# Detection of broken bars in induction motor using the Extended Kalman Filter (EKF)

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***Abstract*—Estimation of rotor resistance rapidly and accurately has received a lot of attention due to its significance in improving the performance of Induction Motor (IM). This paper deals with the diagnostic of broken bars in induction motors. The hypothesis on which detection is based is that the apparent rotor resistance of an induction motor will increase when a rotor bar breaks. To detect rotor fault we propose an Extended Kalman Filter approach for rotor resistance estimation. In particular, rotor resistance is estimated and compared with its nominal value to detect broken bars.**

***Keywords-Diagnosis; Observers; broken bars; Estimation; Kalman; EKF; Induction motor.***

## I. Introduction

Nowadays, no one can deny the important role of the induction motor in industry applications particularly high-tech due to their power-to-weight ratio and low price... It is well-known that an interruption of a manufacturing process due to a mechanical or electrical problem induces a significant financial loss for the firm [1]. The failure of the induction motor may be caused because of many reasons like manufacturing fault, designing fault of the engineer, overloading, environment and poor technical knowledge of the job about in handling the machine. In order to avoid such problems, we have to detect these faults to prevent a major failure from occurring.

In this context, a variety of sensors could be used to collect measurements from an induction motor for the purpose of failure monitoring. We can quote vibration measurement, temperature measurement, Park's Vector currents monitoring, artificial intelligence based techniques and observers estimation such as EKF… [5].

To detect broken rotor bars, the measurements of stator voltages and currents are processed by an EKF for the rotor resistance estimation. In particular, this resistance is estimated and compared with its nominal values to detect the broken rotor bars [4]. However, the rotor resistance changes with temperature and excitation conditions [1].

Several approaches have been proposed, in the available literature, for rotor resistance estimation. The purpose of using an EKF approach is to improve the rotor resistance sensorless estimation using only stator voltages and currents measurements. Moreover, the advantage of using stator currents as state variable is that they are directly measurable [1].

## II. Induction Motor Model

### *1- Time domain induction machine model*

Induction motor can be described by fourth order nonlinear differential equations with four electrical variables (currents and fluxes and two control variables (stator voltages) [2] [6]. In d-q axes where the motor is assumed symmetrical and flux distribution is sinusoidal [8].

- The state equation is

$$\dot{x} = A(\omega_r)x(t) + Bu(t) \tag{1}$$

- The output equation is

$$y = Cx(t) \tag{2}$$

Where

- The state vector is

$$x(t) = [i_{sd} \quad i_{sq} \quad \varphi_{rd} \quad \varphi_{rq}]^T \tag{3}$$

- The input and output vectors are

$$y(t) = [i_{sd} \quad i_{sq}]^T \tag{4}$$

$$u(t) = [u_{sd} \quad u_{sq}]^T \tag{5}$$

- The state matrix 'A' is

$$\begin{pmatrix} -\left(\frac{Rs}{\sigma Ls}+\frac{1-\sigma}{\sigma\tau_r}\right) & \omega_s & \frac{M}{\sigma L_s L_r \tau_r} & \frac{M(\omega_s-\omega_r)}{\sigma L_s L_r} \\ \omega_s & -\left(\frac{Rs}{\sigma Ls}+\frac{1-\sigma}{\sigma\tau_r}\right) & -\frac{M(\omega_s-\omega_r)}{\sigma L_s L_r} & \frac{M}{\sigma L_s L_r \tau_r} \\ \frac{M}{\tau_r} & 0 & \frac{-1}{\tau_r} & \omega_r \\ 0 & \frac{M}{\tau_r} & -\omega_r & \frac{-1}{\tau_r} \end{pmatrix}$$

- The input 'B' and output 'C' matrices are

$$B = \begin{pmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad , \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{7}$$

