



Research article

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ABSTRACT

There are many factors that influence the day-ahead market bidding strategies of a generation company (GenCo) within the framework of the current energy market. Environmental policy issues are giving rise to emission limitation that are becoming more and more important for fossil-fueled power plants, and these must be considered in their management. This work investigates the influence of the emissions reduction plan and the incorporation of the medium-term derivative commitments in the optimal generation bidding strategy for the day-ahead electricity market. Two different technologies have been considered: the high-emission technology of thermal coal units and the low-emission technology of combined cycle gas turbine units. The Iberian Electricity Market (MIBEL) and the Spanish National Emissions Reduction Plan (NERP) defines the environmental framework for dealing with the day-ahead market bidding strategies. To address emission limitations, we have extended some of the standard risk management methodologies developed for financial markets, such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), thus leading to the new concept of Conditional Emission at Risk (CEaR). This study offers electricity generation utilities a mathematical model for determining the unit's optimal generation bid to the wholesale electricity market such that it maximizes the long-term profits of the utility while allowing it to abide by the Iberian Electricity Market rules as well as the environmental restrictions set by the Spanish National Emissions Reduction Plan. We analyze the economic implications for a GenCo that includes the environmental restrictions of this National Plan as well as the NERP's effects on the expected profits and the optimal generation bid.

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1. Introduction

1.1. EU National Emissions Reduction Plan (NERP)

The share of fossil fuels in the world's energy production is more than 85% and in electricity generation more than 60% (Bogdan et al., 2007). Although they provide a reliable and affordable source of energy, the use of fossil-fuelled power plants harm the global

ecosystem by emitting noxious gases and toxic substances into the atmosphere, thus causing the greenhouse effect, that is thought to be responsible for climate change. The EU sets limits for emissions of pollutants from large combustion plants through the so-called National Emissions Reduction Plan (NERP) Directive 2001/80/EC (2003). This directive applies to combustion plants (technical equipment in which fuels are oxidized in order to use the heat generated) with a rated thermal input equal to or greater than 50 MW, irrespective of the type of fuel used (solid, liquid or gaseous). This directive limits the amount of sulphur dioxide (SO₂) and nitrogen oxides (NO_x) emitted from large combustion plants each year. Following these commitments, the Spanish public administration launched the Spanish National Emissions Reduction Plan in 2004 (NERP, Real Decreto 430/2004 National emission reduction plan (2007)). The Spanish NERP imposes, for the period 2008–15, a global reduction of 81% of SO₂ and 15% of NO_x emissions

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as compared to 2001 emissions. NERP directive must be inevitably and explicitly considered when elaborating the generation units' optimal sale bid to the wholesale electricity market, and we will see in this study how strongly this directive reshapes the optimal bid, changing the balance between the production of the different generation technologies and the commitment of the medium-term derivatives. The commitment of this work is to propose a new mathematical formulation that extends the current NERP-violation risk-averse models with a more flexible risk-accepting formulation.

1.2. The Iberian Electricity Market (MIBEL)

The Iberian Electricity Market (MIBEL) is the result of a joint initiative by the Governments of Portugal and Spain to integrate their markets. This market is organized by the Iberian Market Operator of Energy (OMIE, according to its Spanish initials), which has to match supply with demand in real time. The day-ahead market (DAM) of day D consists of a series of twenty-four hourly auctions which are cleared simultaneously between 10:00 h and 10:30 h of the previous day (D-1). The clearing price λ_t^D of each hourly auction for time t is determined by the intersection of the aggregated offer and demand curves: Fig. 1. All the sale/purchase bids with a lower/greater bid price are matched and will be remunerated at the same clearing price λ_t^D , whichever the original bid price.

Bilateral contracts (BC) are agreements between a GenCo and a qualified consumer to provide a given amount of electrical energy at a stipulated price along with a delivery period. The agreement terms are, namely, the energy, the price and the delivery period, all of which are negotiated several days before the DAM with the restriction that the energy destined to the BC cannot be included in the DAM. A futures contract (FC) is an exchange-traded derivative that represents agreements to buy/sell some underlying asset in the future at a specified price (Hull, 2008). The DAM's operator demands that every GenCo commit to the quantity designated to each FC through the DAM bidding of a given set of generation units. This commitment is made through a sale offer with a bid price of 0€/MWh, the so-called *price accepting bid*. All price accepting bids will be matched (i.e., accepted) in the clearing process meaning that the energy shall be produced and will be remunerated at the DAM spot price.

1.3. Generation units

This work considers a GenCo with a set of coal thermal units (high emission technology) and combined cycle gas turbine (CCGT) generation units (low emission technology). The combined cycle

gas turbine units represent a combination of combustion and steam turbines in a power plant (Heredia et al., 2012). The CCGT plants employ more than one thermodynamic cycle thus improving the efficiency of electricity generation (Bachmann et al., 1999). Currently, most of the new generating unit installations in Europe are CCGT units. They are between 20 and 30% more efficient than thermal power plants, and can reach up to 60% efficiency. According to Bachmann et al. (1999), gas turbines in combined-cycle plants produce a practically complete combustion with very low concentrations of unburned elements such as CO or hydrocarbons. Consequently, they cause less climate-damage because they do not produce SO₂ emissions at all, and the NO_x emissions are negligible in comparison with those of thermal units.

2. Literature review and contributions

2.1. Literature review

The greater part of the published works concerning the relationship between energy production and pollutant emissions are devoted to studying the impact of CO₂ emissions being traded in the power industry, especially through medium-term models (Gnansounou et al., 2004; Reneses and Centeno, 2008; Sousa et al., 2005) but also some that are short-term (Kockar et al., 2009). None of this papers consider the SO₂ and NO_x emissions, which is the goal of the NERP, although these rules substantially modifies the shape of the optimal bid strategy of an electricity producer. Actually, quite a bit of attention has been paid in the bibliography to the optimal generation bid strategies under SO₂ and NO_x emission limits. Most of the research production related to SO₂ and NO_x emissions has been dedicated to general long-term studies of different aspects of the impact of reducing the SO₂ and NO_x emissions in the wholesale electricity production system but without any explicit mention to the NERP (Funakin et al., 2011; Tang and Zhou, 2012). Among the few papers that study optimal generation under emission limits, (Liu, 2011) develops a load dispatch model to minimize NO_x emissions only by taking fuel cost and stochastic wind power availability as constraints, thus disregarding the electricity market entirely. The model in Lu and Shahidehpour (2005) considers a classical deterministic unit commitment of both thermal and combined cycle units; this minimizes the generation (fuel) costs (no electricity market) by satisfying simple bounds on SO₂ and NO_x emissions. A quite common approach by several recent papers to the handling of emission limits involves using multiobjective optimization techniques in which both profits and emissions are minimized (Shayanfar et al., 2012; Peng et al., 2012; Jadid and Vahidinasab, 2011), sometimes with additional emissions limit constraints (Catalao and Mendes, 2011). Despite the interest of all these studies, it is worth mentioning that none of them are optimal-bid models, as their formulations do not incorporate the bid rules of the electricity market, and the influence of the electricity market is reduced either to a deterministic forecasting of the electricity prices (Shayanfar et al., 2012; Catalao and Mendes, 2011; Peng et al., 2012) or to the use of spot price scenarios (Jadid and Vahidinasab, 2011). Some recent works have taken into account CO₂ emission constraints in the self-scheduling of thermal units (not CCGT units) that operate in electricity markets through two-stage stochastic programming (Geng et al., 2017; Laia et al., 2015; Rebennack et al., 2012). A flaw of these studies is that the emission limits are imposed as hard constraints that avoids the violation of emission limits for every scenario, even for the most unlikely scenarios, which, as we will show in this paper, is quite a restrictive modelization. A general flaw of the revised works is that they neglect both the specific rules of the day-ahead markets, including the handling of futures and bilateral contracts, and the NERP rules.

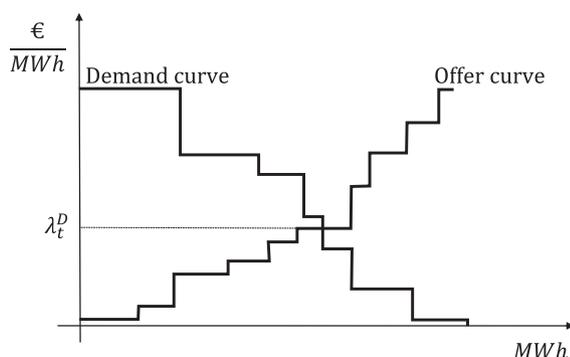


Fig. 1. Market clearing for a certain hour: intersection of the aggregated offer and demand curves.

