Tracking Control of an Inverted Pendulum

Using Computed Feedback Linearization Technique

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***Abstract*— The paper presents an output tracking technique for a balanced rod inverted pendulum based on computed feedback linearization. For any given trajectory of pendulum, which is the output, the trajectory of rod position, which is the internal state, is determined such that the system is input state linearizable without state transformation. Both output and state are used to stabilize and also linearize the system. Once the system is linear, the trajectory control effort can be superimposed such that the system tracks the trajectory. The trajectory control effort is determined from the inverse of the feedback linearized system and thus bounded. In this way, the tracking error is asymptotically decreasing. Both simulation and experiment based on the ECP 505 balanced rod inverted pendulum are used to demonstrate and verify the technique.**

***Keywords*—Inverted Pendulum, Feedback linearization, Trajectory Following Control.**

I. INTRODUCTION

One of the major challenges in control which the controlling of a nonlinear mechanical plant such that its output is tracking a specified trajectory. This problem has been widely investigated by several researchers. There are two major approaches to construct the trajectory controller for nonlinear system. The first one is based on system inversion [1], [2]. The controller consists of state feedback, which stabilize the system, and the feedforward input solved from system inversion. Determination of a feedforward input requires a precise mathematic model of the plant and a pre-specified trajectory. The main problem is that the input tends to be unbounded for non-minimum phase system when using classical inversion technique.

The other approach is based on output regulation theory [3],[4],[5]. This approach requires that the plant is locally exponentially stable nonlinear closed loop system while the reference is generated by stable exosystem. A set of nonlinear PDE's is solved in order to determine the controller and the internal dynamics must be proved that its stable.

In the paper, the trajectory-following controller is to be designed for a balanced rod inverted pendulum. The inverted pendulum has been widely used to investigate and develop new control strategies that can effectively deal with nonlinearities. The main challenge is that the plant is a nonlinear, under-actuated mechanical system with unstable zero dynamics and must be controlled such that the position is at its unstable equilibrium. There are two control modes; stabilizing and swing up modes. The authors have proposed a technique, called "computed feedback linearization" to stabilize this system at any desired position in [6]. The technique is based on pole placement over the first order linearized model along a trajectory. The strategy is to keep the closed loop system's roots fixed by nonlinear state feedback, thus the system becomes approximately linear. If the roots are on the left half plane, i.e., their real part is less than zero, the system is asymptotically stable.

Once the system is approximately linear and its model is known, a trajectory controller can be superimposed. The trajectory control effort is computed from the inverse of the feedback linearized system. For a given trajectory of output, the corresponding trajectory of internal state is determined such that the system is input state linearizable along the trajectory. Both output and internal state are used to regulate the output to follow a desired trajectory, thus the internal state is stable. Both simulation and experiment based on the ECP 505 balanced rod inverted pendulum are used to demonstrate and verify the technique.

This paper is organized into seven sections. In the next section, the balanced rod inverted pendulum is explained and its model is shown. In section 3, the stabilization controller based on LQ technique is discussed. In section 4, the proposed computed feedback linearization and the tracking controller is explained. Simulation and experimental results are demonstrated in section 5 and 6 respectively. Section 7 gives conclusion.

II. MODEL OF THE ECP 505 INVERTED PENDULUM

The ECP 505 inverted pendulum consists of a pendulum rod which supports the sliding balance rod. The DC servo motor, below the pendulum rod, is used to drive the sliding balance rod through a drive shaft, a pulley and a belt. This sliding rod is to be steered horizontally in order to control the vertical pendulum rod. The center of gravity, and thus the system dynamics, can be altered by adjusting the brass counter weight position. The position of the sliding rod and the pendulum rod are sensed by two encoders, one at the back of the motor and the other one at the pivoting base of the pendulum. The kinematics model of the plant and the actual plant are shown in Fig 1 and Fig 4 respectively.

