Received: 22 January 2018 | Revised: 6 January 2019 | Accepted: 10 January 2019
DOI: 10.1002/rnc.4481

RESEARCH ARTICLE

WILEY

Event-triggered control for a class of strict-feedback nonlinear systems

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Funding information
National Natural Science Foundation of China, Grant/Award Number: 61621004 and 61420106016; Research Fund of State Key Laboratory of Synthetical Automation for Process Industries, Grant/Award Number: 2018ZCX03

Summary

This paper is concerned with the event-triggered control problem for a class of strict feedback nonlinear networked systems. Different from the existing design methods, a novel user-adjustable event-triggered mechanism is first developed to determine the sampling state instants using the negative definite property of the derivatives of Lyapunov functions. Then, an event-triggered control strategy is devised based on the sampled state vectors and backstepping techniques. It is proved that the proposed control scheme ensures the global convergence of the closed-loop systems via Lyapunov analyses and the correlation criteria of real variable functions. Finally, two examples are performed to illustrate the effectiveness of the provided control approaches.

KEYWORDS

event-triggered control, networked control systems, strict-feedback nonlinear systems

1 | INTRODUCTION

In recent years, network control is widely used in various areas because of its many advantages, such as high efficiency, real-time, and reliability.[1,2] Crucially, communication resources are shared with multiple devices or different network nodes, which result in the congestion of communication channels and the increase of maintenance cost. Therefore, how to tackle this problem is of great significance both in theory and in the application field. Many attempts and efforts have been made to reduce the waste of computation or communication resources and some excellent results have been obtained. For example, the classical periodic data sampling control based on fixed time alleviates the pressure on the communication channels to some extent (see other works[3-5] and the references therein). However, it is sometimes less preferable, especially during the peak of communications, because the sampling takes place periodically regardless of whether the current behaviors of the system states needs or not. In order to make up for the shortcomings of periodic data sampling, another control method, namely, event-based control strategy, emerges as the times require.[6-16] More specifically, an introduction of event-triggered control and self-triggered control for linear systems was provided in the work of Heemels et al.[6] Subsequently, Wu et al[7] investigated the problem of event-triggered control for directly observable discrete-time linear systems subject to exogenous disturbances. In the work of Peng and Han,[10] an event-triggered $\mathcal{L}_2$ control for sampled-data linear systems was studied. The work reported by Heemels and Donkers[15] designed an observer-based controller for linear systems and proposed advanced event-triggering mechanisms that reduced communication in both the sensor-to-controller channels and the controller-to-actuator channels.

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Nevertheless, most of the articles mentioned earlier about event-triggered control focused on the study of linear systems. In fact, nonlinear systems account for a large proportion in practical industrial systems. Therefore, it is meaningful to study the event-triggered control for nonlinear systems and some outstanding achievements have been emerged. For example, a classical result was given about an event-triggered scheduler for nonlinear systems in the work of Tabuada,[17] which was based on a feedback paradigm and it relaxed the more traditional periodic execution requirements. Meanwhile, Liu and Jiang[18] introduced a new observer-based control design and converted the control system to interconnected input-to-state stable (ISS) systems to design an event-triggered rule. It should be noted that the aforementioned results are obtained under the assumption regarding ISS-Lyapunov functions. Fortunately, Xing et al[19] provided an effective method, that is, by introducing the hyperbolic tangent function into the controller, the event-triggered mechanism is designed by using the nature of hyperbolic tangent function. Furthermore, the same result was obtained by using the output feedback method in the work of Xing et al.[20] However, the event-triggered mechanism was presented for the controller-to-actuator channels, it depends on the difference between the controller and the continuously transmitted counterpart rather than on the difference between the state vectors and the continuously transmitted counterparts. It can be seen that the measures were proposed to reduce the amount of communication in the sensor-to-controller channels in the works of Li and Yang[21] and Zhang and Yang.[22] Nevertheless, the nonlinearities of the systems in these works were approximated by adaptive neural network-based or fuzzy-based control schemes, which led to the estimation residuals remaining in the design process and affected the final convergence of the systems.