As a consequence, these models represent a rough approximation to the complex decision making problems faced by any GenCo that must decide its optimal bid abiding by both the electricity market and NERP regulation, thus providing solutions that actually may be infeasible. Another general lack of the published papers is that they usually omit the CCGT units, which are quite more difficult to represent than regular thermal units, but play an important role in any environmental friendly power system. Finally a common drawback of the existing models is that they are NERP-violation risk-averse models, as they prevent completely the violation even in the most unlikely scenarios, contrary to the more flexible risk-accepting formulation proposed in this study. The combination of the rationale use of CCGT units together with that risk-accepting formulation allow keeping the emissions under acceptable levels without a critical reduction in total profits.

2.2. Contribution

This work presents a new stochastic programming model to cope with the optimal generation bid to the next day's auctions of the MIBEL day-ahead market (DAM) taking into account the SO_2 and NO_x emission limits of the Spanish NERP. We consider a price taker GenCo with a set of thermal coal and CCGT generation units subject to SO_2 and NO_x emissions limits. The objective is to find the generation scheduling and sales bid of each of the generators that maximize the expected value of the net profit of a GenCo including the start-up, shut-down and generation costs together with the incomes from the day-ahead market, futures and bilateral contracts. Several characteristics distinguish this paper from the previous works in this area. In contrast to other studies, our model considers the ex-ante negotiated Futures Contracts (FC) and Bilateral Contracts (BC) of the GenCo, that are integrated into the optimal bidding strategy according to the MIBEL directives, and it provides the optimal generation bid for each of the generation units by assuming the optimal offer curve model developed in Corchero and Heredia (2011) and Corchero et al. (2013). Moreover, the day-ahead electricity market bid with futures and bilateral contracts model (DAMB-FBC) proposed in Corchero et al. (2013) has been improved upon in the present work by modeling the CCGT unit commitment together with the explicit consideration of the NERP emission limits, specifically by means of a new measure of risk called Conditional Emission-at-Risk (CEaR), which is one of the most important contributions of this paper. The resulting model for the optimal day-ahead market bid with emission risk (DAMB-ER) has been validated with real data from generation units operating in the MIBEL and with real prices from the Spanish day-ahead market, all of which have been used to study the impact that the Spanish NERP directives have on the optimal generation bid and expected profit of utilities. The results can be easily extrapolated to any country with similar NERP regulations.

The remainder of this paper is organized as follows. Section 3 develops the proposed day-ahead market bid model with emission risk (DAMB-ER). In Section 4 a case study with real MIBEL data is solved and analyzed. Finally, Section 5 presents the conclusions of the work.

3. Emission risk-constrained model for the optimal electricity generation bid

As a consequence of the deregulation of the countrywide energy production system through the settlement of liberalized electricity markets, the price of electricity has become a significant risk factor because it is unknown at the moment when generation companies have to take operational decisions. This means that the market price has to be considered as a random variable whose realization is

known only once the market has been cleared. Stochastic programming (Birge and Louveaux, 1997) provides a powerful and well established methodology for tackling this uncertainty, as it incorporates into a single mathematical optimization model the available statistical information on the relevant random variables. In this section we begin with a brief description of the so-called day-ahead market bid model with futures and bilateral contracts (DAMB-FBC). This is a two-stage stochastic optimization model developed in Corchero et al. (2013) which allows a GenCo to optimally decide the unit commitment of its generation units, the economic dispatch of the bilateral and futures contracts, and the optimal generation bid to the day-ahead market. The model (DAMB-FBC) is then extended in the second part of the section to cope with the Spanish NERP through a new emission risk measure called *Conditional Emission-at-Risk*.

3.1. Base model: optimal day-ahead market bid with futures and bilateral contracts

The basic day-ahead market bid model with futures and bilateral contracts (DAMB-FBC) considered in this work was developed in Corchero et al. (2013), and it is extended in this section to include combined cycle gas turbine (CCGT) units as a previous step towards developing of the final model with emission risk constraints. Let us consider a price-taker GenCo that owns a set of thermal generation units \mathcal{A} and a set of CCGT units that bid to the $t \in \mathcal{T} = \{1, 2, \dots, 24\}$ hourly auctions of the DAM. Therefore, this is a day-ahead horizon optimization problem. Each of the different operation modes of the CCGT units described in Section 1.3 can be conceptually considered as an individual generation unit, similar to the thermal generation units \mathcal{A} but with a different behavior. Each of these special generation units will be called a *pseudo-unit*, with \mathcal{P} being the set of pseudo-units of all the CCGT units considered (see Appendix A for more details). As a consequence, the total set of generation units considered by the model is $\mathcal{U} = \mathcal{A} \cup \mathcal{P}$, and the parameters for the generation unit $i \in \mathcal{U}$ are:

- c_i^b, c_i^l, c_i^q : constant, linear and quadratic coefficients of the generation cost function ($[\text{€}]$, $[\text{€/MWh}]$ and $[\text{€/MWh}^2]$ respectively).
- $\bar{P}_i, \underline{P}_i$: upper and lower bounds on the energy generation: [MWh].
- c_i^{on}, c_i^{off} : start-up and shut-down costs [€].
- t_i^{on}, t_i^{off} : minimum operation and minimum idle time [h].
- st_i^0 : number of hours the unit has been on ($st_i^0 > 0$) or off ($st_i^0 < 0$) prior to the first time period.

The parameters defining a base load physical futures contract $j \in \mathcal{F}$ are:

- $\mathcal{A}_j \in \mathcal{U}$: the set of generation units allowed to cover the FC j .
- L_j^F : the amount of energy [MWh] to be procured at each interval of the delivery period by the set \mathcal{A}_j of generation units to cover contract j .
- λ_j^F : the price of contract j [€/MWh].

And the parameters defining a base load bilateral contract $k \in \mathcal{B}$ are:

- L_{tk}^B : the amount of energy [MWh] to be procured during hour t of the delivery period by the set of available generation units to cover the BC k .
- λ_k^B : the price of the contract k [€/MWh].

The random variable λ_t^D , which is the clearing price of the t^{th} hourly auction of the DAM, is represented in the two-stage stochastic model by a set of scenarios $s \in \mathcal{S}$, each with its associated clearing price for each DAM auction $\lambda_t^{D,s}$, $t \in \mathcal{T}$ and the corresponding probability P^s (Corchero, 2011).

The first-stage (*here and now*) variables of the model (DAMB-FBC) are, for every time period $t \in \mathcal{T}$ and generation unit $i \in \mathcal{U}$:

- $u_{ti} \in \{0, 1\}$: the unit commitment binary variables, expressing the on-off operating status of the i^{th} unit.
- c_{ti}^u, c_{ti}^d : the start-up/shut-down cost variables [€].
- q_{ti} : the energy of the price accepting offer bid [MWh].
- f_{tij} : the scheduled energy for futures contract $j \in \mathcal{F}$ [MWh].
- b_{ti} : the scheduled energy for the bilateral contract [MWh].

Finally, the second stage (*wait and see*) variables are, for each time period $t \in \mathcal{T}$, generation unit $i \in \mathcal{U}$ and scenario $s \in \mathcal{S}$:

- g_{ti}^s : the total generation [MWh].
- p_{ti}^s : the matched energy in the day-ahead market [MWh].

