

Backstepping Based Control of Heave-induced Pressure Deviations in Managed Pressure Drilling

Saqib Nazir¹, Nouman Ashraf², Iftikhar Ahmad¹

¹National University of Sciences and Technology, Pakistan

²Frederick University, Cyprus

Abstract— Managed Pressure Drilling (MPD) is a control technique for controlling pressure in the presence of wave disturbance. It works as an insurance against delays due to weather conditions by saving time and increasing the efficiency of offshore drilling operations. During MPD, a circulating drilling fluid is released in well through control choke which helps to dynamically control fluid pressure in the presence of undesirable pressure deviations. Pressure deviations are caused by vertical motion of drill string because of wave disturbance (Heave Motion) which may damage the well. Control system should be able to keep fluid pressure on the desired set value by releasing controlled fluid through control choke. In this paper, backstepping based controller is designed for five volumes model of well pressure dynamics. The controller attenuates pressure fluctuations because of disturbance and keeps pressure near to desired value. Results will be compared with previously used controllers for pressure regulation. Matlab simulation results show that backstepping based controller designed for five volumes model restricts pressure variations within ± 1 bar in the presence of heave disturbance of ± 10 m. Backstepping based controller is simple to implement on higher order well models. It attenuates heave disturbance of higher amplitudes as compared to previously proposed controllers.

Index Terms— Managed Pressure Drilling, Heave Disturbance, Backstepping, Control.

I. INTRODUCTION

INITIALLY drilling was done on shore only. After the advent of modern motion control systems, offshore drilling explorations are done using under-water moving platforms. Sea waves move the platform in horizontal and vertical directions. Such movements result in disturbances and large load variations in the drilling pipe and winch systems. To compensate for such heave up and down movements, Heave Compensator Systems are used. Heave Compensator Systems work as an insurance against weather delays by saving time and increasing efficiency of offshore drilling and lifting operations. Heave compensation has changed the nature of drilling and lifting operations at sea by protecting offshore assets from dangerous sea waves. Heave Compensators thus reduce the operation and financial risk from offshore operations. A heave compensator is a type of motion compensator. Motion compensators compensate for motion in all directions but heave compensator normally compensates motion in vertical direction. Heave compensator is sometimes also known as Drill String Compensator.

The amplitude of heave motion can be more than 3 meters. This results in disturbance in the bottom of the well. The drill string moves up and down because of motion of waves.

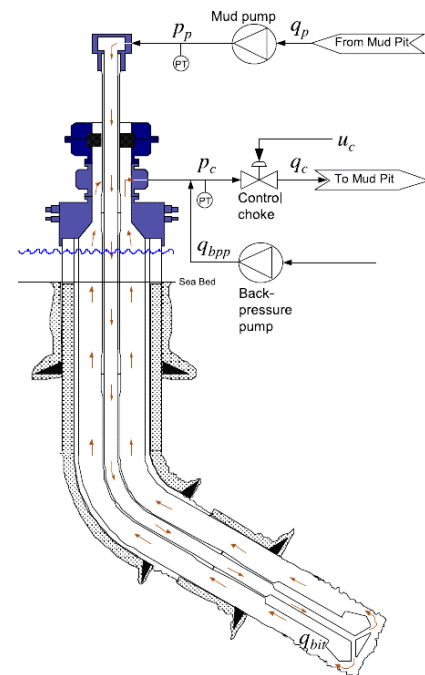


Fig.1. Schematic of MPD System [20]

This increase and decrease in pressure can result in high pressure exceeding the formation fracture pressure or less pressure resulting in uncontrolled flux into reservoir or collapse of well [1]. These issues can result in costly and time taking loss of drilling fluid called mud. These risks also result in environmental damage.

Heave motion of offshore rigs complicates the pressure control in MPD [2], [3]. As mentioned earlier, the drill string moves up and down with the heaving rig causing down-hole pressure variations. Therefore, it is important to design control systems which attenuate the bottom-hole pressure fluctuations. Objective is to reduce the amplitude of variations and keep drilling operation as smooth as possible for better economy and less downtime.

