$, $\varsigma_1 = \xi_1\varrho(\varepsilon) + \frac{\bar{\delta}_M^2}{2} + \frac{\xi_2^2}{2}\bar{D}_M^2 + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2} + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2}\bar{\delta}_M^2 + \frac{\xi_0}{2}\bar{W}_{0M}^2$. By suitably choosing the parameters to guarantee $\kappa_{s1} > 0, \kappa_{z1} > 0, \kappa_{w1} > 0$, then (66) can be rewritten as

$$\dot{L} \leq -\kappa_{s1}\|S\|^2 - \kappa_{z1}\|\nabla V_1\|^2 - \kappa_{w1}\|\tilde{W}_0\|^2 - \xi_1\psi_1\|\tilde{W}_c\|^4 + \varsigma_1 \tag{68}$$

Case 2: In this case, $\Sigma(z, \hat{u}_z) = 1$, based on *Assumption 5*, we know $\nabla V_1^T(F_z + Gu_z^*) = -\nabla V_1^T\Lambda(z)\nabla V_1$ then (65) can be rewritten as

$$\begin{aligned}\dot{L} \quad &\leq \quad -CS^T\text{sgn}(S) + CS^T\delta + S^T\Pi D - S^T\Pi K_s S - \frac{\xi_0}{2}\|\tilde{W}_0\|^2 + \frac{\xi_0}{2}\|W_0\|^2 \\ &\quad +\xi_2\nabla V_1^T D - \xi_2\nabla V_1^T K_z\nabla V_1 - \xi_2\nabla V_1^T C\Pi^{-1}\text{sgn}(S) + \xi_2\nabla V_1^T C\Pi^{-1}\delta \\ &\quad -\xi_2\nabla V_1^T K_s S - \xi_1\psi_1\|\tilde{W}_c\|^4 + \xi_1 K^*\psi_2\|\nabla V_1\| + \xi_2\nabla V_1^T(F_z + Gu_z^*) \\ &\quad +\xi_1\varrho(\varepsilon) - \frac{\xi_2}{2}\tilde{W}_c^T\nabla\varphi_c\Xi\nabla V_1 - \xi_2\nabla V_1^T\tilde{W}_0^T\varphi_0 + \frac{\xi_2}{2}\nabla V_1^T\Xi\nabla\varphi_c^T\tilde{W}_c \\ &\quad +\frac{\xi_2}{2}\nabla V_1^T\Xi\nabla\varepsilon_c \end{aligned} \tag{69}$$

And

$$\begin{aligned}\dot{L} \quad &\leq \quad -\left(\lambda_{min}(C) - \lambda_{max}(\Pi)\bar{D}_M\right)\|S\| + \frac{\|C\|^2}{2}\|S\|^2 + \frac{\delta_M^2}{2} - \lambda_{min}(\Pi K_s)\|S\|^2 \\ &\quad -\frac{\xi_0}{2}\|\tilde{W}_0\|^2 + \frac{\xi_0}{2}\bar{W}_{0M}^2 + \frac{1}{2}\|\nabla V_1\|^2 + \frac{1}{2}\xi_2^2\bar{D}_M^2 + \frac{1}{2}\|\nabla V_1\|^2 + \frac{\xi_2^2}{2}\|C\Pi^{-1}\|^2 \\ &\quad +\frac{1}{2}\|\nabla V_1\|^2 + \frac{\xi_2^2}{2}\|C\Pi^{-1}\|^2\|\delta\|^2 - \xi_2\lambda_{min}(K_z)\|\nabla V_1\|^2 + \xi_1\varrho(\varepsilon) - \xi_1\psi_1\|\tilde{W}_c\|^4 \\ &\quad +\xi_1 K^*\psi_2\|\nabla V_1\| + \frac{\xi_2^2}{2\lambda_2}\|\nabla V_1\|^2 + \frac{\lambda_2}{2}\bar{\varphi}_{0M}^2\|\tilde{W}_0\|^2 - \xi_2\bar{\Lambda}_m\|\nabla V_1\|^2 \\ &\quad +\frac{\xi_2}{2}\bar{\Xi}_M\bar{\varepsilon}_{cM}'\|\nabla V_1\| \end{aligned} \tag{70}$$