Considering the parameters and variables described below, the (DAMB-FBC) model is:

(DAMB – FBC)

$$\begin{cases} \max h(u, c^u, c^d, g, p, b, f) \\ \text{s.t.:} \\ \sum_{i \in \mathcal{G}_j} f_{tij} = L_j^F & j \in \mathcal{F}, t \in \mathcal{T} & (1a) \\ \sum_{i \in \mathcal{U}} b_{ti} = \sum_{k \in \mathcal{B}} L_{tk}^B & t \in \mathcal{T} & (1b) \\ f_{tij} \geq 0 & i \in \mathcal{U}, j \in \mathcal{F}, t \in \mathcal{T} & (1c) \\ 0 \leq b_{ti} \leq \bar{P}_i u_{ti} & i \in \mathcal{U}, t \in \mathcal{T} & (1d) \\ q_{ti} \geq \sum_{j \in \mathcal{G}_j} f_{tij} & i \in \mathcal{U}, t \in \mathcal{T} & (2a) \\ q_{ti} + b_{ti} \geq \bar{P}_i u_{ti} & i \in \mathcal{U}, t \in \mathcal{T} & (2b) \\ p_{ti}^s + b_{ti} \leq \bar{P}_i u_{ti} & i \in \mathcal{U}, t \in \mathcal{T}, s \in \mathcal{S} & (2c) \\ q_{ti} \leq p_{ti}^s & i \in \mathcal{U}, t \in \mathcal{T}, s \in \mathcal{S} & (2d) \\ g_{ti}^s = b_{ti} + p_{ti}^s & i \in \mathcal{U}, t \in \mathcal{T}, s \in \mathcal{S} & (3a) \\ \bar{P}_i u_{ti} \leq g_{ti}^s \leq \bar{P}_i u_{ti} & i \in \mathcal{U}, t \in \mathcal{T}, s \in \mathcal{S} & (3b) \\ c^u, c^d, u \in P_{UC} & & (4) \end{cases} \times$$

where the interpretation of the different set of constraints is as follows:

- (1a)–(1d) guarantee the coverage of both the physical futures and bilateral contract obligations.
- (2a)–(2d) incorporate into the model the MIBEL rules through which the energies L_j^F and L_{tk}^B of the futures and bilateral contracts are integrated into the day-ahead market bid of a generation unit. The first rule is that if generator $i \in \mathcal{U}$ contributes with f_{tij} MWh at period t to the coverage of the FC j , then the energy f_{tij} must be offered to the pool for free and embedded into the price acceptance sale bid (2a). The second rule establishes that if generator $i \in \mathcal{U}$ contributes with b_{ti} MWh at period t to the coverage of any of the BCs, then only the remaining production capacity $\bar{P}_i - b_{ti}$ can be bid to the DAM (constraints (2b) and (2c)).
- (3a), (3b) define the total generation level of a given unit i , g_{ti}^s , as the addition of the allocated energy to the BC, plus the matched energy in the DAM, and it restricts the total generation output to $g_{ti}^s \in \{0\} \cup [\underline{P}_i, \bar{P}_i]$.

- (4) restricts the unit commitment variables (those related to the on-off state of each generation unit) to being part of the feasible unit commitment polyhedron P_{UC} . This feasible polyhedron contains all the values of the binary unit commitment variables u that satisfy the minimum operation and minimum idle time t^{on} and t^{off} and the initial state s^0 . It also conveniently defines the value of the start-up/shut-down cost variables c^u, c^d . The original formulation proposed in Corchero et al. (2013) has been improved upon in this paper and extended to include combined cycle units. Appendix A describes the detail of this formulation.

The objective function h of the model accounts for the expected value of the total profit obtained by the GenCo and is represented by the following expression:

$$h(u, c^u, c^d, g, p, b, f) = E_{\lambda^D}[\text{profit}] = |\mathcal{T}| \left[\sum_{j \in \mathcal{F}} \lambda_j^F L_j^F \right] + \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{B}} \lambda_{tk}^B L_{tk}^B \quad (5a)$$

$$- \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{G}} [c_{ti}^u + c_{ti}^d + c_i^b u_{ti}] \quad (5b)$$

$$- \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}} \left[c_{tP_c(1)}^u + c_{tP_c(2)}^u + \sum_{i \in P_c} c_i^b u_{ti} \right] \quad (5c)$$

$$+ \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{U}} \sum_{s \in \mathcal{S}} P^s \left[\lambda_t^{D,s} p_{ti}^s - (c_i^u g_{ti}^s + c_i^d (g_{ti}^s)^2) \right] \quad (5d)$$

where:

- (5a) corresponds to the incomes of the FCs and the BCs, where λ_k^F and L_k^F are the prices and energies of the FCs and λ_{ij}^B and L_{ij}^B are the prices and energies of the BCs. Both the energy and prices of these contracts have been fixed long before the moment when the bid to the day-ahead market has been decided: therefore, this is a known constant term in our objective function.
- (5b) accounts for the on/off fixed cost of the unit commitment of the thermal units. This term is independent of the realization of the random variable λ_t^D . c_i^b are the constant coefficients of the generation costs (€).
- (5c) represents the start-up and fixed generation costs of the CCGTs. Only start-up costs are associated to the PU, and no cost is associated to the transition from state 2 to state 1 (see Appendix A). This term does not depend on the realization of the random variable λ^D .
- (5d): represents the expected value of the benefits from the day-ahead market, where P^s is the probability of scenario s . The term between brackets corresponds to the expression of the quadratic generation costs with respect to the total generation of the unit, g_{ti}^s .

3.2. Conditional Emission at Risk (CEaR)

The Spanish National Emissions Reduction Plan imposes limits - \overline{SO}_2 and \overline{NO}_x [kg/day] - on the joint emission of the thermal units (CCGT units are excluded). These limitations could of course be included in the model (DAMB-FBC) by simply imposing an emission limit at every scenario s through the following set of constraints:

$$\sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s \leq \overline{SO_2} \quad s \in \mathcal{S} \quad (6)$$

$$\sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{NO_x} g_{ti}^s \leq \overline{NO_x} \quad s \in \mathcal{S} \quad (7)$$

where, as usual, the emissions of a thermal unit are assumed to be linear w.r.t. the total generation g_{ti}^s with emission coefficients $e_i^{SO_2}$ and $e_i^{NO_x}$ [kg/MWh] that depend on the generation technology. This formulation would actually be a simple stochastic extension of the deterministic approach adopted by the previous works that explicitly incorporate the emission limits by imposing an upper bound to the accumulated emissions along the complete optimization horizon (Catalao and Mendes, 2011; Geng et al., 2017; Laia et al., 2015; Lu and Shahidehpour, 2005; Rebennack et al., 2012). Although valid, this risk-averse approach is quite restrictive, as it forces the optimal bid to abide by the NERP rules even in the most extreme (i.e. less likely) scenarios.

The risk management ideas developed for the financial markets offer a new and more flexible approach to addressing the SO_2 and NO_x emission constraints. The risk management theory is about how to handle risks. Classical risk management methods in portfolio optimization theory, include *Value-at-Risk* (VaR) and *Conditional Value-at-Risk* (CVaR) (Rockafellar and Uryasev, 2000; Uryasev, 2000). Following Rockafellar and Uryasev (2000), the VaR of a portfolio with respect to a specified probability β is the lowest amount α such that, with probability β , the loss will not exceed α . The CVaR is defined as the conditional expectation of losses above amount α . Rockafellar and Uryasev (2000) show that, under certain assumptions, the minimization of the CVaR of a given portfolio can be formulated as a continuous optimization problem where the value of VaR is computed endogenously during the optimization process. By analogy to the use of CVaR in the portfolio optimization, the Conditional Emission-at-Risk (CEaR) is proposed in this work as a tool for measuring and controlling the risk of violating the NERP emission limits. In contrast to what happens in hedging a portfolio of financial instruments, where the focus is on the minimization of the CVaR (and thus, CVaR is in the objective function), the goal of this study is not to minimize emission risk, the CEaR, but to maximize the expectation of profits. Indeed, our concern is to use CEaR as a criteria to measure the risk of violating of the NERP limits, not the risk associated to the GenCo's profit. Thus, CEaR does not appear in the objective function of our problem. That fact introduces two relevant differences between the use of CVaR models in portfolio optimization and that of CEaR in our formulation. The first one is that the value of VaR is no longer obtained implicitly as in the models in Rockafellar and Uryasev (2000), but must be given explicitly as an exogenous parameter. Actually, as we will see, the equivalent to VaR in our models is the emission limits imposed by the NERP, $\overline{SO_2}$ and $\overline{NO_x}$. Second, in order to compute the value of CVaR, our formulation cannot avoid the use of binary variables (variables y^s in (8)–(9) below) to account for the scenarios with emission levels above the NERP limits.

Let us start by formulating the constraints that identify the scenario s that violates the NERP limit on SO_2 :

$$\sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s - \overline{SO_2} \leq M^{SO_2} y^s \quad s \in \mathcal{S} \quad (8)$$

$$\sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s - \overline{SO_2} \geq M^{SO_2} (y^s - 1) \quad s \in \mathcal{S} \quad (9)$$

$$\sum_{s \in \mathcal{S}} P^s y^s \leq \gamma \quad (10)$$

The first two equations (8) and (9) conveniently classify the scenarios in which the SO_2 emissions exceed the limit. y^s , $s \in \mathcal{S}$ is a binary variable that takes value 1 if the emissions are higher than $\overline{SO_2}$ and 0 otherwise, and parameter M^{SO_2} is an upper bound of the emission violation, that is:

$$-M^{SO_2} \leq \sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s - \overline{SO_2} \leq M^{SO_2}$$

Of course, a trivially valid value for M^{SO_2} could be $|\mathcal{T}| \sum_i e_i^{SO_2} \bar{P}_i$.