Where

- $\sigma = 1 - \frac{M^2}{L_s L_r}$ : Leakage coefficient (8)
- $\tau_r = \frac{L_r}{R_r}$ : Rotor time constant (9)
- $u_{sd}, u_{sq}$ : Stator voltages in d-q axes (V)
- $i_{sd}, i_{sq}$ : Stator currents in d-q axes (A)
- $\varphi_{rd}, \varphi_{rq}$ : Rotor flux components in d-q axes (Wb)
- $Rs(Rr)$ : Stator(rotor) resistance(Ohm)
- $L_s(L_r)$ : Stator(rotor) self inductance (H)
- $M$ : Mutual inductance (H)
- $\omega_s, \omega_r$ : Stator(rotor) angular velocity (rad/s)

*2- Discretized Induction Machine Model*

The discretized machine equations are [2]

$$X_{(k+1)} = A_{dk} x_{(k)} + B_{dk} u_{(k)} \tag{10}$$

$$y_{(k)} = C x_{(k)} \tag{11}$$

Where

$$A_{dk} = e^{AT_e} \approx I + AT_e \tag{12}$$

$$B_{dk} = BT_e \tag{13}$$

- The input' $B_{dk}$' and output' $C_{dk}$' matrices are

$$B_{dk} = \begin{pmatrix} \frac{T_s}{\sigma L_s} & 0 \\ 0 & \frac{T_s}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad , \quad C_{dk} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{14}$$

- The state matrix' $A_{dk}$' is

$$\begin{pmatrix} 1 - T_s\left(\frac{Rs}{\sigma Ls} + \frac{1-\sigma}{\sigma \tau_r}\right) & T_s\omega_s & \frac{T_s M}{\sigma L_s L_r \tau_r} & T_s \frac{M(\omega_s - \omega_r)}{\sigma L_s L_r} \\ T_s\omega_s & 1 - T_s\left(\frac{Rs}{\sigma Ls} + \frac{1-\sigma}{\sigma \tau_r}\right) & -T_s \frac{M(\omega_s - \omega_r)}{\sigma L_s L_r} & T_s \frac{M}{\sigma L_s L_r \tau_r} \\ T_s \frac{M}{\tau_r} & 0 & 1 - \frac{T_s}{\tau_r} & T_s\omega_r \\ 0 & T_s \frac{M}{\tau_r} & -T_s\omega_r & 1 - \frac{T_s}{\tau_r} \end{pmatrix}$$

## III. THE EXTENDED KALMAN FILTER THEORY

The Extended Kalman Filter (EKF) is the most popular algorithm for estimating physical parameters together with state variables [3][9].

It allows the state-vector to be extended to the rotor resistance [10][11].

The stator-vector is formed by six variables

$$X = [i_{sd} \quad i_{sq} \quad \varphi_{rd} \quad \varphi_{rq} \quad R_r]^T \tag{15}$$

To implement the EKF algorithm we linearise the non-linear model around the most recent estimate. The linearization process requires that partial derivative or Jacobean matrice be obtained for the non-linear functions in the model. In the augenement motor model given by (10) the state equations are non-linear and therefore Jacobean matrices is required as defined below [7][9][11]:

$$F(k) = \frac{\partial f(x(k), u(k))}{\partial x^T(k)}\Big|_{x(k) = \hat{x}(K/k)} , H(k) = \frac{\partial C(x(k))}{\partial x^T(k)}\Big|_{x(k) = \hat{x}(k+1/k)} \tag{16}$$

EKF algorithm consists of two sections: '***Prediction process***' and '***Correction process***' equations. The filtering process is summarized as follows. [6]

Where $K_k$ is the Kalman gain matrix, $k+1/k$ denotes prediction at time $k+1$ based on data at $k$; *x(0)* initial values.