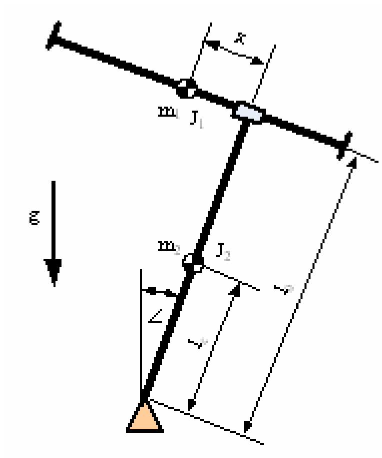


Fig 1. Kinematics diagram of ECP 505 inverted pendulum.

The mathematical model of the ECP 505 system can be derived using Euler-Lagrange equation or the Newtonian approach. However, the first approach is used in this paper as follows;

The Lagrange equation is:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}}\right] - \frac{\partial L}{\partial q} = Q_q ,$$

where

L=T-V

T: kinetic energy

V: potential energy

Q_q: generalized forces

q: generalized coordinates

The q is selected as $[\theta, x]^T$. Thus, the kinetic energy, T, is

$$T = \frac{1}{2} J_0(x)\dot{\theta}^2 + \frac{1}{2} m_1 \dot{x}^2 + m_1 l_0 \dot{x}\dot{\theta} , \quad (1)$$

where,

$$J_0(x) = J_1 + J_2 + m_1(l_0^{\,2} + x^2) + m_2 l_c^{\,2}$$

The potential energy, V, is

$$V = m_1 g(l_0 \cos(\theta) - x\sin(\theta)) + m_2 g l_c \cos(\theta) .$$

The Euler-Lagrange equations result in

$$m_1\ddot{x} + m_1 l_0 \ddot{\theta} - m_1 x\dot{\theta}^2 - m_1 g\sin(\theta) = F(t) \quad (2)$$

$$m_1 l_0 \ddot{x} + J_0 \ddot{\theta} + 2m_1 x\dot{x}\dot{\theta} - (m_1 l_0 + m_2 l_c) g\sin(\theta) - m_1 gx\cos(\theta) = 0 \quad (3)$$

This system is highly nonlinear, having two degrees of freedom with only one actuator. There is also a nonlinear coupling between the actuated and the un-actuated degrees of freedom.

The system can be written as

$$m_{11}\ddot{x} + m_{12}\ddot{\theta} + F_1(x,\dot{x},\theta,\dot{\theta}) = u(t) , \quad (4)$$

$$m_{21}\ddot{x} + m_{22}\ddot{\theta} + F_2(x,\dot{x},\theta,\dot{\theta}) = 0 , \quad (5)$$

where

$$m_{11} = m_1, m_{12} = m_1 l_0, m_{21} = m_1 l_0, m_{22} = J_{0e}$$

$$F_1 = m_1 x\dot{\theta}^2 - m_1 g\sin(\theta)$$

$$F_2 = 2m_1 x\dot{x}\dot{\theta} - (m_1 l_0 + m_2 l_c) g\sin(\theta) - m_1 gx\cos(\theta)$$

$$u(t) = F(t)$$

TABLE I

PLANT PARAMETER

Symbol	Value	Description
m_1	0.213 kg	Mass of sliding rod
m_2	1.785 kg	Mass of complete assembly minus m_1
J_0	0.036 kg.m^2	The equivalent J of the system
l_0	0.330 m	Length of pendulum rod
l_c	0.0281 m	The position of center of m_2
g	9.81 m/s^2	Gravity

The system parameters used in the simulation are given in Table I. This is the nominal parameters for the ECP 505 plant and the controller is design based on these values.