Equation (10), in turn, limits the joint probability of those scenarios exceeding the upper bound $\overline{SO_2}$. Thus, instead of imposing an emission limit at every individual scenario s , this approach allows exceeding the limit, with a given probability of it being not greater than γ . It is worth mentioning that when $\gamma = 0$ constraints (8)–(10) are equivalent to constraints (6): that is, they impose that no scenario can exceed the emission limit. Furthermore, taking $\gamma = 1$ is equivalent to not imposing any limit at all (and then reverting to the base model (DAMB-FBC)).

The above three constraints (8)–(10) are the basis in the development of a CVaR-like model for limiting the average amount by which the emissions can exceed the limit. Analogous to the CVaR function, we will develop the so-called Conditional Emission-at-Risk (CEaR) in order to establish a new measure of risk associated with the expected value of the $\overline{SO_2}$ violation. To this end, let us define first for every scenario s , a new set of variables eS^s whose value will be equal to the value of the SO_2 emissions ($\sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s$) if the emissions in scenario s exceed the limit (that is, if $y^s = 1$), or 0 if the emissions in scenario s are below the limit (that is, whenever $y^s = 0$):

$$eS^s = \begin{cases} \sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s & \text{if } y^s = 1 \\ 0 & \text{if } y^s = 0 \end{cases} \quad s \in \mathcal{S}$$

Eq. (11)–(13) below express variables eS^s in a way that is amenable to the optimization model:

$$eS^s - \sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s \leq M^{SO_2} (1 - y^s) \quad s \in \mathcal{S} \quad (11)$$

$$eS^s - \sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s \geq -M^{SO_2} (1 - y^s) \quad s \in \mathcal{S} \quad (12)$$

$$eS^s \leq M^{SO_2} y^s \quad s \in \mathcal{S} \quad (13)$$

Let us consider now a solution g, y satisfying Eq. (8)–(13). Then, for any given probability level γ and emission limit $\overline{SO_2}$, the conditional emission at risk $CEaR_\gamma^{\overline{SO_2}}$ associated to g, y is defined as the expectation of the SO_2 emissions for those scenarios exceeding $\overline{SO_2}$:

$$CEaR_\gamma^{\overline{SO_2}} = \frac{1}{\sum_{s \in \mathcal{S}} P^s y^s} \sum_{s \in \mathcal{S}} P^s eS^s$$

Then, the following risk-limiting constraint allows controlling the amount by which the expectation of the violating emissions $CEaR_\gamma^{\overline{SO_2}}$ surpasses the limit $\overline{SO_2}$:

$$CEaR_{\gamma}^{\overline{SO_2}} \leq (1 + \beta)\overline{SO_2}$$

where the parameter $\beta \geq 0$ (usually < 1), the violation factor, represents the maximum permitted violation as a fraction of the emission limit $\overline{SO_2}$. The last inequality ensures that the expected violation will be less than a fraction β of $\overline{SO_2}$. Note that when $\beta = 0$ no scenario can exceed the emission limit, irrespective of the value of γ . In order to incorporate the last two equations into a mathematical programming model, it is convenient to combine them in the following single linear inequality:

$$\sum_{s \in S} P^s e^s \leq (1 + \beta)\overline{SO_2} \sum_{s \in S} P^s y^s \quad (14)$$

It is worth mentioning that the value $\overline{SO_2}$ imposed by the NERP plays the role of the VaR level in the classical CVaR definition. The definition of $CEaR_{\gamma}^{\overline{SO_2}}$ is illustrated graphically in Fig. 2 where $f(\text{Emissions})$ represents the probability density function of the SO_2 emissions.

Similarly to what has been done in the case of SO_2 emissions, it is possible to formulate the NO_x CEaR risk constraints through the following set of constraints:

$$\sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{NO_x} g_{ti}^s - \overline{NO_x} \leq M^{NO_x} z^s \quad s \in S \quad (15)$$

$$\sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{NO_x} g_{ti}^s - \overline{NO_x} \geq M^{NO_x} (z^s - 1) \quad s \in S \quad (16)$$

$$\sum_{s \in S} P^s z^s \leq \gamma \quad (17)$$

$$eN^s - \sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{NO_x} g_{ti}^s \leq M^{NO_x} (1 - z^s) \quad s \in S \quad (18)$$

$$eN^s - \sum_{t \in T} \sum_{i \in \mathcal{I}} e_i^{NO_x} g_{ti}^s \geq -M^{NO_x} (1 - z^s) \quad s \in S \quad (19)$$

$$eN^s \leq M^{NO_x} z^s \quad s \in S \quad (20)$$

$$\sum_{s \in S} P^s eN^s \leq (1 + \beta)\overline{NO_x} \sum_{s \in S} P^s z^s \quad (21)$$

Variables z^s and eN^s , and parameter M^{NO_x} are defined analogously to y^s , e^s and M^{SO_2} respectively.

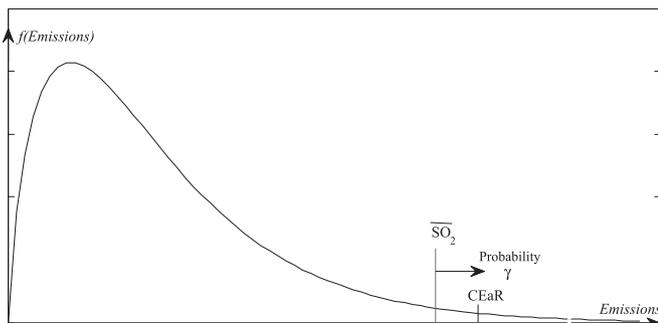


Fig. 2. Graphical representation of the CEaR concept.

3.3. Optimal day-ahead market bid model with emission risk constraints

We are now ready to extend the basic optimal day-ahead market bid model presented in Section 3.1 to the Conditional Emission-at-Risk formulation of the national emissions reduction plan developed in Section 3.2. The resulting optimal day-ahead market bid model with emission risk (DAMB-ER) $_{\gamma, \beta}$ can be expressed through the following mathematical optimization problem:

$$\begin{cases} \max h(u, c^u, c^d, g, p, b, f) \\ \text{s.t. :} \\ \text{Eq. (1a) - (1d)} & \text{FC and BC} \\ \text{Eq. (2a) - (2d)} & \text{Day - ahead market} \\ \text{Eq. (3a) - (3b)} & \text{Total generation} \\ \text{Eq. (A.1) - (A.6)} & \text{Unit commitment, thermal units} \\ \text{Eq. (A.9) - (A.15)} & \text{Unit commitment, CC units} \\ \text{Eq. (8) - (14)} & \text{CEaR for } \overline{SO_2}, \text{ risk level } \gamma, \beta. \\ \text{Eq. (15) - (21)} & \text{CEaR for } \overline{NO_x}, \text{ risk level } \gamma, \beta. \end{cases}$$

Problem (DAMB-ER) $_{\gamma, \beta}$ is a mixed-integer concave quadratic maximization problem with linear constraints and a well defined global optimal solution.

4. Case study

The model (DAMB-ER) $_{\gamma, \beta}$ developed in the previous section depends parametrically on the confidence probability γ and the violation factor β . The different combinations of values for γ and β between these two extreme cases provide information that will be used in this section to assess the impact of the emission limits on the optimal generation bid and the expected profits. The model (DAMB-ER) $_{\gamma, \beta}$ has been implemented with the AMPL modeling language (Fourer et al., 2003) and solved with CPLEX 12.4 (CPLEX, 2008) (mipgap = 0.01) on a Fujitsu RX200 S6 (2 x CPUs Intel Xeon X5680 six core - 12 threads 3.33 GHz, 64Gb RAM), allowing us to take advantage of the multithreading capabilities of CPLEX (threads = 20).