With suitable values of Π and C such that $\lambda_{min}(C) > \lambda_{max}(\Pi)\bar{D}_M$. Recombining the inequality (70), we have

$$\begin{aligned} \dot{L} \leq & -\left(\lambda_{min}(\Pi K_s) - \frac{\|K_s\|^2}{2} - \frac{\|C\|^2}{2}\right)\|S\|^2 \\ & -\left(\xi_2\lambda_{min}(K_z) + \xi_2\bar{\Lambda}_m - \frac{\xi_2^2}{2} - \frac{\xi_2^2}{2\lambda_2} - \frac{3}{2}\right)\|\nabla V_1\|^2 \\ & +(\xi_1 K^*\psi_2 + \frac{\xi_2}{2}\bar{\Xi}_M\bar{\varepsilon}'_{cM})\|\nabla V_1\| - \xi_1\psi_1\|\tilde{W}_c\|^4 \\ & -\left(\frac{\xi_0}{2} - \frac{\lambda_2}{2}\bar{\varphi}_{0M}^2\right)\|\tilde{W}_0\|^2 + \xi_1\varrho(\varepsilon) + \frac{\xi_0}{2}\bar{W}_{0M}^2 \\ & +\frac{\bar{\delta}_M^2}{2} + \frac{\xi_2^2}{2}\bar{D}_M^2 + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2} + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2}\bar{\delta}_M^2 \end{aligned} \quad (71)$$

Let $\kappa_{s2} = \lambda_{min}(\Pi K_s) - \frac{\|K_s\|^2}{2} - \frac{\|C\|^2}{2}$, $\kappa_{z2} = \xi_2\lambda_{min}(K_z) + \xi_2\bar{\Lambda}_m - \frac{1}{2}\xi_2^2 - \frac{\xi_2^2}{2\lambda_2} - \frac{3}{2}$, $\kappa_{w2} = \frac{\xi_0}{2} - \frac{\lambda_2}{2}\bar{\varphi}_{0M}^2$, $\varsigma_2 = \xi_1\varrho(\varepsilon) + \frac{\bar{\delta}_M^2}{2} + \frac{\xi_2^2}{2}\bar{D}_M^2 + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2} + \frac{\xi_2^2\|C\Pi^{-1}\|^2}{2}\bar{\delta}_M^2 + \frac{\xi_0}{2}\bar{W}_{0M}^2$. By suitably choosing the parameters to guarantee $\kappa_{s2} > 0, \kappa_{z2} > 0, \kappa_{w2} > 0$, then (71) can be rewritten as

$$\begin{aligned} \dot{L} \leq & -\kappa_{s2}\|S\|^2 + (\xi_1 K^*\psi_2 + \frac{\xi_2}{2}\bar{\Xi}_M\bar{\varepsilon}'_{cM})\|\nabla V_1\| \\ & -\kappa_{w2}\|\tilde{W}_0\|^2 - \kappa_{z2}\|\nabla V_1\|^2 - \xi_1\psi_1\|\tilde{W}_c\|^4 + \zeta_2 \\ \leq & -\kappa_{s2}\|S\|^2 - \kappa_{w2}\|\tilde{W}_0\|^2 - \xi_1\psi_1\|\tilde{W}_c\|^4 + \varsigma_2 \\ & -\frac{\kappa_{z2}}{2}\left(\|\nabla V_1\| - \frac{2\xi_1 K^*\psi_2 + \xi_2\bar{\Xi}_M\bar{\varepsilon}'_{cM}}{2\kappa_{z2}}\right)^2 \\ & -\frac{\kappa_{z2}}{2}\|\nabla V_1\|^2 + \frac{8\xi_1^2 K^{*2}\psi_2^2 + 2\xi_2^2\bar{\Xi}_M^2\bar{\varepsilon}'^2_{cM}}{8\kappa_{z2}} \\ \leq & -\kappa_{s2}\|S\|^2 - \kappa_{w2}\|\tilde{W}_0\|^2 - \xi_1\psi_1\|\tilde{W}_c\|^4 + \varsigma_2 \\ & -\frac{\kappa_{z2}}{2}\|\nabla V_1\|^2 + \frac{8\xi_1^2 K^{*2}\psi_2^2 + 2\xi_2^2\bar{\Xi}_M^2\bar{\varepsilon}'^2_{cM}}{8\kappa_{z2}} \end{aligned} \quad (72)$$