Therefore, based on the motivation to contribute to a more accurate well pressure management, this paper is a contribution in controlling the magnitude of pressure variations (surge and swab). This paper presents nonlinear controller for controlling pressure fluctuations because of heave motion. Nonlinear backstepping controller is implemented for controlling pressure because of heave motion. Because of its ability to perform drilling with small pressure margins, active control of the well helps in drilling from places which were not drillable using manual drilling operations.

MPD modelling has been investigated in literature [2], [4]–[7]. Design of Control System and estimation is also presented in literature [8]–[12]. Different MPD methods for controlling high and low pressure are presented by Rasmussen [13]. Pavlov presented two control system designs for disturbance rejection of full scale drilling rig [14]. Mahdianfar presented a model representing flow and pressure fluctuations due to drill string movements [15]. Mahdianfar also estimated heave disturbance to design a controller for disturbance rejection of bottom hole pressure [15].

Bottom hole pressure should be kept greater than pore pressure and less than fracture pressure. Low pressure can result in dangerous situations such as gas leakage in the mud creating extreme pressure called kicks, unwanted drilling into neighboring wells, mixing of oil or water in mud, and collapse of well around the heave motion. On the other hand, high pressure can result in leakage of mud in the formation, creation of mud-wall around the holes of the well resulting in slow production, and over-burdening of well because of pressure on the well [13], [16]. Managed Pressure Drilling is also sometimes known as constant bottom hole pressure [16]. It keeps bottom hole pressure within desired limits.

Model predictive control can also be used for MPD [3], [6], [9], [13], [16], [17]. The model presented by Ingar [2] was later used by Nikoofard [18] in the year 2014 for comparing PID controller with Model Predictive Control (MPC) for heave disturbance reduction in offshore MPD systems which shows that MPC works better as compared to PID controller for keeping pressure near to desired value in the presence of disturbance.

Kaasa model of well hydraulics [20] is commonly used for designing existing MPD systems. In Kaasa Model [20] is that there is no distinction between choke pressure and bottom hole pressure. This similarity results in decreased degree of freedom for controlling bottom hole pressure even if the heave compensation problem is resolved in most of the cases. By increasing amplitude of heave pressure, the Kaasa model fails to control choke pressure [2].

Landet et al presented a heave compensation controller [22] using new hydraulic model of well dynamics [2]. The model used by Landet represents main dynamics of MPD System in case of heave disturbance in a well from 2000 meters long Ulrigg testing facility with water as test-bed [2].

Backstepping is a popular technique used for control design of special class of dynamical systems. Using recursive Lyapunov [26] based structure, a controller is designed which guarantees stability in the presence of disturbances. Step by step procedure results in simple mathematical calculations and Lyapunov convergence analysis. Backstepping is simple to implement on higher order models because of its recursive procedure.

This paper is organized as follows. In section 2, mathematical modeling of well hydraulics is presented. In section 3, backstepping based control technique for attenuation of heave-induced pressure deviations is presented. Simulation results are presented in section 4. Finally, our research is summarized and future work is proposed in the last section.

II. MATHEMATICAL MODELING

In this section we present mathematical model of annulus flow. Initially a generic model will be presented for dynamical pressure and flow. The generic mathematical model will be used to derive five volumes, ninth order model representing dynamic

pressure at different locations in the well. The equations representing annulus flow are given by [2]:

$$\frac{\partial p}{\partial t} = -\frac{\beta}{A} \frac{\partial q}{\partial x} \quad (1)$$

$$\frac{\partial q}{\partial t} = -\frac{A}{\rho_0} \frac{\partial p}{\partial x} - \frac{F}{\rho_0} + Ag \cos(\alpha(x)) \quad (2)$$

Where $p(x, t)$ and $q(x, t)$ are pressure flow rates and volumetric flow rates, at location x and time t , respectively. β is bulk modulus of mud, $A(x)$ is cross-sectional area, ρ is mass density, F is Friction force per unit length, g is gravitational constant and α is angle between gravity and position flow direction at location $x(t)$

Equation (1) and (2) are discretized by using finite volumes method [1], [18]. The annulus is divided into a number of control volumes in order to solve this problem as shown in Figure 2.