III. LQ Controller based upon linearized plant

A. Local Linearization about equilibrium

A linearized approximation of the system about the equilibrium point $[x_e \;\; \theta_e]$ = [0 0] which only the first two (zeroeth and first order) terms of Taylor's series expansion are used is found as

$$m_1\ddot{x} + m_1 l_0 \ddot{\theta} - m_1 g\theta = F(t) , \quad (6)$$

$$m_1 l_0 \ddot{x} + J_0 \ddot{\theta} - (m_1 l_0 + m_2 l_c) g\theta - m_1 gx = 0 , \quad (7)$$

where

$$J_{0e} = J_1 + J_2 + m_1 l_0^{\,2} + m_2 l_c^{\,2}$$

The result is the same as setting *sin(θ)* and *cos(θ)* equal to θ and 1 respectively and the $\dot{x}$ and $\dot{\theta}$ are setting equal to zero.

The linearized approximation can be written in state space form as

$$\dot{x} = Ax + BF(t)$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{m_2 l_c g}{J^*} & \frac{m_1 g}{J^*} & 0 & 0 \\ \frac{(J^* - m_2 l_0 l_c) g}{J^*} & \frac{-m_1 l_0 g}{J^*} & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ -\frac{l_0}{J^*} \\ \frac{J_{0e}}{m_1 J^*} \end{bmatrix}, \quad (8)$$

where $x = \begin{bmatrix}\theta & x & \dot{\theta} & \dot{x}\end{bmatrix}^T$ and $J^* = [J_{0e} - m_1 l_0^{\,2}]$

B. LQG Controller

Since the open loop system is both naturally unstable and non-minimum phase, the full state feedback is recommended to control the system. The LQR synthesis, via matrix Riccati equation, is used to determine the optimal gains which the following cost function is minimized

$$J = \int \left(x'Qx + Ru^2 \right) dt \ ,$$

subjected to the linear time invariant dynamics

$$\dot{x} = Ax + Bu \ . \tag{9}$$

The matrices Q and R are positive semi-definite and definite respectively.

Once the optimal gains is determined, the feedback control law is

$$u = -Kx \ . \tag{10}$$

In the simulation, the Q and R are set to 1 and 10 respectively. This results in the optimal gains K = [4.6314 1.4197 8.8969 2.6202]. The roots of the closed loop system are the eigenvalues of [A-BK] and is found to be

$r_{1,2}$ = -0.2998 ± 6.2053i, $r_{3,4}$ = -3.3318 ± 0.1275i

IV. COMPUTED FEEDBACK LINEARIZATION

A. Partial Input-Output Linearization

The controller can be designed based on input feedback linearization to tracking control the under-actuated nonlinear pendulum system. From (4), $\ddot{x}$ can be solved as

$$\ddot{x} = \frac{m_{22}}{m_{21}}\ddot{\theta} + \frac{F_2(x,\dot{x},\theta,\dot{\theta})}{m_{21}} \tag{11}$$

substitution into (3) yields,

$$\left[\frac{m_{11}m_{22}}{m_{21}} + m_{12}\right]\ddot{\theta} + \left[\frac{m_{11}}{m_{21}}F_2(x,\dot{x},\theta,\dot{\theta}) + F_1(x,\dot{x},\theta,\dot{\theta})\right] = u(t) \tag{12}$$

The partial input-output feedback linear zing controller is designed as

$$u = \left[\frac{m_{11}}{m_{21}}F_2(x,\dot{x},\theta,\dot{\theta}) + F_1(x,\dot{x},\theta,\dot{\theta})\right] + \left[\frac{m_{11}m_{22}}{m_{21}} + m_{12}\right]v \tag{13}$$

The feedback linearized system then becomes

$$\ddot{\theta} = v \ . \tag{14}$$

It's noted that the dynamics of the internal state, x, is expressed in (9)

The tracking control law can be designed as

$$v = \ddot{\theta}_d + 2\lambda(\dot{\theta}_d - \dot{\theta}) + \lambda^2(\theta_d - \theta) \ , \tag{15}$$

where λ is a positive number.

The tracking error is now becomes,

$$\ddot{e} + 2\lambda\dot{e} + \lambda^2 e = 0 \ . \tag{16}$$

The tracking error exponentially converges to zero since the roots are on the left half plane. It's noted that the stability of the system also depends on the stability of the internal state, x, which is difficult to proof this.