Based on the analysis results of (68), (72) in *Case 1* and *Case 2*. If the closed loop signals including the trajectory of sliding mode surface $S(t)$, the estimation value of NN weights $\tilde{W}_0, \tilde{W}_c$ and the defined function ∇V_1 are out of the bounds described in (53)-(51), we can derive that the time derivative of Lyapunov function $\dot{L}(t)$ is less than zero. And all the signals will converge to the defined bounds. Then, according to the standard Lypunov extension theorem [49], we can derive that the trajectory of sliding mode surface $S(t)$, the estimation value of NN weights $\tilde{W}_0, \tilde{W}_c$ and the defined function ∇V_1 are proved to be UUB, and the ultimate convergence sets for each signal are small enough with suitable parameters. This completes the proof.

In the next, the bound of auxiliary system state ζ will be given, we construct a Lyapunov candidate function $L_\zeta = \frac{1}{2}\zeta^T\zeta$. Taking the derivative of L_ζ, we have

$$\begin{aligned} \dot{L}_\zeta &= -\zeta^T A\zeta + \zeta^T G\triangle u \\ &\leq -(\lambda_{min}(A) - \frac{1}{2})\|\zeta\|^2 + \frac{1}{2}\bar{g}_M^2\bar{\Delta}_M^2 \\ &\leq -\kappa_\zeta\|\zeta\|^2 + \frac{1}{2}\bar{g}_M^2\bar{\Delta}_M^2 \end{aligned} \tag{73}$$

where $\kappa_\zeta = \lambda_{min}(A) - \frac{1}{2} > 0$.

Then, $\dot{L}_\zeta < 0$ yields

$$\|\zeta\| \leq \sqrt{\frac{\bar{g}_M^2\bar{\Delta}_M^2}{2\kappa_\zeta}} \tag{74}$$

Remark 2: Generally, the Lyapunov candidate function $V_1(z)$ defined in *Assumption 5* is selected in form of polynomials with respect to z, one can derive that ∇V_1 is also a polynomial with respect to z. And based on the result of *Theorem 2* such that ∇V_1 is UUB, we can easily know that the trajectory of the closed-loop system (8) is UUB. Therefore, it is reasonable to assume that there exists a positive constant $\bar{\aleph}$ such that $\|z\| \leq \bar{\aleph}$.

Define the system tracking error $e = x - x_r$, based on (6), we have $e = x - x_r = z + \zeta$, then

$$\|e\| = \|x - x_r\| \leq \|z\| + \|\zeta\| \leq \bar{\aleph} + \sqrt{\frac{\bar{g}_M^2\bar{\Delta}_M^2}{2\kappa_\zeta}} \tag{75}$$

Therefore, the tracking error e of closed-loop system is proved to be bounded.

4. Simulation Study

In this section, we firstly provide the attitude tracking controller design of NSHV, then the simulation tests will be given to illustrate the effectiveness of the proposed optimal integral sliding mode tracking control scheme.