4.1. Data set

This study uses the same set of 50 scenarios generated in Corchero et al. (2011) for the random day-ahead market spot prices λ^D , which resulted from applying a scenario reduction algorithm (Grove-Kuska et al., 2003) to the complete set of historic data available from June 2007 to May 2010 that are available at the website of the Independent Iberian Market Operator OMIE OMEL (2010). As we will see, the imposition of emissions constraints could change the optimal bid of the market participant and, consequently, the series of clearing prices. Nevertheless, as our objective is to compare the optimal bid of models (DAMB-FBC) and (DAMB-ER) $_{\gamma, \beta}$ we need to use the same price scenarios for both models in order to conduct a fair comparison. The generation units in this study correspond to four thermal units and two combined cycle units that are currently operating in the MIBEL, and their technical characteristics are shown in Tables 1 and 2. Table 3 shows the number, energy and price of the bilateral and futures contracts. All data related to SO_2 and NO_x can be obtained from Table 4. The emission limits $\overline{SO_2}$ and $\overline{NO_x}$ derive from the National Emission Reduction Plan (2007). The SO_2 and NO_x emissions rates shown in Table 4 correspond to the values published by the Intergovernmental Panel on Climate Change Emission (2012) for coal thermal units. Further details can be obtained from Cifuentes-Rubiano (2012).

Table 1
Operational characteristics of the thermal units.

\mathcal{A}	c_i^b €	c_i^l €/MWh	c_i^q €/MWh ²	P_i MW	\bar{P}_i MW	st_i^0 hr	c_i^{pn} €	c_i^{off} €	t_i^{pn}, t_i^{off} hr
1	159.24	42.55	0.016	160.0	350.0	+3	435.09	435.09	3
2	901.70	59.38	0.038	250.0	563.2	+1	1307.70	1307.70	3
3	344.68	30.41	0.038	160.0	370.7	-1	462.07	462.07	3
4	322.04	60.04	0.032	160.0	364.1	+1	682.04	682.04	3

Table 2
Operational characteristics of the combined cycle units.

CCGT	\mathcal{P}	c_i^b €	c_i^l €/MWh	c_i^q €/MWh ²	P_i MW	\bar{P}_i MW	st_i^0 hr	c_i^{pn} €	t_i^{pn} hr	t_i^{off} hr
1	5	151.08	50.37	0.023	160.0	350.0	-2	803.75	2	3
1	6	224.21	32.50	0.035	250.0	563.2	-2	412.80	2	3
2	7	163.11	55.58	0.019	90.0	350.0	-3	320.50	2	3
2	8	245.32	31.10	0.022	220.0	700.0	-3	510.83	2	3

Table 3
Characteristics of futures and bilateral contracts.

j	$L_{j,t=1..24}^B$ MW	$\lambda_{j,t=1..24}^B$ €/MWh	$L_{j,t=1..24}^F$ MW	$\lambda_{j,t=1..24}^F$ €/MWh
1	164	43.35	120	45.6
2	50	43.35	120	46.1
3	150	43.35	120	51.2

Table 4
Data for daily emission limits and for thermal unit emissions.

$\overline{SO_2}$ kg/day	$\overline{NO_x}$ kg/day	$e_i^{SO_2}$ kg/MWh	$e_i^{NO_x}$ kg/MWh
3900	11,460	0.7848	1.368

4.2. Impact of the NERP on the expected profits: parameterized efficient frontier

As mentioned earlier, the risk constrained model (DAMB-ER) $_{\gamma,\beta}$ defines a family of problems parameterized by the risk factors γ and β that can be used to assess the dependence of expected returns on the risk level. The *efficient frontier* (Fig. 3) defines the maximum expected profit that can be achieved by a GenCo for a given risk level, defined in our model by the two parameters γ and β (actually, the efficient frontier is an “efficient surface” in our case). It can be observed in Fig. 3 that, as γ increases, emissions may exceed the limit with greater probability and, consequently, the expected profit increases. Moreover, for any given confidence probability γ , if there is an increase in the average percentage at which emissions exceed the limit (i.e., if β increases), then the expected value of the profits increases accordingly. There are two extreme cases in Fig. 3

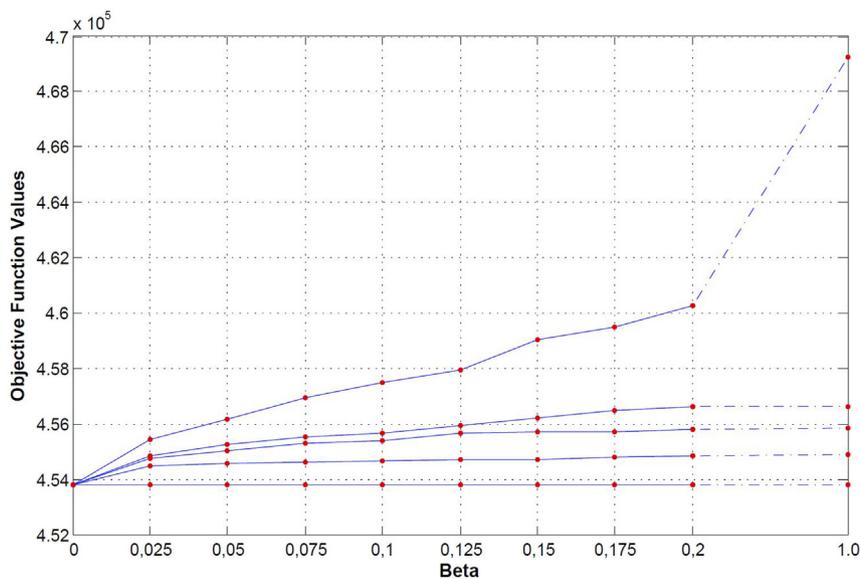


Fig. 3. Parameterized efficient CEaR frontier of the problem (DAMB-ER) $_{\gamma,\beta}$, showing the change in the value of the expected profits as a function of parameters γ and β . From bottom to top, the different curves correspond to the values $\gamma \in \{0, 0.1, 0.3, 0.5, 1\}$. The dots on each of these curves denote the computed optimal expected value of the problems (DAMB-ER) $_{\gamma,\beta}$ for $\beta \in \{0, 0.025, 0.05, 0.075, 0.1, 0.125, 0.15, 0.175, 0.2, 1.0\}$.

- The bottommost, flat curve is associated to $\gamma = 0$ which corresponds to the most restrictive optimization problem (DAMB-ER)_{0,β}, where no scenario is allowed to violate the limit (the value of β therefore being irrelevant). It is worth mentioning that this case is equivalent to the base model (DAMB-FBC) plus the emission constraints (6)–(7).
- The topmost curve associated to $\gamma = 1$, which corresponds to the less restrictive optimization problem (DAMB-ER)_{1,β}, where any scenario is allowed to violate the emissions limit by an amount that is on average not greater than a fraction β of the maximum emission. In practice, the optimal solution of the case $\beta = 1$, (DAMB-ER)_{1,1}, coincides with the base model (DAMB-FBC).

The economic information provided by the parameterized efficient frontier in Fig. 3 is an example of how the model (DAMB-ER)_{γ,β} can be used by a GenCo as a tool for assessing several decisions related to electricity generation under NERP regulations. An example of this would be in determining the convenience of deploying SO₂ and NO_x capture technology with a given capacity and fault probability γ , whereby the GenCo could compare the cost of the installation with the increase in the expected profits between the risk-averse case (DAMB-ER)_{0,0} and the risk-neutral case (DAMB-ER)_{γ,β}, with a violation β representing the capacity of the capture device.

4.3. Impact of the NERP on the optimal generation bid

The purpose of this section is to study in detail the effect of the risk constraints on the optimal generation bid. The study will be based on comparing the optimal, risk-seeking solution of the original problem (DAMB-FBC) - where no emission limits are considered - with the optimal, risk-neutral solution of problem (DAMB-ER)_{0,3,0.15} as a representative element of the parameterized family (DAMB-ER)_{γ,β}. Problem (DAMB-ER)_{0,3,0.15} allows violating limits with a probability of 0.3, and the expected violation cannot exceed the limit by more than 15%. We also report the results for the risk-averse case (DAMB-ER)_{0,0}, where no violation of the NERP limits is allowed. The dimensions and execution time of these problems are indicated in Table 5.