B. Computed Feedback Linearization

In contrast to output regulation technique where the controller is based on the error of output θ, we are to regulate both output, θ, and internal state, x, simultaneously.

Rewrite the (3)-(4) in the matrix form,

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} F_1(x,\dot{x},\theta,\dot{\theta}) \\ F_2(x,\dot{x},\theta,\dot{\theta}) \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix} . \tag{17}$$

Linearize the system around x_0 and θ_0 yields,

$$M\begin{Bmatrix} \ddot{\theta} \\ \ddot{x} \end{Bmatrix} + A(\theta_0,x_0)\begin{Bmatrix} \theta \\ x \end{Bmatrix} + \left(\begin{bmatrix} F_1(x_0,\theta_0) \\ F_2(x_0,\theta_0) \end{bmatrix} - A(\theta_0,x_0)\begin{Bmatrix} \theta_0 \\ x_0 \end{Bmatrix}\right) = \begin{bmatrix} u \\ 0 \end{bmatrix} \tag{18}$$

where $M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$, $A(\theta_0,x_0) = \begin{bmatrix} \frac{\partial F_1}{\partial \theta} & \frac{\partial F_1}{\partial x} \\ \frac{\partial F_2}{\partial \theta} & \frac{\partial F_2}{\partial x} \end{bmatrix}_{\substack{\theta=\theta_0 \\ x=x_0}}$

In contrast to input-state linearization where the input and state are transformed into new variables to gives the complete linearized system based on these new variables. The Computed Feedback Linearization completely linearizes the nonlinear system without using state transformation. Since the system is under-actuated, it cannot be input-state linearized at all positions. However, we are able to determine where we can completely linearized the system as follows,

For any given θ_0, the corresponding internal state, x_0, can be determined such that

$$F_2(x_0,\theta_0) - \begin{bmatrix} A_{21} & A_{22} \end{bmatrix}\begin{Bmatrix} \theta_0 \\ x_0 \end{Bmatrix} = 0 \ . \tag{19}$$

In other word, for any given trajectory of θ, we can determine the trajectory of x that where we can linearize the system. If the system are moving along this trajectory, the full state linearization can be applied. The control effort is chosen as

$$u = \left[F_1(x_0,\theta_0) - \begin{bmatrix} A_{11} & A_{12} \end{bmatrix}\begin{Bmatrix} \theta_0 \\ x_0 \end{Bmatrix} \right] + v = 0 \ . \tag{20}$$

Now, the system becomes,

$$M\begin{Bmatrix} \ddot{\theta} \\ \ddot{x} \end{Bmatrix} + A(\theta_0,x_0)\begin{Bmatrix} \theta \\ x \end{Bmatrix} = \begin{bmatrix} v \\ 0 \end{bmatrix} . \tag{21}$$

Rewrite,

$$\begin{Bmatrix} \ddot{\theta} \\ \ddot{x} \end{Bmatrix} + M^{-1}A(\theta_0,x_0)\begin{Bmatrix} \theta \\ x \end{Bmatrix} = M^{-1}\begin{bmatrix} v \\ 0 \end{bmatrix} . \tag{22}$$

Rewrite the system in full state form as

$$\frac{d}{dt}\begin{Bmatrix}\theta\\ x\\ \dot{\theta}\\ \dot{x}\end{Bmatrix}+\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \multicolumn{2}{c}{M^{-1}A(\theta_0,x_0)} & \begin{matrix}0 & 0\\ 0 & 0\end{matrix}\end{bmatrix}\begin{Bmatrix}\theta\\ x\\ \dot{\theta}\\ \dot{x}\end{Bmatrix}=\begin{bmatrix}0\\ 0\\ M^{-1}\begin{bmatrix}1\\ 0\end{bmatrix}\end{bmatrix}v\,, \tag{23}$$

or

$\dot{x}=A'(x)x+B'v$,

where $x=\begin{bmatrix}\theta & x & \dot{\theta} & \dot{x}\end{bmatrix}^T$

$$A'(x_0)=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \multicolumn{2}{c}{M^{-1}A(\theta_0,x_0)} & \begin{matrix}0 & 0\\ 0 & 0\end{matrix}\end{bmatrix},\; B'=\begin{bmatrix}0\\ 0\\ M^{-1}\begin{bmatrix}1\\ 0\end{bmatrix}\end{bmatrix}$$