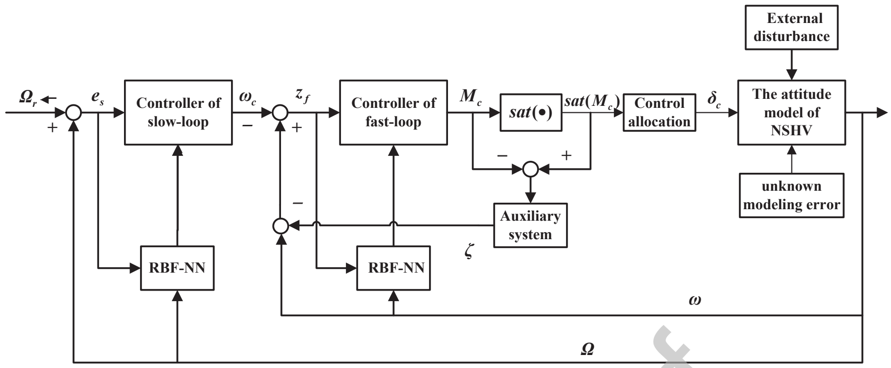

Figure 1: The structure of the optimal adaptive sliding mode attitude control of NSHV

4.1. Attitude tracking control of NSHV

The attitude motion dynamics of NSHV are shown as follows [50]:

$$\begin{cases} \dot{\Omega} = f_s(\Omega) + g_s(\Omega)\omega_c + \phi_s(\Omega) + d_s \\ y_\Omega = \Omega \\ \dot{\omega} = f_f(\Omega,\omega) + g_f sat(M_c) + \phi_f(\Omega,\omega) + d_f \\ y_\omega = \omega \end{cases} \tag{76}$$

where $\Omega = [\alpha, \beta, \mu]^T$ denotes the attitude angle vector of slow-loop system, including attack, sideslip and roll angles, respectively. $\omega = [p, q, r]^T$ is the angle rate vector of fast-loop system, which are roll, pitch and yaw rates, respectively. $f_s = [f_\alpha, f_\beta, f_\mu]^T$ and $f_f = [f_p, f_q, f_r]^T$ are known system state functions. g_s and g_f are known system control gain matrices. $M_c = [l_c, m_c, n_c]^T$ denotes the control input vector. ϕ_s and ϕ_f denote the system unmodeled dynamics, d_s and d_f are unknown external disturbances. The concrete forms of f_s, f_f, g_s, g_f can reference to [51]. The whole control structure block diagram is shown in Figure 1.

For slow-loop system, we only consider the effects of unknown modeling error and external disturbance, and the robust optimal tracking controller can be design as

$$\omega_c = \omega_s + \omega_o \tag{77}$$

where ω_s is the adaptive integral sliding mode control input, which can be designed in form of (25) as

$$\omega_s = -C_s(\Pi_s g_s)^{-1}\varpi(S_s) - g_s^{-1}(\hat{W}_{0s}^T\varphi_{0s} + K_{ss}S_s) \tag{78}$$

and ω_o is the optimal tracking controller in form of

$$\omega_o = g_s^{-1}(\dot{\Omega}_r - f_s(\Omega_r)) - \frac{1}{2}R_s^{-1}g_s^T\nabla\varphi_{cs}\hat{W}_{cs} - g_s^{-1}K_{zs}\nabla V_{1s} \tag{79}$$

For fast-loop system, not only the effects of unknown modeling error and external disturbance, but also the input saturation is considered, and the robust optimal tracking controller can be design as

$$M_c = M_s + M_o \tag{80}$$

where ω_s is the adaptive integral sliding mode control input, which can be designed in form of (25) as

$$M_s = -C_f(\Pi_f g_f)^{-1}\varpi(S_f) - g_f^{-1}(\hat{W}_{0f}^T\varphi_{0f} + A_f\zeta_f + K_{sf}S_f) \tag{81}$$

and the optimal tracking controller ω_o can be designed in form of (56) as

$$M_o = g_f^{-1}(\dot{\omega}_c - f_f(\omega_c)) - \frac{1}{2}R_f^{-1}g_f^T\nabla\varphi_{cf}\hat{W}_{cf} - g_f^{-1}K_{zf}\nabla V_{1f} \tag{82}$$