Inspired by the aforementioned analyses, an interesting question is presented: *Without presupposing the existence of the ISS-Lyapunov functions, how to design an event-triggered controller based on the sampled state vectors such that all signals of the closed-loop systems converge globally, not just bounded?* In this paper, we attempt to deal with the problem of event-triggered control design for a class of nonlinear networked systems. To determine the sampling state instants, a new event-triggered mechanism with user-adjustable parameter is first designed using the negative definite property of the derivatives of Lyapunov functions. Based on this, an event-triggered control strategy is developed using the sampled state vectors and backstepping techniques. The main contributions of this paper are emphasized from three-folds.

- A preferable result of global convergence is obtained. The global convergence of the systems is guaranteed by applying Lyapunov analyses and the theories of real variable functions. This is different from using fuzzy logic systems or neural networks technology in related works[21-23] to obtain only semiglobal bounded results, and it is also superior to the globally bounded results obtained in other works.[19,20,24]
- A new method of designing event-triggered mechanism is proposed. Unlike the works of Postoyan et al[8] and Qi et al,[12] this paper utilizes the partial negative definite property of the derivative of Lyapunov functions to design the event-triggered conditions with user-adjustable parameter, instead of presupposing ISS-Lyapunov functions.
- The assumption about the nonlinearities of the systems is relaxed. Compared with the work of Li and Yang,[23] the assumption regarding the boundedness of the nonlinearities of the systems is not required, which expands the scope of application of the systems.

This article is composed of the following parts. Section 2 presents the objective of this study and gives some preliminaries. In Section 3, the controller is designed off-line based on backstepping method. Section 4 provides the event-triggered control design schemes. Two examples are proposed in Section 5 to verify the validity of the control schemes, whereas Section 6 concludes this paper.

Notation. $\mathbb{Z}^+$ represents the set of all positive integers, and $\mathbb{R}^n$ denotes n-dimensional Euclidean space. For a real vector $\underline{x}_i \triangleq [x_1, \dots, x_i]^T \in \mathbb{R}^i$, $i = 1, \dots, n$, especially $\underline{x}_n = x$, and the norm of x is defined by $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$. The expression of functions are sometimes simplified, for example, a function $f(x(t))$ is denoted by $f(x)$, $f(\cdot)$ or f.

2 | PROBLEM FORMULATION AND PRELIMINARIES

Consider the following strict-feedback nonlinear systems:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\underline{x}_i), \quad i = 1, \dots, n-1, \\ \dot{x}_n = u + f_n(x), \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u(t) \triangleq x_{n+1} \in \mathbb{R}^1$ are system state and control input, respectively. $f_i(\underline{x}_i) : \mathbb{R}^i \to \mathbb{R}$, are continuously differentiable functions with respect to $x_1, \dots, x_i$, and $f_i(0) = 0, i = 1, \dots, n$.

The purpose of this article is to design an event-triggered controller based on the system state vectors, which is released $x_i(t_k)$ at time $t = t_k$, $k \in \mathbb{Z}^+$, such that all the signals are globally bounded and $x_i(t), i = 1, \dots, n$, are convergent as $t \to \infty$. Accordingly, the zero-order hold is used to retain the latest event-sampled state vectors until the next sampling event takes place. The latest held state $\hat{x}_i(t)$ at the zero-order hold is updated with the current state in each $t = t_k$ with $t_0 = 0$. Therefore, the latest state can be written as

$$\hat{x}_i(t^+) = x_i(t_k), \quad t = t_k,$$
$$\hat{x}_i(t) = x_i(t_k), \quad t \in [t_k, t_{k+1}), \quad i = 1, \dots, n.$$

Event-triggered errors refer to the difference between the current measured state vectors $x_i(t)$ and the state vectors at the zero-order hold $\hat{x}_i(t)$, that is,

$$e_i(t) = x_i(t) - \hat{x}_i(t), \quad t \in [t_k, t_{k+1}), \quad i = 1, \dots, n. \tag{2}$$

The following definitions are presented in the work of Evans and Gariepy[25] and they are used in the proof of the main results.

Definition 1. The Lebesgue measure is defined on the Lebesgue σ−algebra, it is the collection of all sets E, which satisfy the condition that, for every $A \subseteq \mathbb{R}$,

$$m^*A = m^*(A \cap E) + m^*(A \cap E^c).$$

For any set in the Lebesgue σ−algebra, its Lebesgue measure is given by its Lebesgue outer measure, that is, $m(E) = m^*(E)$. If $m(E) = 0$, then E is a zero measure set.