A stabilization control law is chosen as,

$$v=-\begin{bmatrix}K_1 & K_2 & K_3 & K_4\end{bmatrix}\begin{bmatrix}\theta & x & \dot{\theta} & \dot{x}\end{bmatrix}^T. \tag{24}$$

Where the gains are adaptive as functions of system position and are chosen such that

$$[A'(x_0)-B'K(x_0)]=\bar{A}\,. \tag{25}$$

The matrix $\bar{A}$ is constant and thus the feedback system is approximately linear and its model is

$$\dot{x}=[A'(x_0)-B'K(x_0)]x=\bar{A}x\,. \tag{26}$$

In this paper, the constant matrix $\bar{A}$ is designed to be the same as *[A-BK]* where *A* is the linearized model of the system at the equilibrium and *K* is chosen by LQ technique as explained earlier. This means that the control performance when the system is at equilibrium will be the same as the LQ controller designed based on linearized plant at this position. The Computed Feedback Linearization is designed to maintain the control performance when the system is not necessary in equilibrium, but anywhere that satisfied (19).

Once the system is approximately linear, a trajectory controller can be designed. In the proposed technique, a trajectory controller is designed based on feedback linearized system and then superimposed into stabilization controller. The trajectory control effort is determined as

$$\dot{x}_d=\bar{A}x_d+u_T(x_d,\dot{x}_d)\,. \tag{27}$$

For a given trajectory of θ_d, the trajectory of x_d can be determined such that (19) is satisfied. Then, the trajectory control effort is computed by inversed dynamics. With the applied trajectory control effort, the system becomes,

$$\dot{x}=\bar{A}x+u_T(t)\,. \tag{28}$$

And thus, results in

$$\dot{e}=\bar{A}e\,, \tag{29}$$

where $e=\begin{bmatrix}[\theta-\theta_d] & [x-x_d] & [\dot{\theta}-\dot{\theta}_d] & [\dot{x}-\dot{x}_d]\end{bmatrix}^T$

The solution of *x* in (28) consists of two terms. The first one is the complementary solution or the tracking error, *e*, while the other one is the particular solution or the nominal trajectory of *x*. The tracking error is stable and converging to zero since all the poles of $\bar{A}$ are stable and the nominal trajectory is also stable since the trajectory control effort is bounded. As we control both output, θ, and internal state, *x*, both are stable. Thus, the system is stable. The main advantage of this technique is that the trajectory control effort is pre-determined from the trajectory and then is used as a reference signal feedforwarding into the feedback linearized system driven by stabilization controller. Both stabilization and trajectory controller are seamlessly integrated. This technique can be applied to the existing system driven by stabilization controller such as the ECP 505 inverted pendulum system.

V. SIMULATION RESULTS

In this section, the matlab® simulation based on 'ode45' is used to demonstrate the technique. In the first simulation, the pendulum is to be stabilized at 10 degrees at time 10 second. The step response is asymptotically stable as shown in Fig 2. The constant feedforward signal is used to stabilize the system at any desired pendulum position.