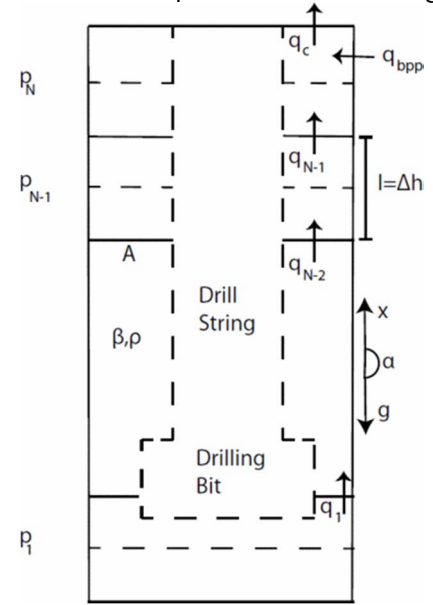


Fig.2. Control Volumes of annulus hydraulic model [2]

Ingar [2] presented that following five control volumes model can be used to represent the main dynamics of 2000 meters long Ulrigg testing facility installed in water based mud.

$$\dot{p}_1 = \frac{\beta_1}{A_1 l_1} (-q_1 - A_d V_d) \quad (3)$$

$$\dot{p}_2 = \frac{\beta_2}{A_2 l_2} (q_1 - q_2) \quad (4)$$

$$\dot{p}_3 = \frac{\beta_3}{A_3 l_3} (q_2 - q_3) \quad (5)$$

$$\dot{p}_4 = \frac{\beta_4}{A_4 l_4} (q_3 - q_4) \quad (6)$$

$$\dot{p}_5 = \frac{\beta_5}{A_5 l_5} (q_4 - q_c + q_{bpp}) \quad (7)$$

$$\dot{q}_i = \frac{A_i}{l_i \rho_i} (p_i - p_{i+1}) - \frac{F_i(q_i) A_i}{l_i \rho_i} - A_i g \frac{\Delta h_i}{l_i} \quad (8)$$

$$q_c = K_c \sqrt{p_c - p_0} G(u) \quad (9)$$

Where

$i=1,2,3,4$ and numbers $1,2,3,4,5$ represent the control volume number, 1 is the lower-most volume number representing bottom hole pressure $p_1 = p_{bit}$, 5 is the upper-most volume number representing the choke pressure $p_5 = p_c$, V_d is heave disturbance, Δh is difference in height, l is the length of each control volume, K_c is choke constant, p_0 is atmospheric pressure and $G(u)$ is a strictly increasing, invertible function representing control input to choke opening.

Based on results of full scale experiments at Ulrigg, frictional force on the i^{th} control volume is given by [2]:

$$F_i(q_i) = \frac{k_{fric} q_i}{A_i} \quad (10)$$

Where k_{fric} is coefficient of friction.

The above model can easily be converted into state space form by considering

$$a_j = \frac{\beta_j}{A_j l_j}, b_j = \frac{A_j}{\rho_j l_j}, c_j = \frac{K_{fric}}{\rho_j l_j} \quad \text{and}$$

$X = [p_1 \ q_1 \ p_2 \ q_2 \ p_3 \ q_3 \ p_4 \ q_4 \ p_5]^T$ in above model, we get:

$$\dot{x}_1 = -a_1 x_2 - a_1 A_d V_d \quad (11)$$

$$\dot{x}_2 = (b_1 x_1 - b_1 x_3) - c_1 x_2 - e_1 \quad (12)$$

$$\dot{x}_3 = a_2 x_2 - a_2 x_4 \quad (13)$$

$$\dot{x}_4 = (b_2 x_3 - b_2 x_5) - c_2 x_4 - e_2 \quad (14)$$

$$\dot{x}_5 = a_3 x_4 - a_3 x_6 \quad (15)$$

$$\dot{x}_6 = (b_3 x_5 - b_3 x_7) - c_3 x_6 - e_3 \quad (16)$$

$$\dot{x}_7 = a_4 x_6 - a_4 x_8 \quad (17)$$

$$\dot{x}_8 = (b_4 x_7 - b_4 x_9) - c_4 x_8 - e_4 \quad (18)$$

$$\dot{x}_9 = a_5 x_8 - a_5 u_a \quad (19)$$

The simulation parameters measured by Iris-Drill Simulator are given in Table 1 [18]:

Parameter	Value	Parameter	Value
a_i	$2.254 \times 10^8 \text{ Pa/m}^3$	g	9.806 m/s^2
b_i	$4.276 \times 10^{-8} \text{ m}^4/\text{Kg}$	A	0.0269 m^2
K_f	$5.725 \times 10^5 \text{ sPa/m}^3$	e_i	$0.2638 \text{ m}^3/\text{s}^2$
c_i	$14.4982 \text{ s}^{-1} \text{ m}^{-2}$	A_d	0.0291 m^2
K_G	0.0650	K_c	2.32
q_{bpp}	$369.2464 \text{ m}^3/\text{s}$	p_0	101325 pa

Table 1: Parameter Values measured by Iris-Drill

Model verification of above model is done by Pavlov [3] and results verify the above model with actual Ulrigg data. Ingar [2] also used same model for comparison with actual Ulrigg data.

III. BACKSTEPPING CONTROLLER DESIGN

In this section we will design a back-stepping based controller for five volumes model. Please note that the parameter ‘c’ is replaced by ‘d’ in order to avoid confusion. Since the system shown above is in strict feedback form, backstepping technique is used in order to design control input u_a for controlling heave motion. Following change of coordinates is made before starting controller design:

$$z_1 = x_1 - x_{des} \quad (20)$$

$$z_i = x_i - \alpha_{i-1} \quad (21)$$

Where $i=2, 3, 4, 5, 6, 7, 8, 9$ and ‘ α ’ is virtual control at previous step, and will be calculated later. x_{des} is the desired bottom hole pressure

Taking derivative of Eq. (20), we get

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_{des} \quad (22)$$

Putting value of \dot{x}_1 from Eq. (21) in Eq. (22), we get:

$$\dot{z}_1 = -a_1 x_2 - a_1 A_d V_d - \dot{x}_{des} \quad (23)$$

Putting the value of x_2 from Eq. (21) in above equation, we get

$$\dot{z}_1 = -a_1(z_2 + \alpha_1) - a_1 A_d V_d - \dot{x}_{des} \quad (24)$$

First Lyapunov function is defined in Eq. 21:

$$V_1 = \frac{1}{2} z_1^2 \quad (25)$$

Following virtual control law α_1 is chosen:

$$\alpha_1 = -\frac{c_1}{a_1} z_1 + A_d V_d - \frac{1}{a_1} \dot{x}_{des} \quad (26)$$

which makes derivative of V_1 as:

$$\dot{V}_1 = -c_1 z_1^2 - a_1 z_1 z_2 \quad (27)$$

Similarly \dot{z}_2 is chosen using Eq. (21) as:

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 \quad (28)$$

Second Lyapunov function V_2 is defined in Eq. (29):

$$V_2 = V_1 + \frac{1}{2} z_2^2 \quad (29)$$

Virtual control law α_2 is designed as shown below:

$$\alpha_2 = \frac{c_2}{b_2} z_2 + A_d V_d - \frac{1}{a_1} \dot{x}_{des} \quad (30)$$

Which makes derivative of V_2 as:

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 - b_1 z_2 z_3 \quad (31)$$

Similarly, Eq. (21) implies:

$$\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_2 \quad (32)$$

Third Lyapunov function is defined in Eq. (33):

$$V_3 = V_2 + \frac{1}{2}z_3^2 \quad (33)$$

Virtual control law ' α_3 ' is designed as shown below

$$\alpha_3 = \frac{c_3}{a_2}z_3 - \frac{b_1}{a_2}z_2 + z_2 + \alpha_1 - \frac{1}{a_2}\dot{\alpha}_2 \quad (34)$$

