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Brief Paper

# Nonlinear Field Voltage Control of a Synchronous Generator using Feedback Linearization*

W. MIELCZARSKI† and A. M. ZAJACZKOWSKI‡

**Key Words**—(Feedback linearization); nonlinear control systems; state estimation; observers; electric power systems; generators.

**Abstract**—The problem of synthesizing a field voltage controller for a synchronous generator is considered. Using the theory of feedback linearization of nonlinear systems, the existence of a linearizing feedback for an improved reduced-order three-dimensional model of a generator is proved and the most simple form of this feedback is derived. An application of the obtained nonlinear control law to synchronous generator stabilization gives very good results, not only for the reduced-order model but also for the 'exact' Park's model of a generator. Simulations of fault and post-fault transients in the obtained control system with the synchronous generator ($P = 360$ MW), in the presence of a nonlinear state observer and field voltage limiters confirmed superiority of this control system as opposed to a system with a linear optimal controller.

## 1. *Introduction*

STABILIZATION OF A synchronous generator is one of the most important problems in power system control. A number of works have shown that linear optimal control (LOC) provides better means than conventional controllers in coordinated stabilization of a generator (Yu, 1983). Implementation of an optimal controller generally requires information about the entire state of a control plant. This cannot be satisfied in the case of a synchronous generator because impossibility of measurement of damper winding fluxes and the angle $\delta$ between the $q$-axis and the power system voltage $U_S$. Therefore the problem of synchronous generator state estimation arises.

Mielczarski (1987a, 1987b) presented the methodology of nonlinear observer design and applied it to the construction of nonlinear observers for four generator models. Digital simulations confirmed the efficiency of the proposed methodology. Further progress in observer investigation allowed the construction of very fast linear and nonlinear observers that can estimate generator state variables in a dozen milliseconds (Mielczarski, 1988a, 1988b).

With efficient observers it is possible to apply LOC to synchronous generator stabilization but it is well known (Okada *et al.*, 1985) that an observer in a feedback loop may spoil robustness properties of LOC and reduce a region of stability. Considering this disadvantage, an attempt was made to design together a linear optimal controller and an observer for a synchronous generator so that the robustness of the resulting closed-loop system is recovered. This goal was accomplished by an additional output feedback loop (Mielczarski and Zajaczkowski, 1987).

Although simulations of the obtained system were quite satisfactory, we believe that the most proper approach to controller design for synchronous generators is an application of the modern theory of nonlinear systems (Isidori, 1985). Our conviction follows the fact that design of linear optimal controllers is based on linear models resulting from the linearization of nonlinear systems about some operating point $P$. Thus a closed-loop system keeps assigned properties only in some neighbourhood of $P$. In many cases, however, faults may cause trajectories of generator state variables to move far from the operating point, and due to the strong nonlinearity of the synchronous generator (Yu, 1983) stability loss is possible.

The first attempt at an application of modern nonlinear control theory to controller design for synchronous generators was the work by Marino (1984). He considered the five-dimensional model of a generator and assumed that two inputs can be controlled: the field excitation voltage and the mechanical power. The last assumption is unrealistic from the technical point of view, because it is impossible to construct a mechanical power source that allows control of a mechanical power directly applied on a generator shaft. If one wants to use mechanical power (or torque) as an input of a generator, one should supplement a generator model by models of a steam or water turbine or a diesel engine (Yu, 1983), but in this case the seven-dimensional model of a generator–turbine system has to be considered and this complicates design. Thanks to the mentioned assumptions, a model used by Marino made controller synthesis easier but his results are rather of a theoretical nature than of practical significance.

Mielczarski and Zajaczkowski (1989) considered a conventional three-dimensional reduced-order model of a generator

$$D_t\boldsymbol{\xi} = \boldsymbol{\phi}(\boldsymbol{\xi}) + \mathbf{b}u_f$$

where $\boldsymbol{\xi}^T = [\delta, \omega, i_f]$ is a state vector.

They employed only one input signal, i.e. the field voltage $u_f$, and assumed that the mechanical power on a generator shaft is constant during control. These assumptions are satisfied for large turbo-generators. Afterwards they proved, using the Hunt *et al.* (1983) method, that this model can be linearized by a nonlinear feedback and they found the most simple and convenient (for practical applications) form of this feedback. They also showed that the same control law can be derived using Korobov's method (1973). The efficiency of the proposed nonlinear controller was evaluated by simulations under a number of conditions appearing in practice. In order to bring the problem closer to practical applications a nonlinear controller with observers was examined. Limitation of the field voltage was also considered during simulations. Although the problem of the nonlinear controller stability with an observer was not discussed and is still open, simulations showed very good transient properties

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† Centre for Electrical Power Engineering, Department of Electrical and Computer Systems Engineering, Monash University, Calyton, Victoria 3168, Australia.