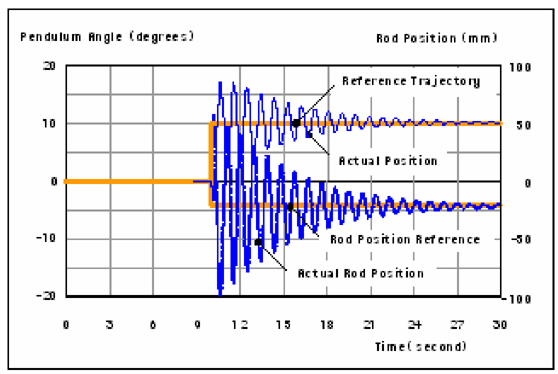


Fig 2. Stabilizing the pendulum at 10 Degrees.

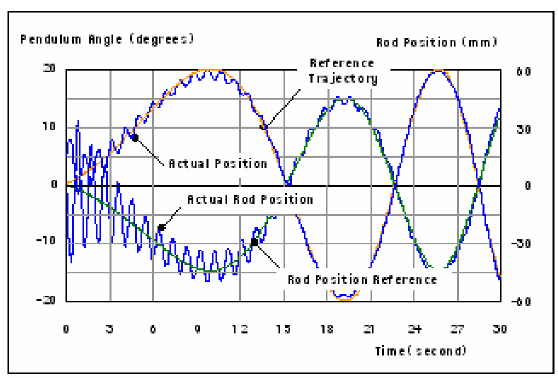


Fig 3. Tracking response to the sine sweep 0.01 – 0.1 Hz in 30 seconds.

In the second experiment, the pendulum is to track the sine sweep trajectory 0.01-0.1 Hz in 30 second. The time varying feedforward signal or the trajectory control effort is used to regulate the system. The tracking error is asymptotically converging to zero as the pendulum is tracking the trajectory as shown in Fig. 3. The simulation is used to verify the control algorithm, which will be implemented to the ECP 505 inverted pendulum plant. The experiment is demonstrated in the next section.

VI. Experimental Results

The ECP 505 inverted pendulum, shown in Fig 4, is used to validate the proposed technique. The control algorithm is implemented through Delta-tau Pmac lite DSP card that comes with the plant. The sampling rate is set to 0.00442 second and digital filter is also implemented.

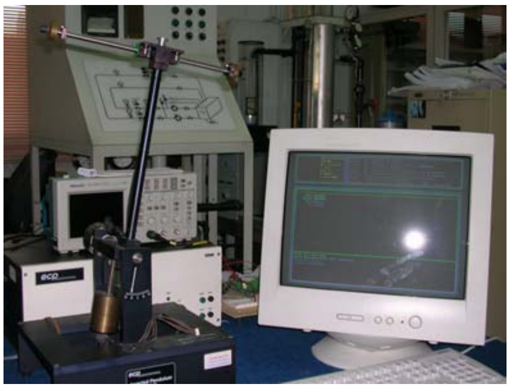


Fig 4. The ECP 505 plant.

In the first experiment, the pendulum is successfully stabilized at 12 and zero degrees using computed feedback linearization technique. The non-minimum phase characteristic is observed in the step response as shown in Fig. 5. We have experimentally demonstrated in [6] that the system is quite linear by this technique.