Definition 2. If (X, σ, m) is a measure space, a property P is said to hold almost everywhere in X if there exists a subset N of X with $m(N) = 0$, and for $\forall x \in X \setminus N$, x has property P.

Assumption 1. The nonlinear functions $f_i(\underline{x}_i)$ satisfy the Lipschitz condition such that $|f_i(\underline{x}_i) - f_i(\underline{y}_i)| \le L_i \|\underline{x}_i - \underline{y}_i\|$, where $\underline{x}_i$, $\underline{y}_i \in \mathbb{R}^i$, and $L_i > 0, i = 1, \dots, n$, are known constants.

Remark 1. In most of the relevant results,[8,17] the design schemes are based on the assumptions that there exists a feedback controller to render closed-loop systems that are ISS about the event-triggered errors. However, for a general nonlinear system, it should not be surprising that such assumptions may not hold because the premise of ISS is that the unforced systems are globally uniformly asymptotically stable. For example, the simple scalar systems $\dot{x} = u + x$ and $\dot{x} = u + x^3$ are clearly unstable, when $u = 0$. Of course, the ISS conditions are not satisfied. Furthermore, even when the origin of the unforced system is globally uniformly asymptotically stable, the ISS conditions are also not always trivial.[26]

Remark 2. Some comparisons with related literature are presented to explain the differences or less conservativeness of the assumption. In the works of Li and Yang[21] and Zhang and Yang,[22] the nonlinear functions are approximated by fuzzy logic systems or adaptive neural networks, which led to the estimation residuals remaining in the design process and did not make $\dot{V} \le 0$ without any restrictions on the state vectors. This limitation also occurs in the work of Li and Yang,[23] thus the states of the systems are only guaranteed to be bounded, instead of convergence. Furthermore, the assumptions on the boundedness of the nonlinear functions are removed compared with the aforementioned work,[23] which extends the scope of application of the systems.

3 | OFF-LINE CONTROLLER DESIGN

In this section, the virtual controllers and the actual controller are designed off-line by using traditional backstepping technique, which is a recursive design with n steps. The following classical coordinate transformation is introduced:

$$\begin{cases} z_1 = x_1, \\ z_i = x_i - \alpha_{i-1}, i = 2, \dots, n, \end{cases} \tag{3}$$

where $\alpha_{i-1}, i = 2, \dots, n$, are virtual control about $x_1, \dots, x_{i-1}$ and $z_i, i = 1, \dots, n$, are the corresponding error variables.

Step 1. Consider the first differential equation in (1), taking the time derivative of the error variable that

$$\dot{z}_1 = \dot{x}_1 = z_2 + \alpha_1 + f_1(x_1). \tag{4}$$

Select the Lyapunov function as $V_1 = \frac{1}{2}z_1^2$, and the virtual control α_1 is designed by

$$\alpha_1 = -c_1 z_1 - f_1(x_1), \tag{5}$$

where c_1 is a positive design parameter. Thus, $\dot{V}_1$ satisfies

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2. \tag{6}$$

Step i ($i = 2, \dots, n$). Considering the ith equation of system (1), the derivative of z_i is

$$\dot{z}_i = \dot{x}_i - \dot{\alpha}_{i-1} = z_{i+1} + \alpha_i + f_i(\underline{x}_i) - \dot{\alpha}_{i-1}. \tag{7}$$

One can take virtual control law α_i as

$$\alpha_i = -c_i z_i - z_{i-1} - f_i(\underline{x}_i) + \dot{\alpha}_{i-1}, \tag{8}$$

where c_i is a positive design constant and $\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(x_{j+1} + f_j(\underline{x}_j))$. Then, taking the time derivative of $V_i = \sum_{j=1}^{i} \frac{1}{2} z_j^2$ yields

$$\dot{V}_i = -\sum_{j=1}^{i} c_j z_j^2 + z_i z_{i+1}, \tag{9}$$

when $i = n$, $z_{n+1} \triangleq 0$, and the actual controller is that

$$u = -c_n z_n - z_{n-1} - f_n(\underline{x}_n) + \dot{\alpha}_{n-1}. \tag{10}$$

It should be noted that the aforementioned virtual controllers and the actual controller are designed off-line.