‡ Department of Fundamental Research in Electrotechnics, Ministry of Industry and Polish Academy of Sciences Institute of Electrotechnics, 04-703 Warsaw, Pozaryskiego no. 28, Poland.

of a synchronous generator with the nonlinear controller and an observer working under fault and post-fault conditions. These results were also presented in Mielczarski and Zajaczkowski (1988).

It is worth mentioning that similar results to those of Mielczarski and Zajaczkowski were obtained independently by Ilic and Mak (1989). They considered the conventional third-order model of a generator described by state variables $(\delta, \omega, e_q)$ and using engineering intuition they found new state coordinates $z_1 = \delta$, $z_2 = \omega$, $z_3 = \dot{\omega}$ and proved that there exists a linearizing feedback controller for the considered model of a generator.

In the conventional three-dimensional model that was used in these studies, the damper windings of a generator have not been considered. The inclusion of damper windings into the generator model requires implementation of five- or seven-dimensional models of a generator (for example Park's model—see Appendix A), but it is rather difficult to find a nonlinear control law for these models. The problem then arises of how to take into account the influence of damper windings in the reduced-order three-dimensional model of a generator.

Sauer *et al.* (1988) showed that the concept of an integral manifold can be used to solve this problem in a rigorous way. They considered the six-dimensional models as an 'exact' model of a generator and using the integral manifold theory they obtained the improved reduced-order three-dimensional model (see Appendix B) in which an additional electromagnetic torque represents approximately the influence of damper windings. It is worth mentioning that this model was found by Kimbark (1956) using induction machine theory. Simulations presented in Sauer *et al.* (1988) showed that there are no significant differences between trajectories of the 'exact' and improved reduced-order model of a generator.

The main objective of this paper is the derivation of a linearizing state feedback for the improved reduced-order model of a generator and the presentation of effectiveness of the resulting closed-loop control system during fault and post-fault processes.

The organization of the paper is as follows. Section 2 gives an introduction to feedback linearization of a single-input nonlinear system. A linearizing feedback control law for the improved reduced-order model of a generator is derived in Section 3. Section 4 presents a number of digital simulation results of the obtained closed-loop control system with the seven-dimensional model of a generator, linearizing feedback, nonlinear observer and field voltage limiter. In Section 5 some conclusions and comments are formulated. Synchronous generator models and nominal parameters of the generator used for digital simulations are given in Appendices A, B and C.

## 2. *Feedback linearization of single-input nonlinear systems*

In this section we discuss briefly the feedback linearization of single-input nonlinear systems. Specifically we consider a finite-dimensional time-invariant control system described by a state equation of the form:

$$\dot{\mathbf{x}} = \mathbf{a}(\mathbf{x}) + \mathbf{b}(\mathbf{x})u \tag{1}$$

where $\mathbf{x} \in \mathbf{X} \subset \mathbf{R}^n$ is a state vector, $\mathbf{X}$ is an open set containing the origin, $u \in \mathrm{R}$ is a control variable, $\mathbf{a}, \mathbf{b}$ are $\mathrm{C}^\infty$ vector fields on $\mathbf{X}, \mathbf{a(0)} = \mathbf{0}$.

Throughout this section some differential geometric concepts such as Lie bracket, Lie derivative and involutiveness of $\mathrm{C}^\infty$ vector fields are used. The reader is referred to Hunt *et al.* (1983) and Isidori (1985) for definitions and notation.

We consider the following feedback linearization problem (Hunt *et al.*, 1983). For the system (1) find a one-to-one $\mathrm{C}^\infty$ transformation $\mathbf{T} = (T_1, T_2, \ldots, T_n, T_{n+1}, T_{n+m}): \mathscr{X} \times \mathscr{U} \to \tilde{\mathscr{X}} \times \mathscr{V}$ is searched, such that

(1) $\mathscr{X} \subset X$, $\tilde{\mathscr{X}} \subset R^n$, $\mathscr{U} \subset R^m$, $\mathscr{V} \subset R^m$ are open sets containing the origins,

(2) $\mathbf{T(0,0)} = \mathbf{(0,0)}$,

(3) $(T_1, T_2, \ldots, T_n): \mathscr{X} \to \tilde{\mathscr{X}}$ is one-to-one and the Jacobian matrix $(D_j T_i(\mathbf{x}))_{i,j=1,\ldots,n}$ is nonsingular for all $\mathbf{x} \subset \mathscr{X}$,

(4) $(T_{n+1}, \ldots, T_{n+m}): \mathscr{X} \times \mathscr{U} \to \mathscr{V}$ has a nonsingular Jacobian matrix with respect to '$u$' for a fixed $\mathbf{x} \subset \mathscr{X}$,

(5)

$$\tilde{x}_i = T_i(\mathbf{x}), \quad i = 1, \ldots, n \tag{2}$$

$$v_i = T_{n+1}(\mathbf{x}, u), \quad i = 1, \ldots, m \tag{3}$$

are state and control variables of the linear system.

$$\dot{\tilde{\mathbf{x}}} = \tilde{\mathbf{A}}\tilde{\mathbf{x}} + \tilde{\mathbf{b}}v \tag{4}$$

in Brunovsky canonical form with Kronecker indices $k_1 \geq k_2 \geq \cdots \geq k_m$.