### *a. Prediction process:*

1- The determination of the $x(0)$ (17)

2- $\hat{x}_{(k+1/k)} = f(\hat{x}_{(k/k)}, u(k))$ (18)

3- $F(k) = \frac{\partial f(x(k), u(k))}{\partial x^T(k)}\Big|_{x(k) = \hat{x}(k/k)}$ (19)

4- $\hat{P}_{(k+1/k)} = F(k)\hat{P}_{(k/k)}F^T(k) + Q$ (20)

5- $H(k) = \frac{\partial h(x(k))}{\partial x^T(k)}\Big|_{x(k) = \hat{x}(k+1/k)}$ (21)

***b. Correction process:***

6- $K_{(k+1)} = \frac{\hat{P}_{(K+1/k)} H^T(k)}{H(k)\hat{P}_{(K+1/k)} H^T(k)+R}$ (22)

7- $\hat{x}_{(k+1/k+1)}$=$\hat{x}_{(k+1/k)}$+$K_{(k+1)}(y_{k+1}$-$C\hat{x}_{K+1/k}$ (23)

8- $\hat{P}_{k+1/k+1} = [I - K_{(k+1)} H(k)]\, \hat{P}_{(K+1/k)}$ (24)

9- *Go to step* 2

## IV. Application to the Induction Machine

The induction motor model described in section (II) is here used to apply the Extended Kalman Filter above reviewed. The state variables are selected as (15).

The induction motor dynamic behavior is modeled as

$$f(x(k), u(k)) = A_{dk} x(k) + B_{dk} u(k) \tag{25}$$

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- State equations $f_k$:

$$\begin{bmatrix} i_{sd}\left(1 - T_s\left(\frac{Rs}{\sigma Ls} + \frac{1-\sigma}{\sigma\tau_r}\right)\right) + i_{sq} T_s\omega_s + \varphi_{rd}\frac{T_s M}{\sigma L_s L_r \tau_r} + \varphi_{rq} T_s \frac{M(\omega_s - \omega_r)}{\sigma L_s L_r} + u_{sd}\frac{T_s}{\sigma L_s} \\ i_{sd} T_s\omega_s + i_{sq}\left(1 - T_s\left(\frac{Rs}{\sigma Ls} + \frac{1-\sigma}{\sigma\tau_r}\right)\right) - \varphi_{rd} T_s\frac{M(\omega_s - \omega_r)}{\sigma L_s L_r} + \varphi_{rq}\frac{T_s M}{\sigma L_s L_r \tau_r} + u_{sq}\frac{T_s}{\sigma L_s} \\ i_{sd} T_s\frac{M}{\tau_r} + \varphi_{rd}\left(1 - \frac{T_s}{\tau_r}\right) + \varphi_{rq} T_s\omega_r \\ i_{sq} T_s\frac{M}{\tau_r} - \varphi_{rd} T_s\omega_r + \varphi_{rq}\left(1 - \frac{T_s}{\tau_r}\right) \\ R_r \end{bmatrix} \tag{26}$$

- The Jacobean matrix, namely $F_k$

$$\begin{bmatrix} 1 - T_s\left(\frac{Rs}{\sigma Ls} + \frac{1-\sigma}{\sigma\tau_r}\right) & T_s\omega_s & \frac{T_s M}{\sigma L_s L_r \tau_r} & T_s\frac{M(\omega_s - \omega_r)}{\sigma L_s L_r} & T_s\frac{M\varphi_{rd} - M^2 i_{sd}}{L_r^2 L_s - M^2 L_r} \\ T_s\omega_s & 1 - T_s\left(\frac{Rs}{\sigma Ls} + \frac{1-\sigma}{\sigma\tau_r}\right) & 1 - T_s\left(\frac{Rs}{\sigma Ls} + \frac{1-\sigma}{\sigma\tau_r}\right) & \frac{T_s M}{\sigma L_s L_r \tau_r} & T_s\frac{M\varphi_{rq} - M^2 i_{sq}}{L_r^2 L_s - M^2 L_r} \\ T_s\frac{M}{\tau_r} & 0 & 1 - \frac{T_s}{\tau_r} & T_s\omega_r & T_s\frac{M i_{sd} - \varphi_{rd}}{L_r} \\ 0 & T_s\frac{M}{\tau_r} & -T_s\omega_r & 1 - \frac{T_s}{\tau_r} & T_s\frac{M i_{sq} - \varphi_{rq}}{L_r} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

- The measurement matrix is given by:

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{28}$$

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## V. Simulation Results

Estimation of rotor resistance has been estimated using the extended fifth order EKF algorithm. In order to test the algorithm in the unhealthy case, the value of Rr is increased respectively at (t=1s, t=2s). The validity of the proposed estimation technique is well verified by simulations as illustrated by Figs. It should be noticed that these results have been obtained for an unloaded induction motor, which is generally more difficult to achieve by classical methods.