4 | EVENT-TRIGGERED CONTROL DESIGN SCHEME

First, the diagrammatic sketch of the strict feedback nonlinear networked systems is presented in Figure 1. Then, we design the event-triggered mechanism and prove that $x(t)$ is globally convergent. Finally, a positive constant bound of the interevent times is displayed to illustrate that the event-triggered control systems can avoid the Zeno behavior. As shown in Figure 1, once joining the network, the controller is updated only when an event is triggered. It means that the event-triggered controller is devised based on the sampled state $\hat{x}_i(t)$, the last transmitted states $\hat{x}_i(t)$, $i = 1, \dots, n$, are kept as constants by the zero-order hold, thus $\dot{\hat{x}}_i(t) = 0$, which results in $\dot{\alpha}_{i-1}(\hat{x}_1, \dots, \hat{x}_{i-1}) = 0$ and $\dot{\hat{z}}_i(t) = 0$ for $t \in [t_k, t_{k+1})$. Therefore, from (5), (8), (10), one can be obtained that

$$\alpha_1 = -c_1 \hat{z}_1 - f_1(\hat{x}_1), \tag{11}$$

$$\alpha_i = -c_i \hat{z}_i - \hat{z}_{i-1} - f_i(\hat{\underline{x}}_i), \qquad i = 2, \dots, n-1, \tag{12}$$

$$u = -c_n \hat{z}_n - \hat{z}_{n-1} - f_n(\hat{\underline{x}}_n), \tag{13}$$

where

$$\hat{z}_1 = \hat{x}_1, \quad \hat{z}_i = \hat{x}_i - \alpha_{i-1}, \quad i = 2, \dots, n. \tag{14}$$

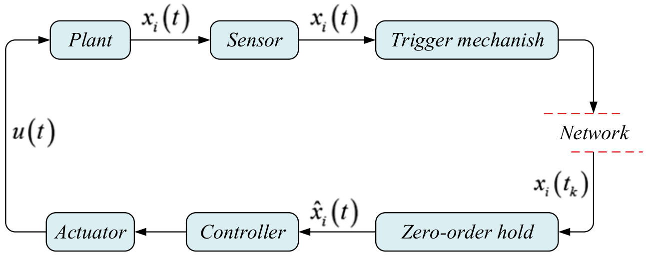

FIGURE 1 The diagrammatic sketch of networked control systems [Colour figure can be viewed at wileyonlinelibrary.com]

Considering that the systems are connected to the network, the virtual control law in (3) is replaced by (12). Therefore, it is easy to get

$$e_i(t) = x_i(t) - \hat{x}_i(t) = z_i(t) - \hat{z}_i(t), \qquad i = 1, \ldots, n. \tag{15}$$

Before giving the main results, the event-triggered conditions are presented as

$$|e_1| \le \frac{c_1}{c_1 + L_1}\lambda|z_1|, \tag{16}$$

$$\|\underline{e}_i\| \le \frac{c_i}{1 + c_i + L_i}\lambda|z_i|, i = 2, \ldots, n, \tag{17}$$

where $\underline{e}_i = [e_1, \ldots, e_i]^T$, the parameter $0 < \lambda < 1$ represents the proportion that $\dot{V}$ is used to offset the event-triggered errors. It can be adjusted independently according to the needs of the users. $c_i, L_i, i = 1, \ldots, n$, are the aforementioned positive design parameters, Lipschitz constants, respectively. The event-triggered mechanism shows that, once (16) and (17) are not satisfied, then the sensor sends the current of the system states to update the event-triggered control law.

Remark 3. The Lipschitz constants are determined immediately when a specific system is given. However, we should pay attention to the following two points when choosing c_i. (i) It can be seen from (16), (17) that c_i play a decisive role if $c_i \gg L_i$ and λ is selected independently. Therefore, one has $\frac{c_i}{c_i+L_i}\lambda \approx \lambda$ and $\frac{c_i}{1+c_i+L_i}\lambda \approx \lambda$, that is, the design parameters c_i do not affect the trigger frequency basically. (ii) L_i play a decisive role if $L_i \gg c_i$ and λ is selected independently, then $\frac{c_i}{c_i+L_i}\lambda \approx 0$, this may result in an increase in the trigger frequency. Therefore, case (ii) should be avoided when selecting parameters c_i.