According to (4) we can write

$$\mathbf{u}(\mathbf{x}, v) = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})v, \tag{5}$$

where $\mathbf{f}, \mathbf{g}: \mathscr{X} \to R^m$, $G: \mathrm{R}^{m \times m}$ are $\mathrm{C}^\infty$ functions and $\mathbf{g}(\mathbf{x})^{-1}$ is invertible $\mathscr{X}$.

The following theorem gives necessary and sufficient conditions for the existence of solution to the feedback linearization problem (we omit arguments for the sake of brevity).

*Theorem* (Su, 1982; Hunt *et al.*, 1983; Isidori, 1985). The feedback linearization problem is solvable if and only if in some open set $\mathscr{X} \subset \mathbf{X} \subset \mathrm{R}^n$ containing the origin the vector fields $\mathbf{a}, \mathbf{b}$ satisfy the following conditions.

(1) The controllability matrix

$$\mathbf{M}_c = [\mathbf{b} \mid [\mathbf{a}, \mathbf{b}] \mid \cdots \mid ad_{\mathbf{a}}^{n-1}\mathbf{b}] \tag{6}$$

has rank '$n$' on $\mathscr{X}$.

(2) The set of vector fields $[\mathbf{b} \mid [\mathbf{a}, \mathbf{b}] \mid \cdots \mid ad_{\mathbf{a}}^{n-2}\mathbf{b}]$ is involutive on X.

In the constructive proof of this theorem it was shown (Su, 1982) that the functions $T_i$, $i = 1, \ldots, n, n+1$ must satisfy the following system of partial differential equations:

$$\begin{aligned} \langle \mathbf{d}T, \mathbf{b} \rangle &= 0, \quad i = 1, \ldots, n-1, \\ \langle \mathbf{d}T, \mathbf{a} \rangle &= L_{\mathbf{a}} T_i = T_{i+1}, \quad i = 1, \ldots, n-1, \end{aligned} \tag{7}$$

$$\langle \mathbf{d}T, \mathbf{a} + \mathbf{b}u \rangle = L_{\mathbf{a}} + \mathbf{b}uT_n = T_{n+1}. \tag{8}$$

Su (1982) also showed that $T_1$ can be found from the over-determined system of partial differential equations of the form:

$$\langle \mathbf{d}T_1, ad_{\mathbf{a}}^k \mathbf{b} \rangle = 0 \quad k = 0, 1, \ldots, n-2, \tag{9}$$

$$\langle \mathbf{d}T_1, ad_{\mathbf{a}}^{n-1}\mathbf{b} \rangle \neq 0. \tag{10}$$

It is easy to notice that if we have $T_1$ then the remaining components of the mapping $\mathbf{T}$ can be calculated from (7) and (8).

## 3. *Feedback linearization of a synchronous generator*

In order to obtain a linearizing state feedback for a synchronous generator we consider the improved reduced-order model of a generator which is described by the state equations given in Appendix B.

In applying the feedback linearization method, at first we must check the necessary and sufficient conditions for solvability of the feedback linearization problem. After simple calculations we find:

(1)

$$\mathrm{rank}\,([\mathbf{b} \mid [\mathbf{a}, \mathbf{b}](\mathbf{x}) \mid \cdots \mid ad_{\mathbf{a}}^{n-1}\mathbf{b}](\mathbf{x})) = 3, \quad \forall \mathbf{x} \in \mathscr{X} \tag{11}$$

where

$$\mathscr{X} = \{\mathbf{x} \in R^3 \mid \sin(\xi_{e1} + x_1) \neq 0\}; \tag{12}$$

(2) the set of vector fields $\{\mathbf{b}, [\mathbf{a}, \mathbf{b}]\}$ is involutive on $\mathscr{X}$.

Using the theory of partial differential equations, John (1975) and Hunt *et al.* (1983) developed a systematic method for the solution of the system (9), (10). This method is applicable to nonlinear systems described by equation (1), but in many cases obtaining the linearizing feedback in the form (5) may be hard or even impossible. However, for a class of systems, called pure-feedback systems (Su and Hunt, 1986), there is no need to apply the method of Hunt *et al.*

A single-input nonlinear system (1) is called a pure-feedback system if vector fields $\mathbf{a}, \mathbf{b}$ are of the form (Su and Hunt, 1986):

$$\mathbf{a}(\mathbf{x}) = \begin{bmatrix} a_1(x_1, x_2) \\ a_2(x_1, x_2, x_3) \\ \vdots \\ a_{n-1}(x_1, x_2, x_3, \ldots, x_n) \\ a_n(x_1, x_2, x_3, \ldots, x_n) \end{bmatrix} \quad \mathbf{b}(\mathbf{x}) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_n(\mathbf{x}) \end{bmatrix}. \quad (13)$$

Korobov (1973) was the first who showed that for these systems the first component function of $\mathbf{T}$ is given as $T_1(\mathbf{x}) = x_1$, and that the remaining components of $\mathbf{T}$ can be obtained from (7) and (8).