4.2. Simulation results

In this paper, we assume that the flight of NSHV is in lower-level near space ($H_0 = 22000m$) with high speed ($V_0 = 3000m/s$). And the initial conditions of NSHV are chosen as $\alpha_0 = \beta_0 = \mu_0 = 1°$, $p_0 = q_0 = r_0 = 0$. The desired flight attitude angles are given as:

$$\begin{cases} \alpha_r = \begin{cases} 3°, t \le 30 \\ 5°, t > 30 \end{cases} \\ \beta_r = 0° \\ \mu_r = 6\sin(0.2t)° \end{cases}$$

In order to avoid the discontinuity of the desired signal α_r, a first order filtering element $5/(s+5)$ is added to the desired signal.

Assume the aerodynamic force and aerodynamic moment coefficients have 20% uncertainties which represent the system unmodeled dynamics, and the external time-varying disturbances acting on slow-loop and fast-loop systems are:

$$\begin{cases} d_s = [0.01\sin(2t), 0.01\sin(t), 0.01\sin(2t)]^T \\ d_f = [0.07\sin(4t), 0.06\sin(3t), 0.06\sin(3t)]^T \end{cases}$$

The saturability of control moment is set to be $M_{cmin} \leq M_c \leq M_{cmax}$, where $M_{cmin} = [-2000, -20000, -20000]^T kN.m$, $M_{cmax} = [2000, 20000, 20000]^T kN.m$. The active functions for critic neural network are designed as:

$$\begin{aligned} \varphi_{cs} = [&z_{s1}, z_{s2}, z_{s3}, \cos(z_{s1}), \cos(z_{s2}), \cos(z_{s3}), \cos(2z_{s1}), \\ &\cos(2z_{s2}),\ \cos(2z_{s3}),\ \sin(z_{s1}), \sin(z_{s2}),\ \sin(z_{s3}), \\ &\sin(2z_{s1}), \sin(2z_{s2}), \sin(2z_{s3}), \tanh(z_{s1}), \tanh(z_{s2}), \\ &\tanh(z_{s3}), \tanh(2z_{s1}), \tanh(2z_{s2}), \tanh(2z_{s1})]^T; \\ \varphi_{cf} = [&z_{f1}, z_{f2}, z_{f3}, \cos(z_{f1}), \cos(z_{f2}), \cos(z_{f3}), \cos(2z_{f1}), \\ &\cos(2z_{f2}),\ \cos(2z_{f3}),\ \sin(z_{f1}), \sin(z_{f2}),\ \sin(z_{f3}), \\ &\sin(2z_{f1}), \sin(2z_{f2}), \sin(2z_{f3}), \tanh(z_{f1}), \tanh(z_{f2}), \\ &\tanh(z_{f3}), \tanh(2z_{f1},), \tanh(2z_{f2}), \tanh(2z_{f3})]^T. \end{aligned}$$

And the initial value of critic neural network weight $W_{cs} \in \Re^{21}$ and $W_{cf} \in \Re^{21}$ are set between $[-1, 1]$. After sufficiently training process, the convergence of the critic NN weights are shown in Fig. 2 and Fig. 3. And we can observe that, after 300 seconds training, the converge value of critic value are

$$\begin{aligned} \hat{W}_{cs} = [&-0.7236, 0.7288, -0.3382, -0.7723, -0.1532, \\ &-0.0374, -1.6790, -1.8112, -1.4846, 0.8601, \\ &-0.7177, -0.0903, 0.4097, -0.131, 0.0415, 0.2294, \\ &0.5546, 0.3687, -0.6062, -0.1493, -0.0707]^T; \\ \hat{W}_{cf} = [&0.7782, 0.26, -0.0276, 0.3969, -0.3139, -0.1386, \\ &-0.0324, -0.7174, -0.4548, 0.2472, -0.127, \\ &0.3011, 0.2358, -0.2391, -0.5878, -0.363, \\ &0.5713, 0.1267, -0.3611, 0.4291, 0.4313]^T. \end{aligned}$$

Based on the converged weight vectors, the optimal tracking controllers of slow-loop system ω_o and fast-loop system M_o can be designed.