Which makes derivative of V_3 as:

$$\dot{V}_3 = -c_1z_1^2 - c_2z_2^2 - c_3z_3^2 - a_2z_3z_4 \quad (35)$$

Similarly, backstepping procedure results in recursive development of new variables in sequel up to last step:

$$\dot{z}_9 = \dot{x}_9 - \dot{\alpha}_8 \quad (36)$$

Ninth Lyapunov function V_9 is defined in Eq. (37):

$$V_9 = V_8 + \frac{1}{2}z_9^2 \quad (37)$$

Following Backstepping based control law u_a is chosen:

$$u_a = \frac{c_9}{a_5}z_9 - \frac{b_4}{a_5}z_8 + z_8 + \alpha_7 - \frac{1}{a_5}\dot{\alpha}_8 \quad (38)$$

which makes derivative of Lyapunov V_9 as shown below:

$$\dot{V}_9 = -c_1z_1^2 - c_2z_2^2 - c_3z_3^2 - c_4z_4^2 - c_5z_5^2 - c_6z_6^2 - c_7z_7^2 - c_8z_8^2 - c_9z_9^2 \leq 0 \quad (39)$$

Closed loop error equations are given below:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \\ \dot{z}_6 \\ \dot{z}_7 \\ \dot{z}_8 \\ \dot{z}_9 \end{bmatrix} = - \begin{bmatrix} c_1 & a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_3 & a_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4 & b_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_5 & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_6 & b_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_7 & a_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_8 & b_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_9 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \\ z_8 \\ z_9 \end{bmatrix} \quad (40)$$

Where $c_1, c_2, c_3 \dots c_9$ are positive design coefficients. Minus sign indicates that errors are converging to zero.

IV. SIMULATION RESULTS

Backstepping based controller is simulated for heave disturbance of ± 10 m. Matlab simulation results are shown in Figure 3-7.

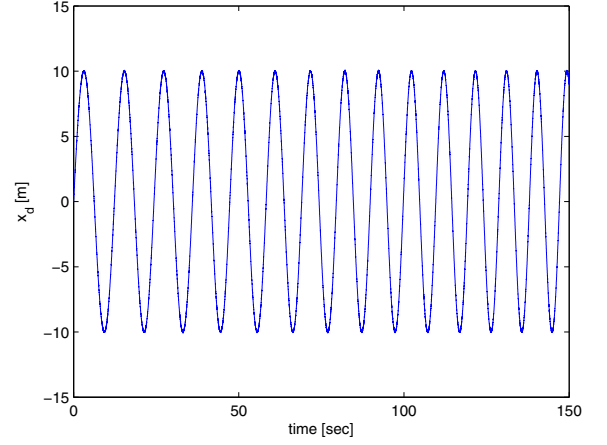


Fig.3. Heave Disturbance used for backstepping

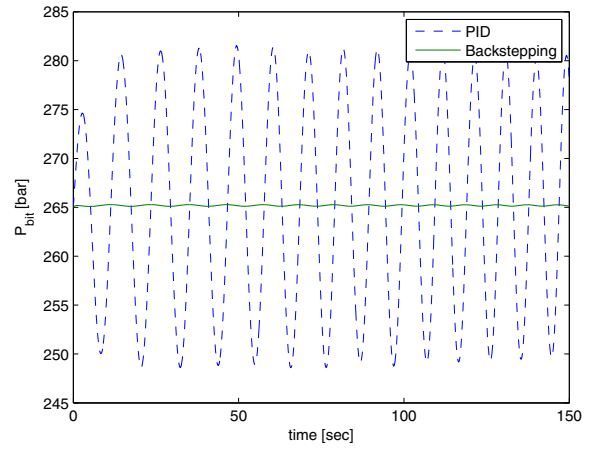


Fig.4. Comparison of Backstepping Controller with PID Controller

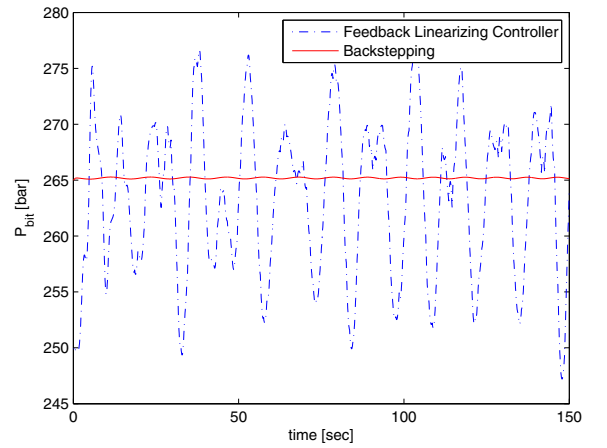


Fig.5. Comparison of Backstepping Controller with Feedback Linearizing Controller [22]