Table 6 depicts the expected value of the SO₂ and NO_x emissions at the optimal solution of the three cases. The results for problem (DAMB-ER)_{0,3,0.15} indicate a reduction in the emissions of 43.2% when compared to the (DAMB-FBC) case. The decrease in the expected profits of the risk-neutral case is merely 2.9% in comparison with the risk-seeking case, showing that a huge decrease in emissions can be attained without a critical reduction in profits. The expected value for SO₂ emissions for problem (DAMB-ER)_{0,3,0.15} is a 4.5% above the limit of 3,900kg/day imposed by NERP, which does not represent any inconsistency because, as $\gamma > 0$, we are allowing some violation of the limits. For its part, the risk-averse problem (DAMB-ER)_{0,0}, which abides strictly by the NERP limits, shows a reduction in emissions that is very similar to the risk-neutral case (DAMB-ER)_{0,3,0.15} (45.6%) but at the expense of lower profits (1307€/day less).

Table 5
Characteristics of the optimization problems.

Cases	Variables		Constraints	Exec. time
	Continuous	Binary		
(DAMB-FBC)	20,160	200	49,458	360 sec.
(DAMB-ER) _{0,3,0.15}	20,260	300	49,962	48 min.
(DAMB-ER) _{0,0}	20,260	300	49,962	32 min.

Table 6
Expected daily emissions and profit variation.

	$E[SO_2]$	$E[NO_x]$	E_p [profit]
	kg/day	kg/day	€
(DAMB-FBC)	7182	12,519	469,597
(DAMB-ER) _{0,3,0.15}	4075	7104	455,699
(DAMB-ER) _{0,0}	3900	6798	454,392

Table 7
Total expected energy production.

	(DAMB-FBC) MWh	(DAMB-ER) _{0,3,0.15} MWh	Difference MWh
Thermal 1	4745.8	776.0	-3969.8 (-84%)
Thermal 2	600.0	600.0	0.0 (0%)
Thermal 3	7083.0	5604.3	-1478.7 (-21%)
Thermal 4	530.0	620.0	90.0 (+17%)
Total thermal	12,958.8 (39%)	7600.3 (27%)	-5358.5 (-41%)
CCGT 1	8915.7	9311.7	396.0 (+4%)
CCGT 2	11,190.8	11,430.7	239.9 (+2%)
Total CCGT	20,106.5 (61%)	20,742.4 (73%)	635.9 (+3%)
Total thermal+CCGT	33,065.3	28,342.7	-4722.6 (-14%)

Bold are totals for thermal Thermal, CCGT and the whole system.

The effect of the NERP regulations on the aggregated daily expected energy generation of each unit can be observed in Table 7. The results show that under the NERP regulations:

- The total expected production is reduced by 4722.6 MWh (-14%), with a reduction of -5258.5 MWh (-41%) of the thermal unit production and an increase of 635.9 MWh (+3%) of the CCGT production.
- The reduction in the thermal units is concentrated in thermal unit 1 (which is switched off as soon as possible), with a decrease of 3969.8 MWh (-84%) and thermal unit 3 (which is kept in operation), with a decrease of 1478.7 MWh (-21%) in its expected production.
- The generation of the CCGT units increases by 396.0 MWh for unit 1 (+4%) and 239.9 MWh for unit 2 (+2%).
- The fraction of the total generation corresponding to the CCGT increases from 61% to 73%, while the fraction of the thermal units is reduced accordingly.

Fig. 4 shows the impact of the emission risk constraints on the individual unit commitment of each generation unit, together with the optimal dispatch of the bilateral and futures contracts among each generation unit for the (DAMB-FBC) problem (left column) and (DAMB-ER)_{0,3,0.15} (right column). The blue area corresponds to the energy allocated to the bilateral contracts (variable b_{ti}); the green area is the energy of the price accepting bid q_{ti} , which includes the energy allocated to the futures contracts f_{ij} . Finally, the yellow area is, for each generation i and period t , the expected value of the matched energy in the day-ahead market $\sum_{s \in S} P^s p_{ti}^s$.

Comparing the generation profiles in Fig. 4 it is clear how the NERP regulation is affecting the unit commitment: all coal thermal

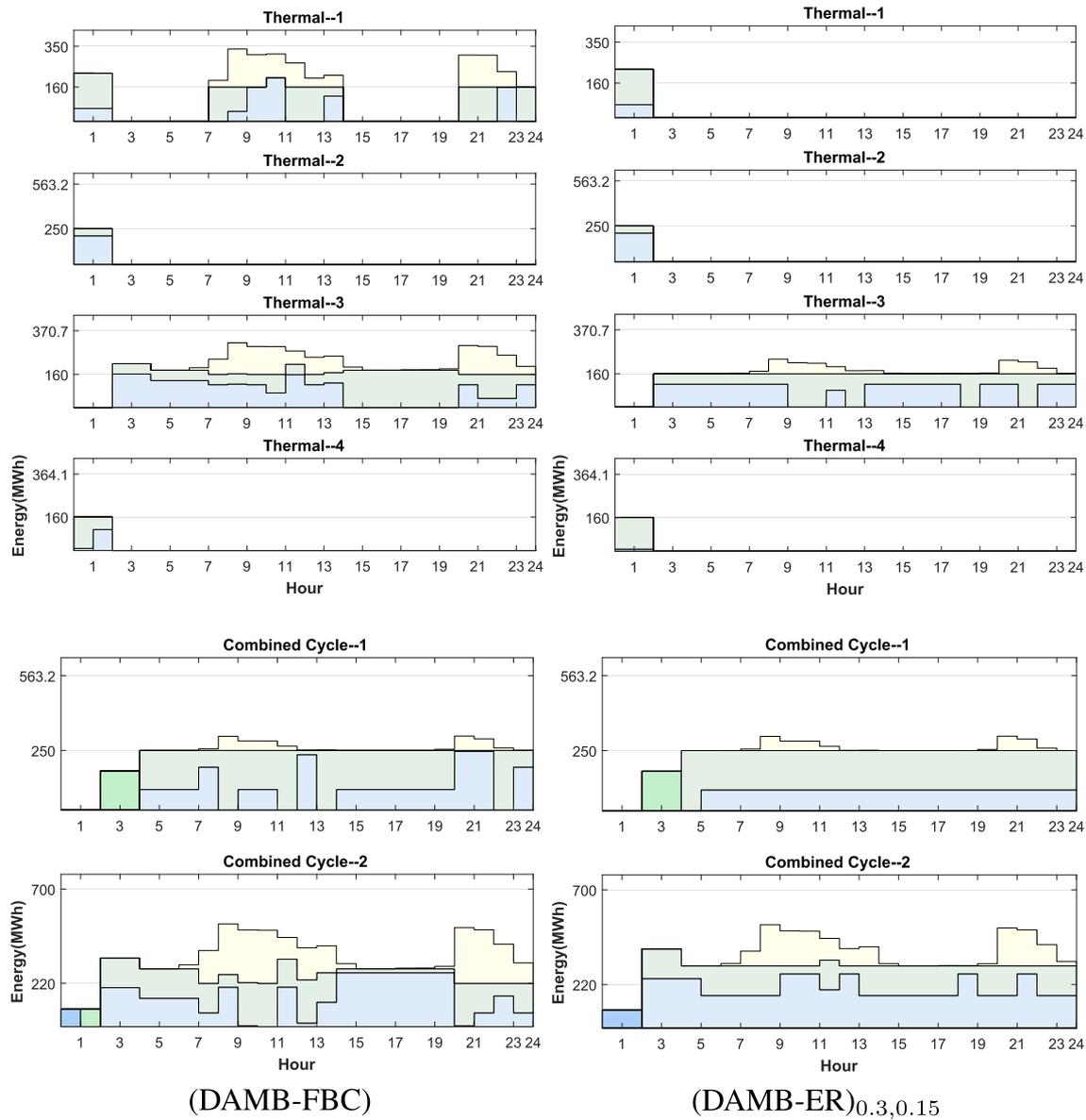


Fig. 4. Comparison of the unit commitment for models (DAMB-FBC) (left) and (DAMB-ER)_{0.3,0.15} (Right): b_{it} (scheduled energy for bilateral contract, blue), q_{it} (price accepting bid, green). Yellow indicates the expected value of the matched energy. For the CCGT units, dark colors are for pseudo-unit 1 and light colors for pseudo-unit 2.

generators (high-emission units) are shut-down early, with the exception of thermal unit 3, which is maintained in operation to satisfy futures contract 3 (in the absence of futures contracts, thermal unit 3 would have been kept shut-down all day long).