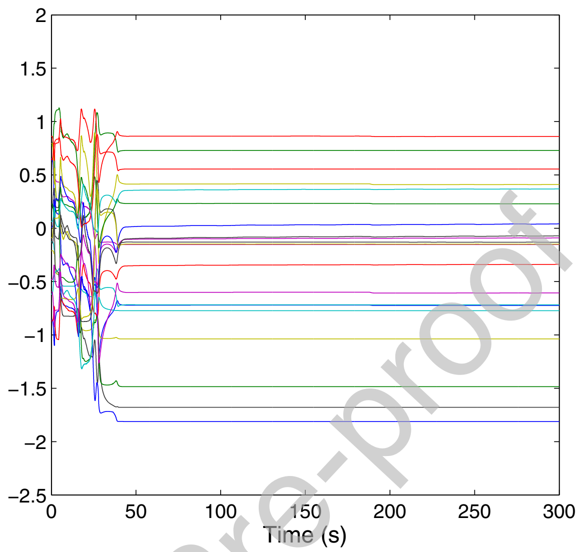

Figure 2: Evalution of critic NN weights for slow-loop system

The RBF basis functions for slow-loop system and fast-loop system are designed as: $\varphi_0(\Omega)_\iota = exp(-\frac{\|\Omega-\nu_{s\iota}\|^2}{2b_{s\iota}^2})$, $\varphi_0(\omega)_\iota = exp(-\frac{\|\omega-\nu_{f\iota}\|^2}{2b_{f\iota}^2})$, where $\iota = 1, \cdots, 9$, $b_{s\iota} = b_{f\iota} = 0.02$, $\nu_{s\iota} = \nu_{f\iota} = [0.01(\iota-5), 0.01(\iota-5), 0.01(\iota-5)]'$. The initial value of $W_{0s} \in \Re^{9\times 3}$ and $W_{0f} \in \Re^{9\times 3}$ are set to 0. The relevant parameters of adaptive integral sliding mode control are $C_s = 2, \Pi_s = 2$, $\Gamma_s = 2\text{eye}(9)$; $K_{ss} = \text{diag}\{2; 2; 2\}$, $K_{zs} = \text{diag}\{3; 3; 3\}$, $C_f = 1, \Pi_f = 2$, $A_f = \text{diag}\{1; 1; 1\}$, $\Gamma_f = 2\text{eye}(9)$, $K_{sf} = \text{diag}\{1; 1; 1\}$, $K_{zf} = \text{diag}\{3; 3; 3\}$. The parameters of optimal tracking control are $R_s = \text{diag}\{1; 1; 1\}$, $Q_s = \text{diag}\{20; 10; 250\}$, $\xi_{0s} = 0.5$ $\xi_{1s} = 0.5$, $\xi_{2s} = 1$; $R_f = \text{diag}\{1; 1; 1\}$, $Q_f = \text{diag}\{10; 10; 10\}$, $\xi_{0f} = 0.5$, $\xi_{1f} = 0.5$, $\xi_{2f} = 1$.

Rest of simulation results are given from Fig.4-7. First of all, the tracking responses of attitude angle are shown in Fig. 4. From the figure, we can observer that the actual attitude angles can well track the desired signals in a