Theorem 1. *Under the event-triggered controller (13), if system (1) satisfies Assumption 1 and the events are triggered when the conditions in (16), (17) are violated, then all the states of the closed-loop systems are globally bounded and convergent as $t \to \infty$.*

Proof. In this proof, we still adopt recursive method to design the event-triggered mechanism and prove that all states of the systems are convergent globally. To this end, two cases are considered. In Case 1, the system is analyzed during interevent periods, and it is analyzed when the measurement errors are zero in Case 2.

Case 1: Consider the interevent periods $t_k \le t < t_{k+1}, k \in \mathbb{Z}^+$, during which time there are nonzero event-sampling errors, but with constant control input. The specific analyses are given as follows.

Step 1. Calculating the derivative of Lyapunov function $V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\hat{z}_1^2$ along solutions of system (1), it follows from (11), (15), Assumption 1, and $\dot{\hat{z}}_1(t) = 0$ that

$$\begin{aligned}
\dot{V}_1 &= z_1\dot{z}_1 = z_1(z_2 - c_1\hat{z}_1 + f_1(x_1) - f_1(\hat{x}_1)) \\
&= z_1z_2 + z_1(-c_1z_1 + c_1(z_1 - \hat{z}_1) + (f_1(x_1) - f_1(\hat{x}_1))) \\
&\le -(1-\lambda)c_1z_1^2 + z_1z_2 + (c_1 + L_1)|z_1||e_1| - \lambda c_1 z_1^2.
\end{aligned} \tag{18}$$

If one restricts the error to satisfy (16), then (18) can be rewritten as

$$\dot{V}_1 \le -(1-\lambda)c_1z_1^2 + z_1z_2. \tag{19}$$

This completes Step 1.

Step 2. According to the definition of the norm, it is easy to know $|e_i| \le \|\underline{e}_j\|, i = 1, \ldots, j, \quad j = i, \ldots, n$. Keeping (19), Assumption 1, and $\dot{\hat{z}}_2(t) = 0$ in mind, computing the time derivative of $V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}\hat{z}_2^2$ along system (1), we can obtain

$$\begin{aligned}
\dot{V}_2 &\le -(1-\lambda)c_1z_1^2 + z_1z_2 + z_2(z_3 - c_2\hat{z}_2 - \hat{z}_1 - f_2(\underline{\hat{x}}_2) + f_2(\underline{x}_2)) \\
&= -(1-\lambda)c_1z_1^2 + z_2z_3 + z_2((z_1 - \hat{z}_1) - c_2z_2 + c_2(z_2 - \hat{z}_2) + f_2(\underline{x}_2) - f_2(\underline{\hat{x}}_2)) \\
&\le -(1-\lambda)\left(c_1z_1^2 + c_2z_2^2\right) + z_2z_3 + (1 + c_2 + L_2)|z_2|\|\underline{e}_2\| - \lambda c_2 z_2^2.
\end{aligned} \tag{20}$$

It follows from (17) and (20) that

$$\dot{V}_2 \le -(1-\lambda)\left(c_1z_1^2 + c_2z_2^2\right) + z_2z_3.$$

Step k ($k = 3, \ldots, n$). Suppose, at Step $k-1$, there exists a suitable threshold strategy to satisfy

$$\dot{V}_{k-1} \le -(1-\lambda)\sum_{j=1}^{k-1} c_jz_j^2 + z_{k-1}z_k. \tag{21}$$