Finally, Fig. 5 show the comparison of the optimal unit commitment and energy allocation between models (DAMB-ER)_{0,0} and (DAMB-ER)_{0.3,0.15}. We observe that in model (DAMB-ER)_{0,0} the contribution of thermal unit 3 to the bilateral contracts (blue blocks) is reduced in time periods 10, 11 and 13 and 22 while the contribution of CCGT 2 is increased in the same time periods. Also, a certain increase in the expected value of the matched energy (yellow blocks) is observed for thermal unit 3.

5. Conclusions

Generation companies have to decide the daily generation bid to be submitted to the day-ahead electricity market, where, in Spain, a

total of more than 30 million Euros are negotiated daily. The GenCo's optimal generation bid is aimed at both maximizing the expected profit and abiding by the different National Emissions Reduction Plans of each country. The new competitive and environmentally constrained electricity supply industry requires new mathematical and computing tools to ensure, first, competitiveness with other generating companies in the electricity market and, secondly, environmental protection by limiting the damaging emissions that are released into the atmosphere. With the goal of advancing in that direction, this work proposes a new two-stage stochastic programming model to cope with the optimal generation bid to the day-ahead electricity market specifically for a GenCo that is operating a pool of thermal and combined cycle generation units with a given set of futures and bilateral contracts to be settled next day. The model takes into account the MIBEL market rules and the SO₂ and NO_x emission limits of the current Spanish NERP regulations through a new measure of risk called Conditional

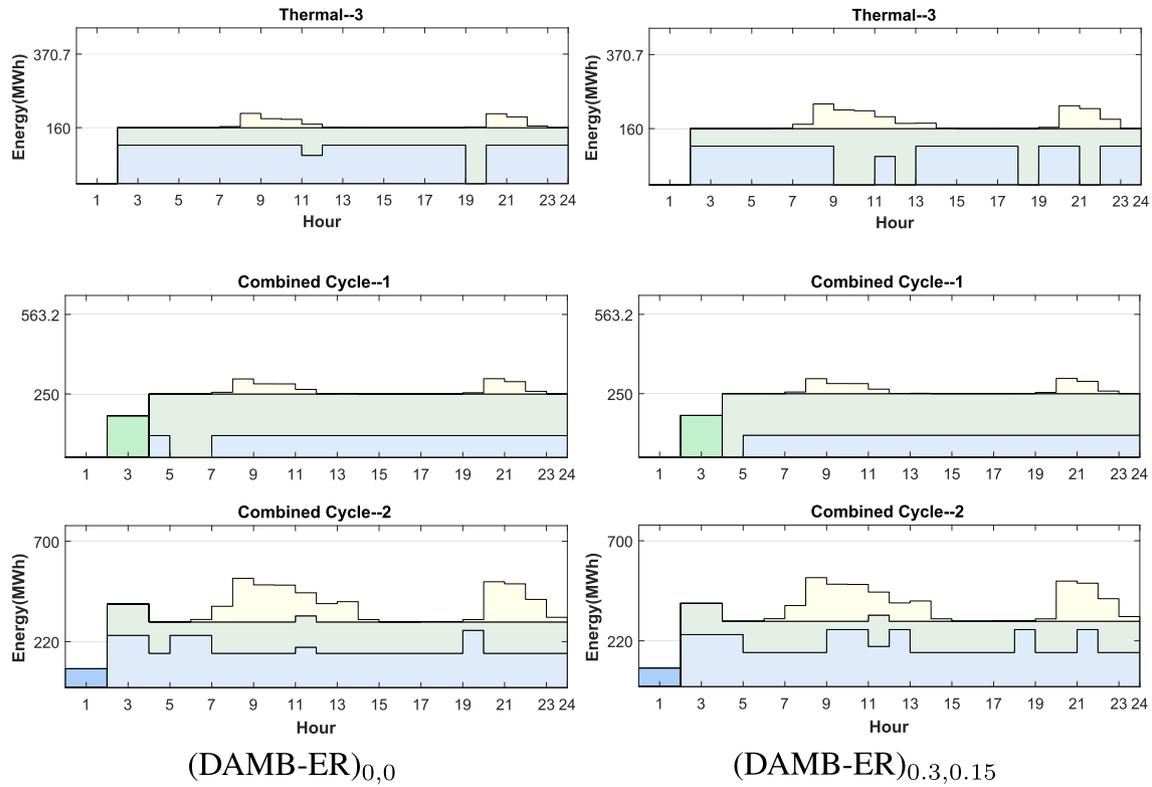


Fig. 5. Comparison of the unit commitment for models (DAMB-ER)_{0,0} (left) and (DAMB-ER)_{0,3,0.15} (Right): b_{ti} (scheduled energy for bilateral contract, blue), q_{ti} (price accepting bid, green). Yellow indicates the expected value of the matched energy. For the CCGT units, dark colors are for pseudo-unit 1 and light colors for pseudo-unit 2.

Emission-at-Risk (CEaR). CEaR allows the formulation of a family of (DAMB-ER) _{γ,β} models parameterized by the emission risk level defined by γ and β , and this provides a flexible tool for assessing a wide range of decisions related to electricity generation under NERP regulations. The computational experiments performed with real data of the Spanish wholesale electricity market provide the optimal dispatch of each individual thermal and CCGT unit among the different energy contracts in the day-ahead market. The numerical results show that, for a given representative risk level, the SO₂ and NO_x NERP obligations can be met by reducing the expected total energy production by 14%, with less than a 3% decrease in the expected profits. This reduction in the total energy production is unevenly distributed among the generation technologies, with a 41% decrease in the thermal production against a 3% increase in the CCGT generation, confirming the central role of the CCGT technology in an environmentally friendly energy production system.

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Appendix. A. Thermal and combined cycle unit commitment constraints

This appendix deals with the development of the explicit formulation of the polyhedron P_{UC} in equation (4). Let u_{ti} be the first-stage binary variable expressing the off-on operating status of the i^{th} unit and let c_{ti}^u, c_{ti}^d be continuous variables representing the startup and shutdown costs, respectively, of unit i in interval t .

Let also constant G_i be the number of periods that unit i must be initially online, due to its minimum up-time t_i^{on} . Analogously let H_i be the number of periods that unit i must be initially offline, due to its minimum down-time t_i^{off} . Finally let parameter u_{0i} stands for the initial state of each thermal unit: $u_{0i} = 1$ if the unit is on and $u_{0i} = 0$ if the unit is off. The following set of constraints conveniently models the start-up and shut-down costs and the minimum operation and idle time for each unit, thereby reducing the number of constraints of the equivalent formulation in Corchero et al. (2013):

$$c_{ti}^u \geq c_i^{on} [u_{ti} - u_{(t-1),i}] \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{I} \quad (A.1)$$

$$c_{ti}^d \geq c_i^{off} [u_{(t-1),i} - u_{ti}] \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{I} \quad (A.2)$$

$$\sum_{j=n}^{G_i} (1 - u_{ji}) = 0 \quad \forall i \in \mathcal{I} \quad (A.3)$$

$$\sum_{j=1}^{H_i} u_{ji} = 0 \quad \forall i \in \mathcal{I} \quad (A.4)$$

$$\min\{t+t_i^{on}-1, |\mathcal{T}|\} \sum_{n=t} u_{ni} \geq \alpha_{ti}^{on} [u_{ti} - u_{(t-1),i}] \quad t = G_i + 1, \dots, |\mathcal{T}|, \forall i \in \mathcal{I} \quad (A.5)$$

$$\min\{t+t_i^{off}-1, |\mathcal{T}|\} \sum_{n=t}^{} (1-u_{ni}) \geq \alpha_{ti}^{off} [u_{(t-1)i} - u_{ti}] \quad t=H_i+1, \dots, |\mathcal{T}|, \forall i \in \mathcal{I} \quad (\text{A.6})$$

where the parameters α_{ti}^{on} and α_{ti}^{off} are defined as:

$$\alpha_{ti}^{on} = \min\{t_i^{on}, |\mathcal{T}| - t + 1\} \quad (\text{A.7})$$

$$\alpha_{ti}^{off} = \min\{t_i^{off}, |\mathcal{T}| - t + 1\} \quad (\text{A.8})$$

Table A8

States of the CCGT unit and its associated pseudo units.