It is quite straightforward to observe that the improved reduced-order model of a generator is an example of the pure-feedback system (see Appendix B) and that in this case the diffeomorphism $\mathbf{T}$ is given by:

$$\hat{x}_1 = T_1(\mathbf{x}) = x_1 \quad (14)$$

$$\hat{x}_2 = T_2(\mathbf{x}) = \omega_b a_1(\mathbf{x}) = \omega_b x_2 \quad (15)$$

$$\hat{x}_3 = T_2(\mathbf{x}) = \omega_b a_2(\mathbf{x}), \quad (16)$$

$$\begin{aligned} v &= T_4(\mathbf{x}, u) = \langle \mathbf{d}T_3(\mathbf{x}), \mathbf{a}(\mathbf{x}) + \mathbf{b}u \rangle \\ v &= \langle \mathbf{d}T_3(\mathbf{x}), \mathbf{a}(\mathbf{x}) \rangle + p_6 D_3 T_3(\mathbf{x}) u \end{aligned} \quad (17)$$

where

$$D_i T_3(\mathbf{x}) = \omega_b D_i a_2(\mathbf{x}), \quad i = 1, 2, 3. \quad (18)$$

From equation (17) we obtain the linearizing state feedback of the form:

$$\begin{aligned} u(\mathbf{x}, v) &= (p_6 D_3 T_3(\mathbf{x}))^{-1}(-\langle \mathbf{d}T(\mathbf{x}), \mathbf{a}(\mathbf{x}) \rangle + v), \\ D_3 T_3(\mathbf{x}) &\neq 0, \quad \forall \mathbf{x} \in \mathscr{X}. \end{aligned} \quad (19)$$

Transformation (14)–(17) is the simplest possible one, and was already used by Mielczarski and Zajaczkowski (1988, 1989) for the conventional three-dimensional model of a generator.

The new state variables $\hat{x}_1, \hat{x}_2, \hat{x}_3$ can be interpreted as generator angle deviation from the desired equilibrium value $\delta_e$, rotor speed deviation from the synchronous speed $\omega_s$ and rotor acceleration, respectively. Furthermore, from (12) it follows that the set $\mathscr{X}$ in which the linearizing feedback (19) exists can be described as follows:

$$\mathscr{X} = \bigcup_{k \in \mathbf{Z}} \mathscr{X}_k \quad (20)$$

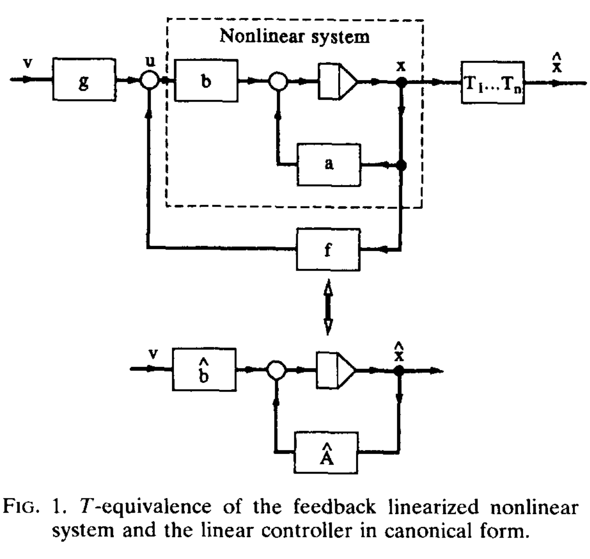


FIG. 1. $T$-equivalence of the feedback linearized nonlinear system and the linear controller in canonical form.

where

$$\mathscr{X} = \{\xi \in R^3 \mid k\pi < \delta < (k+1)\pi\} \quad (21)$$

and $\mathbf{Z}$ denotes the set of integers.