In this step, chose the Lyapunov candidate function as $V_k = V_{k-1} + \frac{1}{2}z_k^2 + \frac{1}{2}\hat{z}_k^2$, with (12), (13), (15), (21), and $\dot{\hat{z}}_k(t) = 0$ in hand, calculating its time derivative along system (1) yields

$$\begin{aligned}
\dot{V}_k &\le -(1-\lambda)\sum_{j=1}^{k-1} c_j z_j^2 + z_{k-1}z_k + z_k\left(\dot{x}_i - \dot{\alpha}_{i-1} - y_r^{(k-1)}\right) \\
&= -(1-\lambda)\sum_{j=1}^{k-1} c_j z_j^2 + z_k z_{k+1} + z_k(-c_k z_k + (z_{k-1} - \hat{z}_{k-1}) + c_k(z_k - \hat{z}_k) + f_k(\underline{x}_k) - f_k(\underline{\hat{x}}_k)) \\
&\le -(1-\lambda)\sum_{j=1}^{k} c_j z_j^2 + z_k z_{k+1} + (1 + c_k + L_k)|z_k|\|\underline{e}_k\| - \lambda c_k z_k^2.
\end{aligned} \tag{22}$$

Under the restriction of (17), the following inequality holds:

$$\dot{V}_k \le -(1-\lambda)\sum_{j=1}^{k} c_j z_j^2 + z_k z_{k+1}. \tag{23}$$

By now, the proof of inductive has been completed. When $k = n$, $z_{n+1} \triangleq 0$, (23) deduces

$$\dot{V}_n \le -(1-\lambda)\sum_{j=1}^{n} c_j z_j^2. \tag{24}$$

Case 2: At the event-sampled instants, the sampling errors are zero, that is, $e_i(t) = 0, t = t_k, k \in \mathbb{Z}^+$. According to (3), (11), (12), it can be seen that

$$z_i(t_k) = x_i(t_k) - \alpha_{i-1}(\underline{x}_i(t_k)), \tag{25}$$

$$z_i^- \triangleq \lim_{t\to t_k^-} z_i(t) = \lim_{t\to t_k^-}(x_i(t) - \alpha_{i-1}(\underline{\hat{x}}_i(t))). \tag{26}$$

In addition, from (14), we have that

$$\begin{aligned}
\hat{z}_i(t_k) &= x_i(t_k) - \alpha_{i-1}(\underline{x}_i(t_k)), \\
\hat{z}_i^- &\triangleq \lim_{t\to t_k^-} \hat{z}_i(t) = x_i(t_{k-1}) - \alpha_{i-1}(\underline{x}_i(t_{k-1})) = \hat{z}_i(t_{k-1}).
\end{aligned}$$

By (24), $\dot{V}_n < 0$ for $\forall\|z(t)\| \neq 0$, where $t \in [t_k, t_{k+1})$, so $\|z(t)\|$ is decreasing monotonically during interevent periods. Considering Lyapunov candidate function $V_n = \sum_{i=1}^{n}(\frac{1}{2}z_i^2 + \frac{1}{2}\hat{z}_i^2)$, according to (27), the difference of V_n in the event-sampled instants is that

$$\begin{aligned}
\Delta V_n &= \sum_{i=1}^{n}\frac{1}{2}\left(\left(z_i^2(t_k) - \left(z_i^-\right)^2\right) + \left(\hat{z}_i^2(t_k) - \left(\hat{z}_i^-\right)^2\right)\right) \\
&= \sum_{i=1}^{n}\frac{1}{2}\left(\left(z_i^2(t_k) - \left(z_i^-\right)^2\right) + \left(\hat{z}_i^2(t_k) - \hat{z}_i^2(t_{k-1})\right)\right) \\
&\le -\sum_{i=1}^{n}\frac{1}{2}\left(z_i^-\right)^2 + \sum_{i=1}^{n}\frac{1}{2}\left(\hat{z}_i^2(t_k) - \hat{z}_i^2(t_{k-1})\right) + D,
\end{aligned} \tag{27}$$

where D is the bound of $\sum_{i=1}^{n} z_i^2(t_k)$. Therefore, as long as $\sum_{i=1}^{n}\frac{1}{2}(z_i^-)^2 + \sum_{i=1}^{n}\frac{1}{2}(\hat{z}_i^2(t_{k-1}) - \hat{z}_i^2(t_k)) > D$ is established, there is $\Delta V_n < 0$, that is, z_i and $\hat{z}_i$, $i = 1, \dots, n$, are also bounded at the sampling instants.