CCGT unit with a CT and HRSG/ST					
State	Composition	Pseudounit 1	$u_{\mathcal{P}_c(1)t}$	Pseudounit 2	$u_{\mathcal{P}_c(2)t}$
0	0CT+0HRSG/ST	off	0	off	0
1	1CT+0HRSG/ST	on	1	off	0
2	1CT+1HRSG/ST	off	0	on	1

Let us now develop the formulation of the equivalent unit commitment constraints (A.1)–(A.6) for the combined cycle units. The operational rules of a typical CCGT unit were formulated in [Lu and Shahidehpour \(2004\)](#), with the help of the so-called *pseudo units*, (PUs) and subsequently improved in [Heredia et al. \(2012\)](#) through a reduction in the number of PUs with the associated savings in the number of variables and constraints. Our approach keeps the reduction in the number of PUs proposed in [Heredia et al. \(2012\)](#), but also reformulates the entire set of CCGT unit commitment constraints similarly to constraints (A.1)–(A.6) in order to obtain an even greater reduction in the number of constraints and variables. As is well known, a CC unit is composed of several combustion turbines (CTs) and a set of heat recovery steam generators (HRSG) with a steam turbine (ST). Each available combination of CTs and HRSG/ST defines a state of the CCGT unit. Table A.8 shows the three possible states (0,1,2) of a CCGT unit with a CT and an HRSG/ST, together with the state of the two pseudo-units that can be used to mathematically represent the operation of this CCGT. In the first state, state 0, both the CT and the HRSG/ST are disengaged, no electricity production is generated, and the two associated pseudo-units are considered to be off. The next state, state 1, corresponds to the situation when the CT is online and thus feeding its output to the system, but the HRSG/ST is not functioning. In the equivalent representation of this state in terms of pseudo-units, we consider that the first pseudo-unit is on and generating the same production as the CT, but the second pseudo-unit is off. Finally, in state 2, when the CCGT simultaneously engages the CT and the HRSG/ST, pseudo-unit 1 is considered to be down while the second pseudo-unit is on with the same generation as the CCGT plant.

The two PUs of each CCGT unit can be treated as a special case of a thermal unit with some dependency among their respective on/off states. The on/off states of these two PUs uniquely determine the state of their associated CCGT, as shown in columns 3 and 5 of Table A.8.

Let \mathcal{P}_c be the set of PUs of the CCGT unit $c \in \mathcal{C}$, and let the whole set of PUs be represented by $\mathcal{P} = \cup_{c \in \mathcal{C}} \mathcal{P}_c$. Let also $\mathcal{P}_c(j)$ be the PU associated with the state $j \in \{1, 2\}$ of the CCGT unit c . Then, the complete set of thermal and pseudo units is $\mathcal{U} = \mathcal{I} \cup \mathcal{P}$. Moreover, we introduce the first-stage binary variables u_{ti} , $i \in \mathcal{U}$ to represent

the on/off state of each thermal and pseudo unit at period t . Table A.8 illustrates the relationships that variables $u_{\mathcal{P}_c(1)}$ and $u_{\mathcal{P}_c(2)}$ have with the state of the associated CCGT unit. Regarding the transition costs, each PU has its own start-up cost, no shut-down costs, and the transition from state 2 to state 1 is costless. The following constraints define the value of the PUs' start-up cost variable $c_{\mathcal{P}_c(j)}^u$:

$$c_{\mathcal{P}_c(1)}^u \geq c_{\mathcal{P}_c(1)}^{on} \left[\left[u_{\mathcal{P}_c(1)} - u_{(t-1)\mathcal{P}_c(1)} \right] - \left[u_{(t-1)\mathcal{P}_c(2)} - u_{t\mathcal{P}_c(2)} \right] \right] \quad t \in \mathcal{T}, c \in \mathcal{C} \quad (\text{A.9})$$

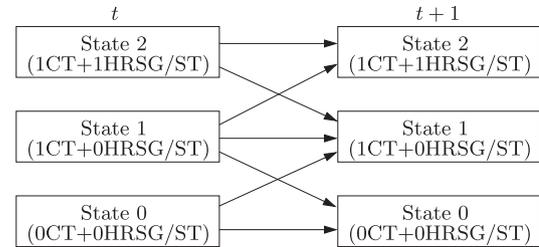


Fig. A6. Feasible transitions of the CCGT unit with a CT and HRSG/ST.

$$c_{\mathcal{P}_c(2)}^u \geq c_{\mathcal{P}_c(2)}^{on} \left[u_{\mathcal{P}_c(2)} - u_{(t-1)\mathcal{P}_c(2)} \right] \quad t \in \mathcal{T}, c \in \mathcal{C} \quad (\text{A.10})$$

The minimum up time for each PU $i \in \mathcal{P}$ is established by the following constraints, t_i^{on} :

$$\min\{t+t_i^{on}-1, |\mathcal{T}|\} \sum_{n=t}^{} u_{ni} \geq \alpha_{ti}^{on} [u_{ti} - u_{(t-1)i}] \quad t = G_i + 1, \dots, |\mathcal{T}|, i \in \mathcal{P} \quad (\text{A.11})$$

where again u_{0i} , α_{ti}^{on} and G_i are defined similarly to the thermal units unit commitment constraints. Then, following Eq. (A.3):

$$\sum_{t=1}^{G_i} (1 - u_{ti}) = 0 \quad i \in \mathcal{P}, t \in \mathcal{T} \quad (\text{A.12})$$

Once a CCGT unit has been shut down it cannot be started up before $(t_c^{off})^C$ periods. If H_c^C represents the number of the initial time periods along which the CCGT unit must remain off the following constraints impose the minimum down time condition for the CCGT units:

$$\min\{t+(t_c^{off})^C-1, |\mathcal{T}|\} \sum_{n=t}^{} \left[1 - (u_{n\mathcal{P}_c(1)} + u_{n\mathcal{P}_c(2)}) \right] \geq \alpha_{tc}^{off} \left[(u_{(t-1)\mathcal{P}_c(1)} + u_{(t-1)\mathcal{P}_c(2)}) - (u_{t\mathcal{P}_c(1)} + u_{t\mathcal{P}_c(2)}) \right] \quad t = H_c^C + 1, \dots, |\mathcal{T}|, c \in \mathcal{C} \quad (\text{A.13})$$

$$\sum_{t=1}^{H_c^C} u_{t\mathcal{P}_c(1)} + u_{t\mathcal{P}_c(2)} = 0 \quad c \in \mathcal{C}, t \in \mathcal{T} \quad (\text{A.14})$$

with $\alpha_{tc}^{off} = \min\{(t_c^{off})^C, |\mathcal{T}| - t + 1\}$. The feasible transitions rules (Fig. A.6) impose additional constraints on the operation of the PUs associated to a given CCGT unit, $c \in \mathcal{C}$. A first consideration is that

the PUs in \mathcal{P}_c are mutually exclusive, i.e., only one of them can be committed at a given period (see Eq. (A.15)(a) below). The second rule establishes that the changes between periods $t - 1$ and t in the commitments of the PUs in \mathcal{P}_c are limited to the feasible transitions depicted in Fig. A6. Consequently, if the CCGT unit c is in state 0 at period $t - 1$ ($u_{(t-1)\mathcal{P}_c(1)} + u_{(t-1)\mathcal{P}_c(2)} = 0$), it cannot be in state 2 at period t ($u_{t\mathcal{P}_c(2)} = 0$) (Eq. (A.15)(b)). Conversely, if $u_{(t-1)\mathcal{P}_c(2)} = 1$, then $u_{t\mathcal{P}_c(1)} + u_{t\mathcal{P}_c(2)} \geq 1$ (Eq. (A.15)(c)). All these operations rules are laid down in the following set of constraints:

$$\left. \begin{aligned} \sum_{m \in \mathcal{P}_c} u_{tm} &\leq 1 & (a) \\ u_{t\mathcal{P}_c(2)} &\leq u_{(t-1)\mathcal{P}_c(1)} + u_{(t-1)\mathcal{P}_c(2)} & (b) \\ u_{(t-1)\mathcal{P}_c(2)} &\leq u_{t\mathcal{P}_c(1)} + u_{t\mathcal{P}_c(2)} & (c) \end{aligned} \right\} \begin{array}{l} c \in \mathcal{C}, \\ t \in \mathcal{T} \end{array} \quad (A.15)$$

Finally, the polyhedron P_{UC} is defined through the set of constraints

$$P_{UC} = \left\{ c^u, c^d \in \mathbb{R}^{|\mathcal{T}| \times |\mathcal{A}| \times |\mathcal{C}|}, u \in \mathbb{0}, 1^{|\mathcal{T}| \times |\mathcal{A}| \times |\mathcal{C}|} : (A.1) - (A.6), (A.9) - (A.15) \right\}$$

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