In accordance with (4) the reduced-order model of a generator with the nonlinear state feedback (19) is $T$-equivalent to the linear system described by a state equation of the form:

$$\dot{\hat{\mathbf{x}}} = \hat{\mathbf{A}}\hat{\mathbf{x}} + \hat{\mathbf{b}}v = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \hat{\mathbf{x}} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v. \quad (22)$$

In order to guarantee the desired dynamic characteristics of the closed-loop system, some external controller $\hat{\mathbf{x}} \to v(\hat{\mathbf{x}})$ should be used. The simple choice is the linear feedback

$$v(\hat{\mathbf{x}}) = \mathbf{a}^T \hat{\mathbf{x}} + w \quad (23)$$

where $w \in \mathrm{R}$ is a new external control variable and $\boldsymbol{\alpha}^T = (\alpha_0, \alpha_1, \ldots, \alpha_{n-1})$ is a vector of constant feedback gains which determine the characteristic polynomial of the closed-loop system,

$$\dot{\hat{\mathbf{x}}} = (\hat{\mathbf{A}} + \hat{\mathbf{b}}\mathbf{a}^T)\hat{\mathbf{x}} + \hat{\mathbf{b}}w. \quad (24)$$

The block diagram of the closed-loop control system is shown in Fig. 1. There are two feedback loops: the inner nonlinear loop responsible for the feedback linearization and the outer linear loop that assigns poles of the system (24) to the desired positions in the complex plane. It is easy to notice that there is considerable freedom in the choice of a form of outer feedback loop. This choice depends on requirements imposed by the designer on dynamic characteristics of a closed-loop system and any controller that fulfils these requirements can be used.

Stability of the linear system (24) and existence of diffeomorphism $(T_1, T_2, T_3)$ imply stability of the generator model with linearizing feedback (19) and external controller (23), providing that an initial state $\boldsymbol{\xi}(\mathbf{0}) = \boldsymbol{\xi}_e + \mathbf{x}(\mathbf{0})$ of the model is in one of the disjoint sets $\mathscr{X}_k$, $k \in \mathbf{Z}$. Thus in simulations it was assumed that

$$\boldsymbol{\xi}(\mathbf{0}) \in \mathscr{X}_0 = \{\boldsymbol{\xi} \in R^3 \mid 0 < \delta < \pi\}. \quad (25)$$

4. *Simulation*

In order to evaluate the effectiveness of the proposed method of controller design, a number of conditions appearing in practice have been simulated. The main goal was to simulate real conditions as precisely as possible. Two generator models were considered: the first one was the improved reduced-order model (three-dimensional model); the second one was the seven-dimensional model (Park's model). The nonlinear controller derived for the improved reduced-order model was applied to the two models, taking into account a number of additional restrictions such as the field voltage limitation and the necessity of generator state estimation by the nonlinear observer.

The full-order nonlinear observer of the three-dimensional model is in the following form

$$\dot{\mathbf{z}} = \mathbf{a}(\mathbf{z}) + \mathbf{b}u + \mathbf{k}(y - y_r) \quad (26)$$

where

$\mathbf{a}, \mathbf{b}$ are defined in Appendix B,
$\mathbf{z}$ is the observer state vector,
$y = i_f$ is the field current,
$y_r = z_3$,
$\mathbf{k}$ is a gain vector.

The gain vector $\mathbf{k}$ was designed using the method developed by Mielczarski (1987a, 1987b).

During simulations the trajectories of the two generator models controlled by the same nonlinear controller designed in Section 3 have been compared. The computations were carried out for fault and post-fault conditions for the system with an observer and a field voltage limiter. The results of simulations are shown in Fig. 2. During a post-fault transient, process trajectories obtained for a seven-dimensional model are similar to trajectories for the reduced-order three-dimensional model. Moreover, generator angle deviations are smaller in the case when a full-dimensional model is employed (NC-M7). The better performance of a full-dimensional model can be explained by an influence of

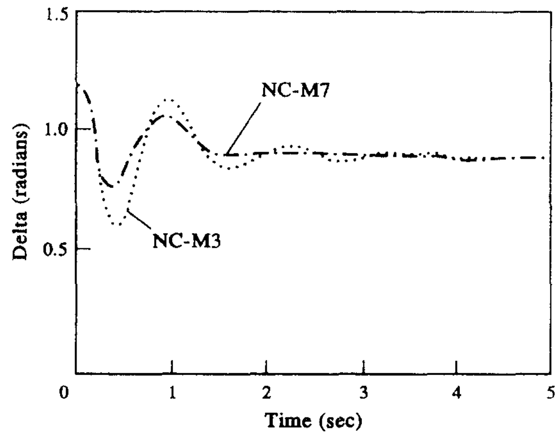


FIG. 2. Post-fault generator angle trajectories for three-dimensional NC–M3 and seven-dimensional NC–M7 models. Controller eigenvalues are equal to −10.0.