Next, we will demonstrate that the states of closed-loop systems (1), (13) are globally bounded and convergent. Notice

$$V_n = \sum_{i=1}^{n}\left(\frac{1}{2}z_i^2 + \frac{1}{2}\hat{z}_i^2\right). \tag{28}$$

According to (24), (27), and (28), one can obtain $z_i(t) \in \mathcal{L}_\infty$, $i = 1, \dots, n$. Let $E = [0, \infty)$ and $M = \{t \mid t = t_k, k \in \mathbb{Z}^+\}$. Because M is a countable set, it is easy to know that $m(M) = 0$ based on measure theory. In the light of Definitions 1, 2, Cases 1 and 2 ensure that $\dot{V}_n < 0$ holds almost everywhere on time interval E, for $\forall z \neq 0$, which means $\|z\|$ is reduced on E. Moreover, in view of the boundedness of $\|z(0)\|$, one can obtain that $\|z(t)\| \to 0$ as $t \to 0$. Therefore, it can be seen that $z \to 0$ as $t \to 0$ on time interval E. Therefore, $x_1 \in \mathcal{L}_\infty$ and converges to 0, this implies $\alpha_1 \in \mathcal{L}_\infty$, and $x_2 = z_2 + \alpha_1 \in \mathcal{L}_\infty$. In this way, one can testify $x_3 \in \mathcal{L}_\infty, \dots, x_n \in \mathcal{L}_\infty$, and $u \in \mathcal{L}_\infty$ according to (13). So far, it has been proved that all signals of the systems are globally bounded on the interval $[0, \infty)$. In the following, we show that $x_i, i = 2, \dots, n$, are convergent. By x_1, z_2 converge to 0 and $f_1(0) = 0$, (5) guarantees α_1 is convergent, which means $x_2 = z_2 + \alpha_1$ is convergent. In the same way, the convergence of $x_i, i = 3, \dots, n$, can be obtained. Finally, it can be concluded that $x_i, i = 1, \dots, n$, are globally bounded and converge to zero. □

Remark 4. The aforementioned proof indicates that the partial negative definite property of $\dot{V}$ is used to offset the error caused by the events. More specifically, without assuming ISS-Lyapunov functions or introducing compensation signals into the controller, the event-triggered errors are offset by sacrificing the more "negative" properties of $\dot{V}$, which improves the existing design methods.

Remark 5. Compared with the existing literature,[19,21-24] Theorem 1 not only shows that all the signals in the systems are bounded but also globally convergent. This is not a straightforward thing, since the classical Barbalat lemma is not suit to prove convergence to the event-triggered control problem. A new way of applying the correlation theories of real variable functions is explored, which increases the difficulty of the theoretical proof.

Next, we will show that the proposed control strategy can avoid the Zeno behavior. This result is presented in the following theorem.

Theorem 2. *Consider the closed-loop systems (1), (13), under Assumption 1 and the event-triggered mechanism (16), (17), then the interevent times $t_{k+1} - t_k$ are lower bounded by a nonzero positive constant t^*.*

Proof. According to Assumption 1, (16), (17), and Theorem 1, there exist constants $M_i > 0, i = 1, \dots, n$, such that

$$|f_i(\underline{x}_i) - f_i(\hat{\underline{x}}_i)| \leq L_i \cdot \|\underline{e}_i\| \leq M_i, \quad i = 1, \dots, n. \tag{29}$$

It is easy to know that $f_i(\hat{\underline{x}}_i)$ are bounded.(29) shows that there exist constants $Q_i > 0, i = 1, \dots, n$, such that

$$|f_i(\underline{x}_i)| \leq Q_i, \quad i = 1, \dots, n. \tag{30}$$

Considering (1), (15), and (30), the event-triggered error dynamics are given as

$$\dot{e}_i(t) = \dot{x}_i(t) - \dot{\hat{x}}_i(t) = \dot{x}_i(t) = x_{i+1} + f_i(\underline{x}_i) \leq x_{i+1} + Q_i, \quad i = 1, \dots, n. \tag{31}$$