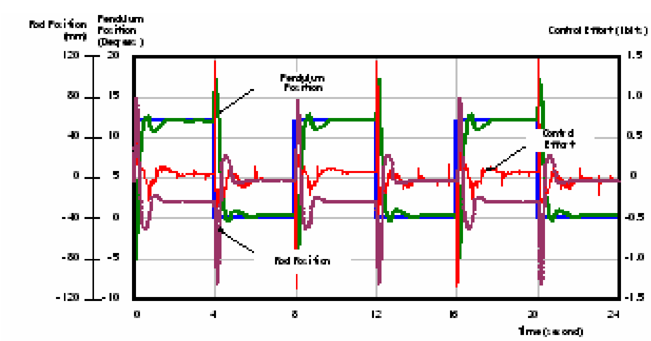
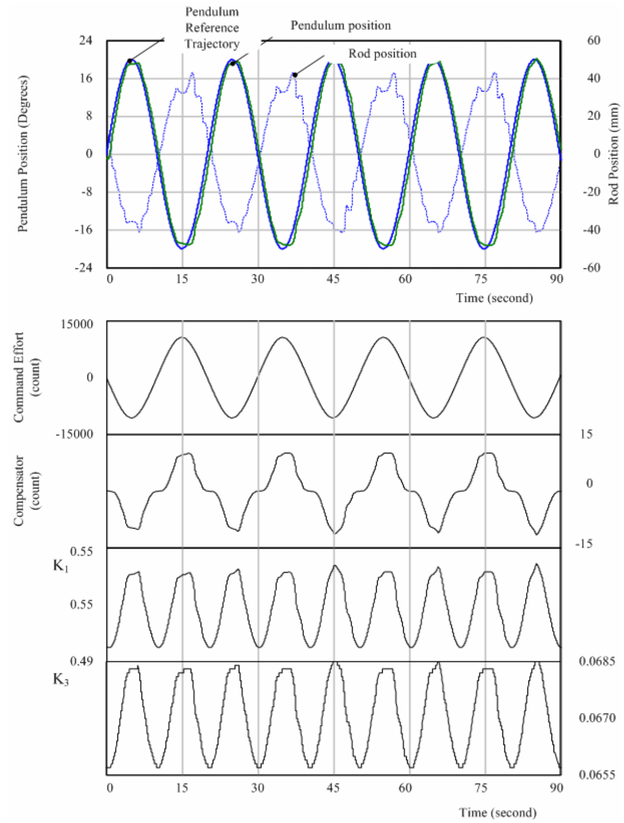
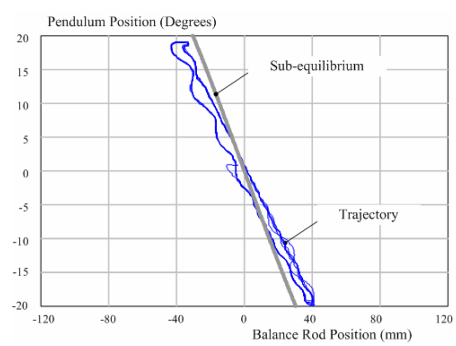


Fig 5. Stabilizing the pendulum at 12 degrees.

In the second experiment, the pendulum is able to track the low frequency sinusoidal command (0.05 Hz in this case) using only the stabilize controller as shown in Fig. 6a. The position command is directly feedforwarding into the feedback linearized system. In this technique, the output of the system is regulated to the position along the trajectory using computed feedback linearization. The nonlinear cancellation term $[f(x_0) - A(x_0)x_0]$ and feedback gains are adapted to pendulum position in real time as shown in Fig. 6a. The system travels around sub-equilibrium as shown in Fig. 6b where the approximate input-state linearization is obtained by computed feedback linearization. Despite the system is able to track a slow trajectory, the tracking performance can be improved as shown in the next experiment.