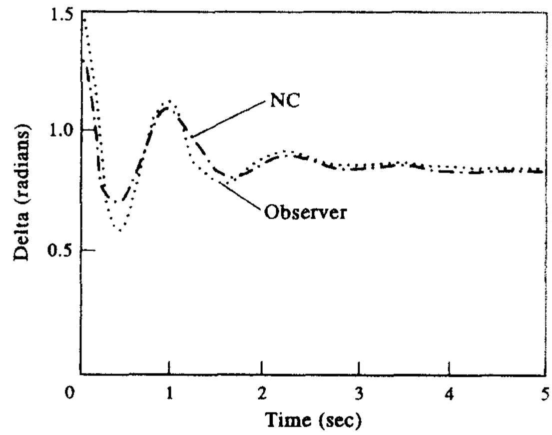


FIG. 4. Post-fault generator angle trajectories for the nonlinear controller and for the observer when an observer initial error is positive. Controller and observer eigenvalues are equal to −10.0.

generator damping windings. These windings were not considered during nonlinear controller design but their existence causes faster stabilization of transient processes.

Further simulations have been carried out for a full seven-dimensional model of a generator. An influence of observer error on generator load angle transients is shown in Figs 3 and 4. The trajectory denoted NC represents a load angle of the synchronous generator in a control system where the synchronous generator was represented by the seven-dimensional model and it was controlled by the nonlinear controller derived in Section 3. The nonlinear controller was supplied by signals obtained from a nonlinear observer described by equation (26). The trajectory denoted OBSERVER represents an estimate of generator angle obtained from the nonlinear observer (26).

When an observer error is negative, i.e. $[z(0) - x(0)] < 0$, Fig. 3, the controller receives incorrect information on generator states. An estimated angle is smaller than a real angle so a controller reaction is not very strong. When an observer error is positive, i.e. $[z(0) - x(0)] > 0$, Fig. 4, then the controller is informed on larger than real deviations and its action is stronger. Position errors lead to faster damping. Even observer errors that can appear in practical applications are not larger than 0.25 rad for the first and the second half-wave. An influence of observer errors on damping time is negligible.

In order to evaluate the proposed method with existing control systems, two power systems have been simulated (Fig. 5). Results of these simulations are shown in Fig. 6 for

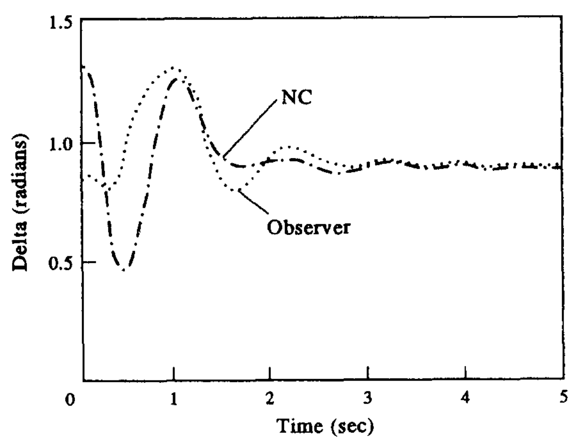


FIG. 3. Post-fault generator angle trajectories for the nonlinear controller and for the observer when an observer initial error is negative. Controller and observer eigenvalues are equal to −10.0.

post-fault conditions and on Fig. 7 for fault and post-fault conditions. The fault was simulated as the power system voltage $U_S$ dropped from the nominal value equal to 1.0 per unit system (pu) to 0.5 pu. Time of the fault is shown as a bold line on the time axis. Both figures show better performance of the proposed controller than a classical voltage controller with PSS. Parameters of a voltage controller and PSS used during simulation are taken from real control systems working with a 360 MW generator in the Belchatow power station.

### 5. *Conclusions*

Using the theory of feedback linearization of single-input nonlinear systems we have shown that for the improved reduced-order three-dimensional model of a synchronous generator there exists a linearizing state feedback and the most simple form of this feedback has been derived. The obtained nonlinear closed-loop system has very good dynamic properties and can stabilize large state vector deviations resulting from severe faults. Additional impediments such as input limitation and observer errors cannot significantly change properties of the system. Simulations show a strong robustness of the nonlinear controller. A very good performance of the controller has not been spoilt by:

- the use of the controller designed for a simplified model for controlling the full-dimensional model that has different structure and parameters;
- the influence of observer errors that result from initial errors and simplification of a generator model used for observer design (26);

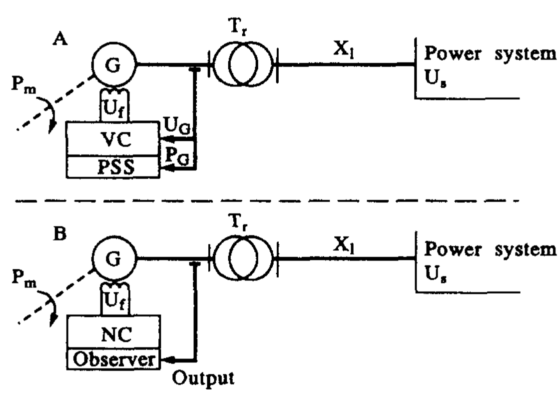


FIG. 5. Two simulated systems. The upper system represents a classical voltage controller with power system stabilizer (PSS). The lower system shows the proposed controller with the observer.