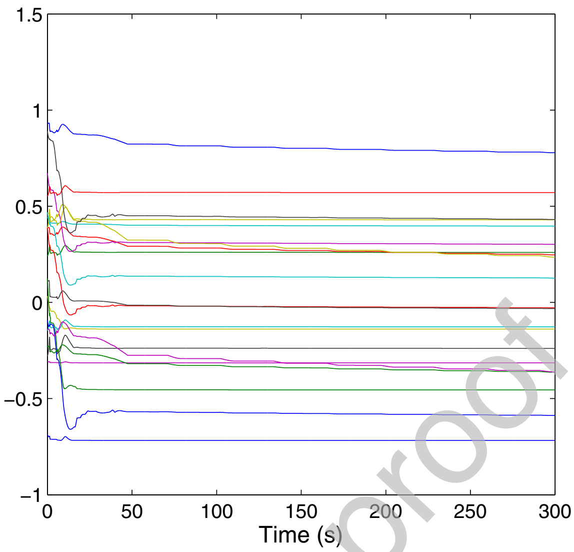

Figure 3: Evalution of critic NN weights for fast-loop system

short time. And the corresponding tracking errors are shown in Fig.5, which can converge to a small bound near zero. Secondly, the values of attitude angler rates are shown in Fig. 6, which are stable signals. Finally, the control input of control moments including roll, pitch and yaw control moments are shown in Fig.7 and we can also observe that the control moment input are eligible and the values of control input signals are restricted to designated areas. In summary, the proposed robust optimal flight control strategy is feasible for the NSHV attitude system with system uncertainty, external disturbance and input saturation from above simulation results.

5. Conclusion

For nonlinear NSHV attitude system with system uncertainty, external disturbance and control input saturation, a novel robust optimal tracking control method based on integral sliding mode method and ADP algorithm has been presented. First of all, the RBFNN approximation technique and integral sliding