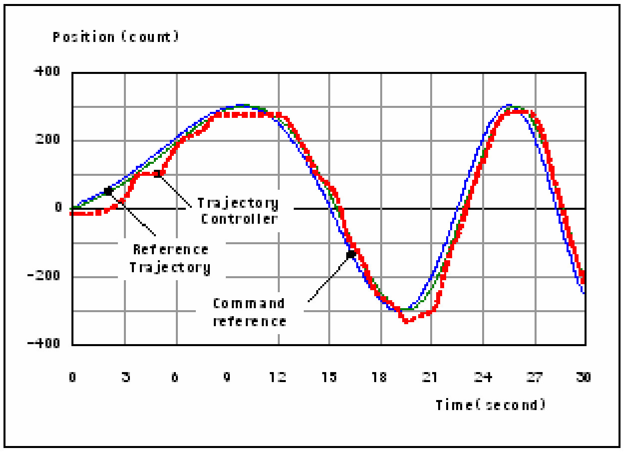
a) Tracking performance



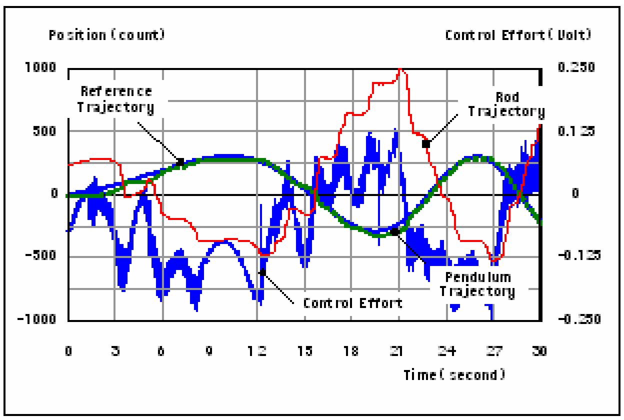
b) The pendulum trajectory

Fig 6. Response to sinusoidal input

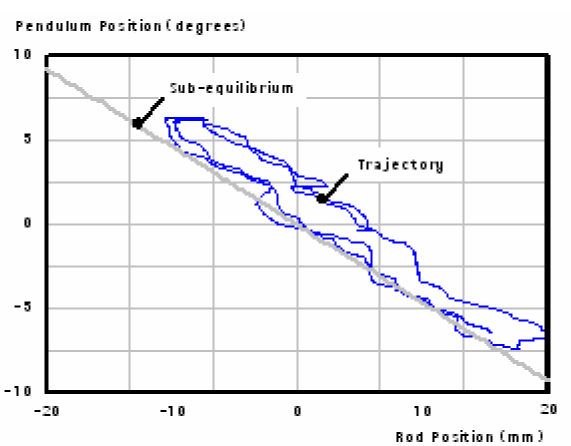
In the third experiment, the trajectory controller is applied and the pendulum is to track the sine sweep trajectory 0.01 to 0.1 Hz in 30 second. The result, compared with when using regulator based on LQ, shows that the system can track the trajectory using both techniques. However, the tracking performance is improved when using the proposed technique since the system can follow the trajectory closely while the tracking response when using LQ lacks the trajectory as shown in Fig. 7a.



a) Pendulum Response to Reference Trajectory



b) Rod and Pendulum Trajectory



c) The trajectory in x-θ plane

Fig 7. Tracking response to the sine sweep

0.01 – 0.1 Hz in 30 seconds.

Since the dynamical model of the feedback linearized system is known, the command reference or feedforward signal is determined from the trajectory which its derivative is taking into account. Fig. 7b shows the feedforward signal compared to the trajectory. When the feedforward signal is applied, the system follows the trajectory as shown.

The rod trajectory is plotted along with the pendulum trajectory. The rod position is the internal state in this case. Since we control both pendulum and rod simultaneously, both states are stable. The control effort is within the hardware limit.

The trajectory in the x-θ plane is shown in Fig 7c. The system is closed to sub-equilibrium all the way the system follow the trajectory. Thus, the input-state linearization is accurate and the system performance is as expected. It is noted that if the system is not near the sub-equilibrium, other control strategy, such as output regulation or swing up controller, must be used to bring the system into sub-equilibrium. The strategy to control the system at these positions is not mentioned in the paper.

VII. CONCLUSION

The tracking controller is successfully designed for the ECP 505 balanced rod inverted pendulum. The system is first feedback linearized along the trajectory using computed feedback linearization technique. The approximate input state linearization is obtained near sub-equilibrium without state transformation. The nonlinear full state feedback is then used to place poles of the system. In this project, the LQ technique is used to determine the optimum poles when the system is at equilibrium and the nonlinear full state feedback is used to tie these poles when the system goes along the trajectory. Thus, the system is approximately linear and its model is as desired. The superposition technique is used to superimpose the feedforward signal such that the system will follow the trajectory. The feedforward signal is determined from inverse dynamic of the feedback linearized system in which the trajectory and also its derivative is taking into account. The experiment demonstrates that the system can track the sine sweep signal quite well.

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