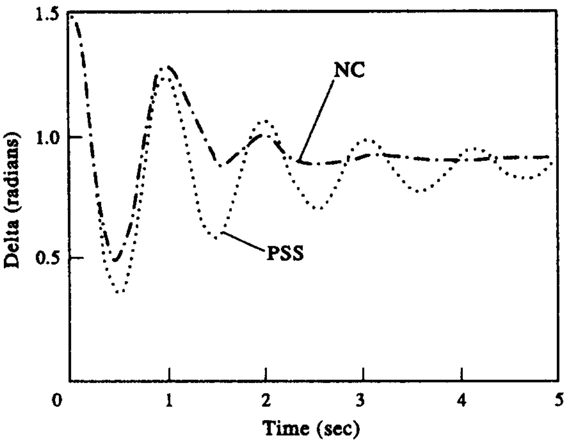


FIG. 6. Post-fault generator angle trajectories for the proposed nonlinear controller and for a voltage controller with PSS.

- limitation of maximum values of input signals. These limitations have not been considered in controller design but they have been implemented in simulations.

The presented approach to the design of field voltage controllers possesses advantages that are both of theoretical and practical significance:

- an application of the necessary and sufficient conditions for solvability of the feedback linearization problem allows us to define precisely a set in which the linearizing feedback for a synchronous generator exists—see (21). It is easy to notice that conditions implied by (21) are in accordance with the classical condition on synchronous generator stability;
- the obtained linearizing feedback is the simplest possible one which is of prime importance to the problem of technical implementation of the field voltage controller;
- although the nonlinear controller design is based on the improved reduced-order model of a generator, simulation results show that the proposed controller works properly with the 'exact' Park's model of a generator.

Obtained results show that a practical application of nonlinear controllers is possible and it should give good transient properties of the power system.

Presented results can be found as the next but not the last step in the construction of the most proper controller for a synchronous generator stabilization. Two problems, i.e. coordinated stabilization of a generator (nonlinear controller for the excitation system, the steam turbine) and comparative studies of conventional voltage controllers with a power system stabilizer, are worked out and the results were published recently (Mielczarski and Zajaczkowski, 1990).

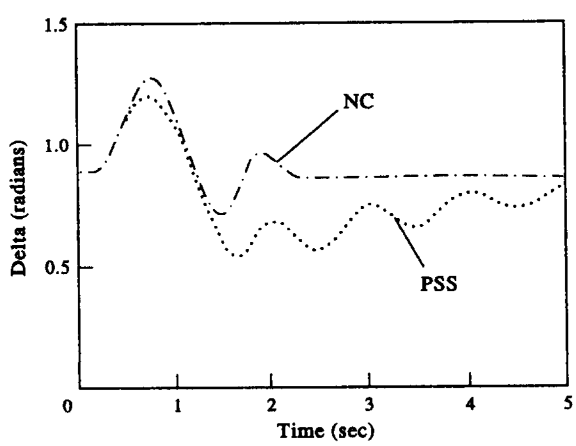


FIG. 7. Fault and post-fault generator angle trajectories for the nonlinear controller and a voltage controller with PSS.

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*Appendix A—Park's model of a synchronous generator*

$$\dot{\delta} = \omega_b(\omega - \omega_s)$$

$$\dot{\omega} = \Psi_d(q_1\Psi_q + q_2\Psi_Q) + \Psi_q(q_3\Psi_f + q_4\Psi_D) + p_3P_{\mathrm{mech}}$$

$$\dot{\Psi} = s_{33}\Psi_d + s_{34}\Psi_f + s_{35}\Psi_D + s_{36}\Psi_Q + s_{37}\Psi_Q - \omega_b\omega\Psi_q + \omega_b U_s \sin\delta$$

$$\dot{\Psi}_f = s_{43}\Psi_d + s_{44}\Psi_f + s_{45}\Psi_D + \omega_b u_f$$

$$\dot{\Psi}_D = s_{53}\Psi_d + s_{54}\Psi_f + s_{55}\Psi_D$$

$$\dot{\Psi}_Q = s_{76}\Psi_q + s_{77}\Psi_Q.$$

Elements $p_3, q_1, q_2, q_3, q_4$ and $s_{ij}$ are functions of generator parameters. For details see Anderson and Fouad (1977).