An abrupt stepwise on rotor resistance corresponds to a broken bar condition. This situation is also well identified by the proposed EKF technique as illustrated by “Fig 1.1”. The rotor resistance error is given by “Fig 1.2”. Moreover, broken bars affect not only the rotor resistance but also the magnetic flux and stator currents as illustrated by “Fig 2.1”, “Fig 3.1” and there error simulations in “Fig 2.2”,”Fig 3.2”. The variation of rotor resistance affects also the rotation speed as illustrated by “Fig 5”.

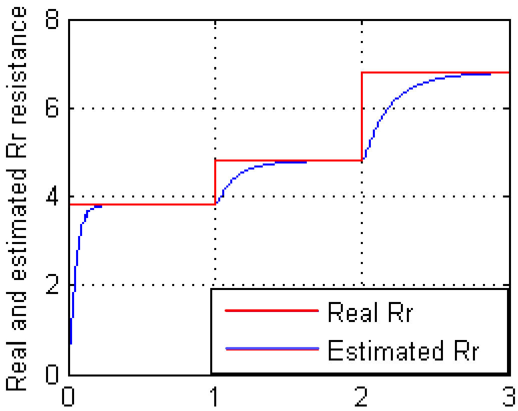


Figure 1.1. Real and estimated Rr(Ohm)
With Rr stepwise at (1, 2)s

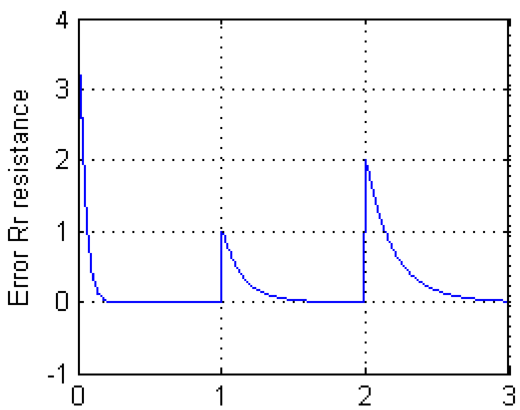


Figure 1.2. Rotor resistance error
With Rr stepwise at (1, 2)s

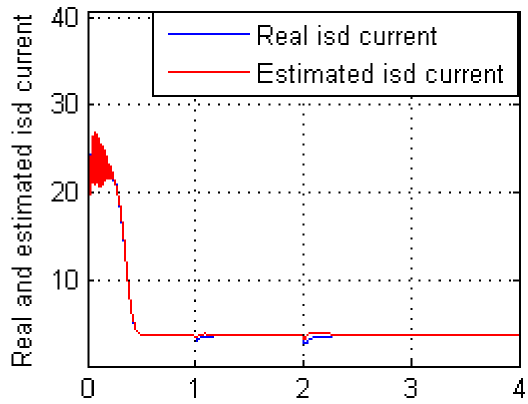


Figure 2.1. Real and estimated $i_{sd}$ (A)
With Rr stepwise at (1, 2)s

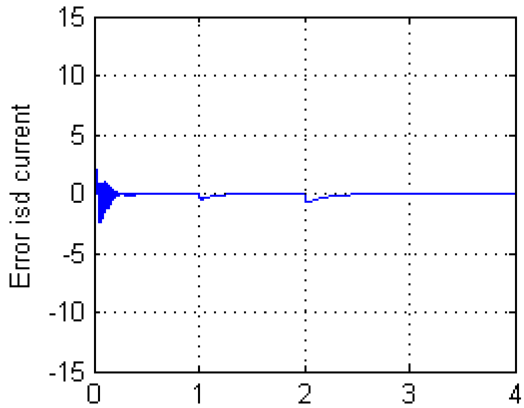


Figure 2.2. $i_{sd}$ current error
With Rr stepwise at (1, 2)s

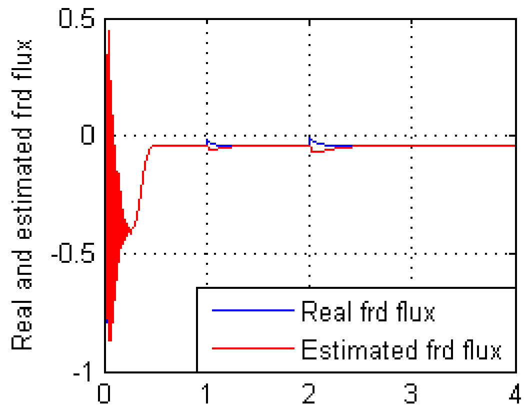