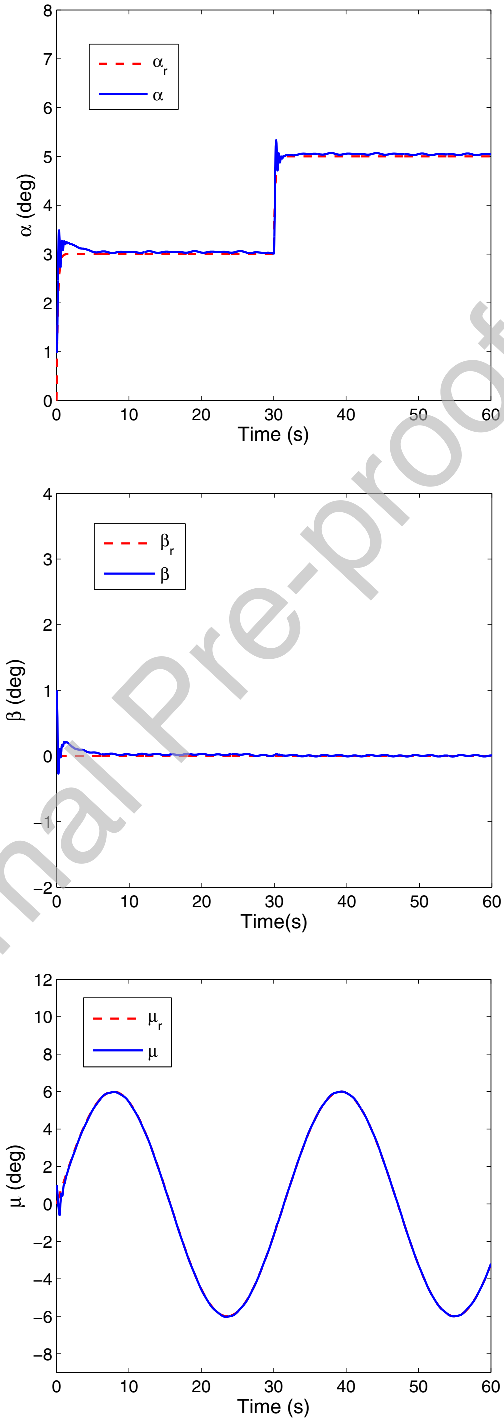

Figure 4: The responses of the attitude angles

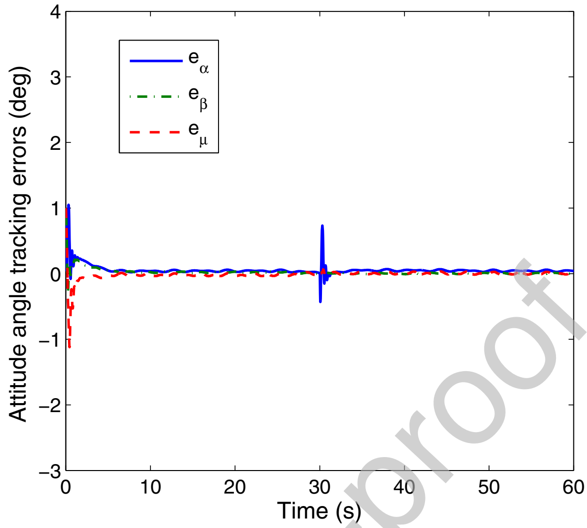

Figure 5: The responses of the attitude tracking errors

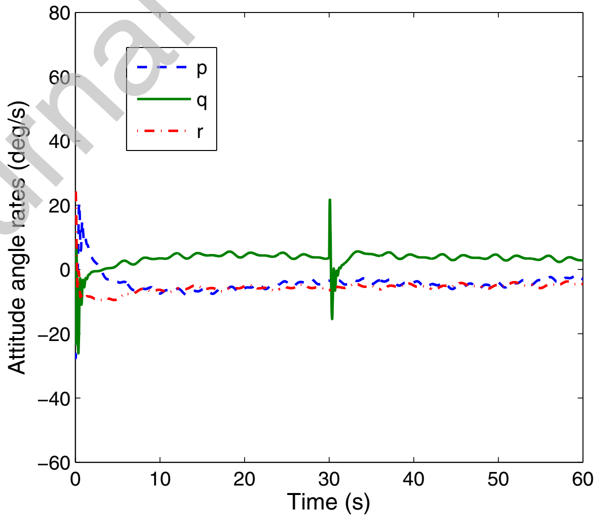

Figure 6: The responses of the attitude angler rates

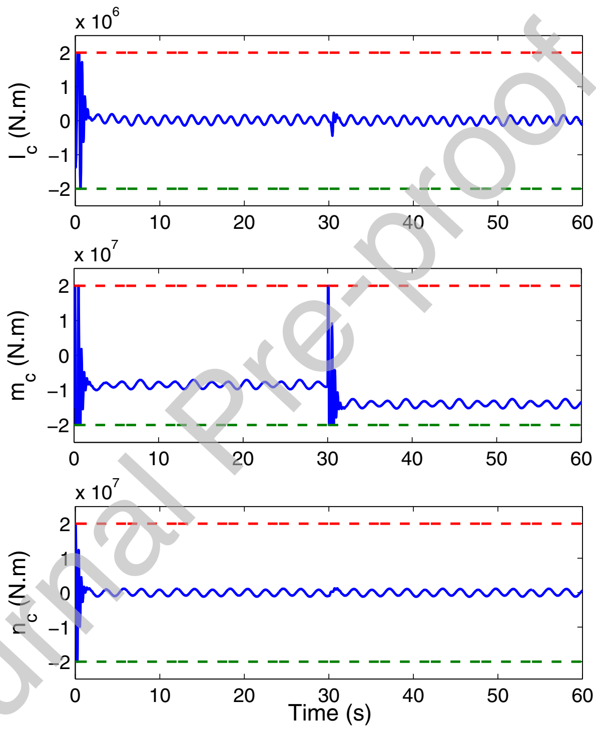

Figure 7: The control input of control moments

mode control theorem are applied to deal with system uncertainty and external disturbance, and an auxiliary system is constructed to compensate the effect of input saturation. Based on the above methods, an AISMC law is presented to force the system trajectories to a defined integral sliding surface, and the effects of system uncertainty, external disturbance and control input saturation can be eliminated. In addition, an ADP based optimal tracking controller is designed for nominal system such that a prescribed level of tracking performance is guaranteed. Finally, all the signals in the closed-loop system are proved to be UUB and simulation results for NSHV attitude system are given to verify the effectiveness of proposed control strategy.

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Conflicts of Interest

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