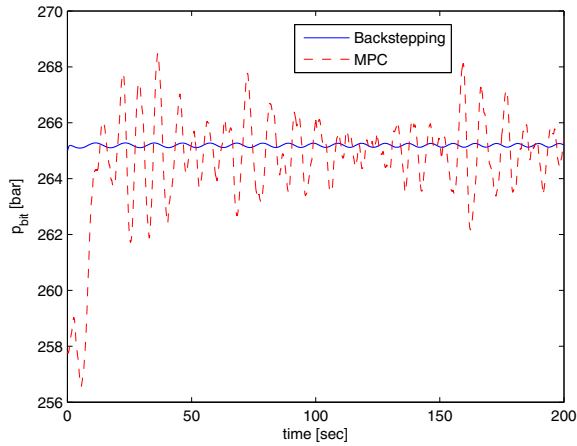


Fig. 6. Comparison of Backstepping Controller with MPC [16]

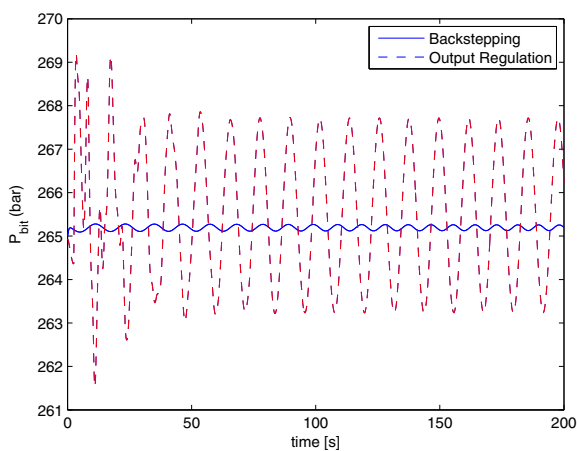


Fig. 7. Comparison of Backstepping Controller with Output Regulation Controller [2]

Table 2 summarizes the results and shows that as compared to PID, Feedback linearising, MPC, and Output regulation controller, Pressure deviations, from desired set-point of 265 bar, are less in case of backstepping based controller.

Controller Type	Pressure Deviations [bar]
Backstepping	± 1
Output Regulation	± 2.5
PID	± 16
Feedback Linearizing	± 12
Model Predictive Controller	± 4

Table 2: Performance Comparison of Controllers

Above simulation results clearly show that backstepping based controller can be used for keeping pressure near to desired value in the presence of disturbance. Deviation from desired pressure in case of backstepping based control is ± 1 bar which is less as compared to Output Regulation (± 2.5 bar), PID (± 16 bar), Feedback Linearizing (± 12 bar) and Model Predictive Controller (± 4 bar).

V. CONCLUSION

Heave disturbance can result in costly delays and dangerous accidents because of pressure variations. Managed Pressure Drilling can decrease such delays and accidents by attenuating heave disturbances. This is done by the help of pressure control system which dynamically opens and closes the control choke. In this paper, backstepping based controller is implemented for keeping pressure near to the desired value in the presence of disturbances. Backstepping based controller can easily be implemented on higher order well models because of its simple step-by-step implementation procedure. Lyapunov based stability analysis guarantees convergence of pressure to desired pressure value. Simulation results show that backstepping based controller attenuates more heave disturbance as compared to previously proposed controllers and keeps pressure near the desired value. This results in more safe and economical Managed Pressure Drilling operations. In future an adaptive backstepping controller for controlling bottom hole pressure can be designed in order to cater parameter variations and uncertainties.

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