Figure 3.1. Real and estimated $\varphi_{rd}$(Wb)
With Rr stepwise at (1, 2)s

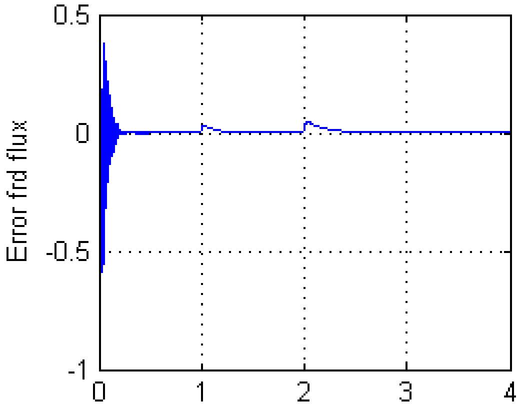


Figure 3.2. $\varphi_{rd}$ flux error
With Rr stepwise at (1,2)s

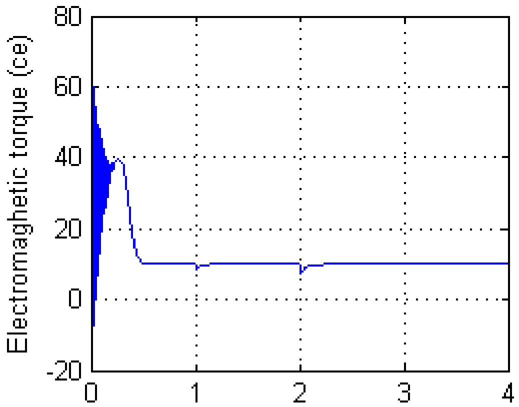


Figure 4. Electromagnetic torque Ce (N.m)
With Rr stepwise at (1, 2)s

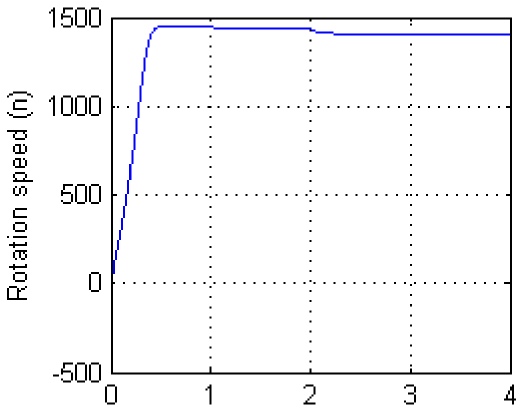


Figure 5. Speed rotation n (tr/mn)
With Rr stepwise at (1, 2)s

## VI. Conclusion

As an estimator, the Extended Kalman Filtering technique is investigated to observe the rotor resistance. For this purpose, appropriate mathematical model of induction machine is studied and discretized for real-time applications. Several simulations are illustrated to examine the performance of EKF. The hypothesis on which detection is based is that the apparent rotor resistance of an induction motor will increase when a rotor bar breaks. Its main advantage is the correct rotor resistance estimation even for an unloaded induction motor.

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