*Appendix B—Improved reduced-order model of a synchronous generator*

We consider the following state equation of a generator

$$\dot{\boldsymbol{\xi}} = \boldsymbol{\Phi}(\boldsymbol{\xi}) + \mathbf{b}u_f$$

where

$\boldsymbol{\xi}^T = [\xi_1, \xi_2, \xi_3] = [\delta, \omega, i_f]$ is the state vector,

$u_f$ is a control variable,

$\mathbf{b}^T = [0, 0, p_6]$

$\boldsymbol{\phi}: R^3 \to R^3$ is given by its component functions:

$$\Phi_1(\boldsymbol{\xi}) = \omega_b(\xi_2 - \omega_s)$$

$$\Phi_2(\boldsymbol{\xi}) = M_{ad} + p_1\xi_3 \sin \xi_1 + p_2 \sin 2\xi_1 + p_3 P_{mech}$$

$$\Phi_3(\boldsymbol{\xi}) = p_4(\xi_2 - \omega_s) \sin \xi_1 + p_5\xi_3$$

where $M_{ad} = d_2(\xi_2 - \omega_s)\cos^2 \xi_1$ is an additional element introduced by Sauer *et al.* (1988) in order to improve the reduced-order model of a generator.

In Mielczarski and Zajaczkowski (1989) the conventional three-dimensional model of a generator was used in which

$$M_{ad} = d_2^*(\xi_2 - \omega_s).$$

The value of $d_2^*$ was chosen experimentally.

A point $(\boldsymbol{\xi}_e, u_{fe}) \in R^3 \times R$ is called an operating point of a synchronous generator if

$$\boldsymbol{\Phi}(\boldsymbol{\xi}_e) + \mathbf{b}u_{fe} = 0.$$

For further considerations it is convenient to have the origin $(\mathbf{0}, 0)$ of an $\mathbf{R}^3 \times \mathbf{R}$ space as an operating point of the generator. A linear change of state and control variables

$$\mathbf{x} = \boldsymbol{\xi} - \boldsymbol{\xi}_e,$$

$$u = u_f - u_{fe}$$

results in the following state equation of a synchronous generator

$$\dot{\mathbf{x}} = \mathbf{a}(\mathbf{x}) + \mathbf{b}u, \quad \text{(A1)}$$

where

$$\mathbf{a}(\mathbf{x}) = \boldsymbol{\Phi}(\boldsymbol{\xi}_e + \mathbf{x}) + \mathbf{b}u_{fe} \quad \text{(A2)}$$

$$a_1(\mathbf{x}) = \omega_b x_2 \quad \text{(A3)}$$

$$a_2(\mathbf{x}) = M_{ad}(\mathbf{x}) + p_1(\xi_{e3} + x_3)\sin(\xi_{e1} + x_1) + p_2 \sin 2(\xi_{e1} + x_1) + d_1\xi_{e2} + p_3 P_{mech} \quad \text{(A4)}$$

$$a_3(\mathbf{x}) = p_4 x_2 \sin(\xi_{e1} + x_1) + p_5 x_3 \quad \text{(A5)}$$

$$M_{ad}(\mathbf{x}) = d_2 x_2 \cos^2(\xi_{e1} + x_l). \quad \text{(A6)}$$

It is easy to see that in $(\mathbf{x}, u)$-space $\mathbf{a}(\mathbf{0}) = \mathbf{0}$ and thus $(\mathbf{0}, 0)$ is an operating point of the synchronous generator.

*Appendix C—Parameters of a synchronous generator model*

$$p_1 = (-U_S X_{AD})/(T_M(x_d + x_L))$$

$$p_2 = 0.5U_S^2(x_d - x_q')/(T_M(x_q + x_L)(x_L + x_d))$$

$$p_3 = 1/T_M$$

$$p_4 = -(x_L + x_d)/((x_L + x_d')\tau_{do}')$$

$$p_5 = (x_d - x_d')U_S\omega_b/((x_d' + x_L)X_{AD})$$

$$p_6 = (x_L + x_d)/(\tau_{do}' r_f(x_d' + x_L))$$

$$d_2 = -\tau_{qo}'(1 - x_q'/x_q)U_S^2/x_q,$$

where

$\delta$—generator angle,

$\omega_s$—power system frequency,

$u_f$—field winding voltage

$r_f$—resistance of field winding

$\omega$—angular rotor speed

$i_f$—field winding voltage

$X_L$—tie line reactance

$U_s$—power-system voltage.

For details of model parameters see Anderson and Fouad (1977).

*Generator nominal parameters*

| | | | |
|---|---|---|---|
| $P_n = 360$ MW | $S_n = 432$ MVA | $X_L = 0.3$ pu | $x_d = 2.459$ pu |
| $x_q = 2.354$ pu | $x_d' = 0.315$ pu | $\tau_{do} = 7.95$ s | $T_M = 8$ s |
| $X_{AD} = 2.28$ pu | $\tau_{qo}' = 0.39$ s | $x_q' = 0.476$ pu | $x_r = 0.191$ pu |
| $r_f = 0.002$ pu | $\tau_{do}'' = 0.01$ s | $\tau_{qo}'' = 0.016$ s. | |