(31) can be rewritten as

$$\dot{x} = Ax + Q, \quad t \in [t_k, t_{k+1}), \tag{32}$$

where $A = \begin{bmatrix} 0 & & \\ \vdots & I_{n-1} & \\ 0 & \dots & 0 \end{bmatrix}$, $I_{n-1} \in \mathbb{R}^{(n-1)\times(n-1)}$ denotes an identity matrix, and $Q = [Q_1, \dots, Q_n]^T$. From (32), one can obtain

$$\|\dot{x}\| \leq \|A\|\|x\| + \|Q\|, \quad t \in [t_k, t_{k+1}). \tag{33}$$

On the other hand, the following inequality holds:

$$\frac{d}{dt}\|e\| = \frac{d}{dt}(e^T e)^{\frac{1}{2}} = \frac{e^T \dot{e}}{\|e\|} \leq \frac{\|e^T\|\|\dot{e}\|}{\|e\|} = \|\dot{e}\|, \quad t \in [t_k, t_{k+1}). \tag{34}$$

Therefore, combining inequalities (31), (33), (34) with condition $e(t_k) = 0$, one has

$$\|e\| \leq \int_{t_k}^{t} e^{\|A\|(t-\tau)}\|Q\|dt = \frac{\|Q\|}{\|A\|}(e^{\|A\|(t-t_k)} - 1), \quad \forall t \in [t_k, t_{k+1}). \tag{35}$$

With (35) in mind, it can be calculated that

$$t_{k+1} - t_k = t^* > t - t_k = \frac{1}{\|A\|}\ln\left(\frac{\|A\|}{\|Q\|}(\|e\| + 1)\right) \geq 0, \quad \forall t \in [t_k, t_{k+1}).$$

Thus, the interevent times $t_{k+1} - t_k$ are lower bounded by a nonzero positive constant t^*, that is, the Zeno behavior does not occur. This completes the proof. □

Remark 6. It is well known that the Zeno phenomenon is common in physics field, such as the bouncing ball can be regarded as a high frequency switched system with jump at the transient. However, it is necessary to avoid the Zeno phenomenon in event-triggered control systems because, once it happens, it will cause trigger accumulation point, which makes the trigger mechanism meaningless.

5 | SIMULATION EXAMPLES

Example 1. Consider the following nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 + 2\sin x_1, \\ \dot{x}_2 = u + x_2. \end{cases} \tag{36}$$

Obviously, the unforced system is not globally asymptotically stable and $f_2 = x_2$ is unbounded on $\mathbb{R}$, so system (36) is neither ISS nor bounded, which leads to the existing event-triggered control schemes cannot be applied to this example. The initial conditions and parameters are chosen as follows: $x_1(0) = -0.6, x_2(0) = 0.6, L_1 = 2, L_2 = 1$ and $c_1 = 10, c_2 = 10, \lambda = 0.5$. The sampling period is chosen as 0.05 seconds for discretization. Under event-triggered control law (13) and continuous time controller (10) with event-triggered conditions (16), (17), the simulation results are displayed in Figures 2 to 6. To be specific, Figures 2 and 3 present the state trajectories of the system under event-triggered control law (blue line) and continuous controller (red line). It can be seen that all the states are globally bounded and convergent. In Figure 4, blue line and red line represent the event-triggered controller and the continuous time controller, respectively. Figure 5 provides the interevent intervals and it is calculated that the number of triggers is 67 times, whereas the number of continuous time sampling is 121 times, which verified the effectiveness of the event-triggered control schemes.

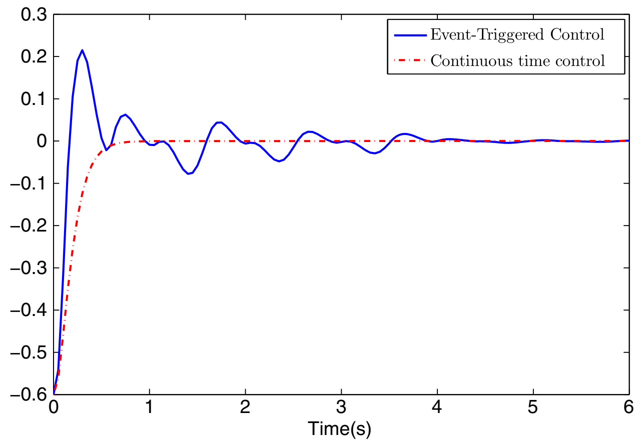

FIGURE 2 The trajectories of the state x_1 [Colour figure can be viewed at wileyonlinelibrary.com]

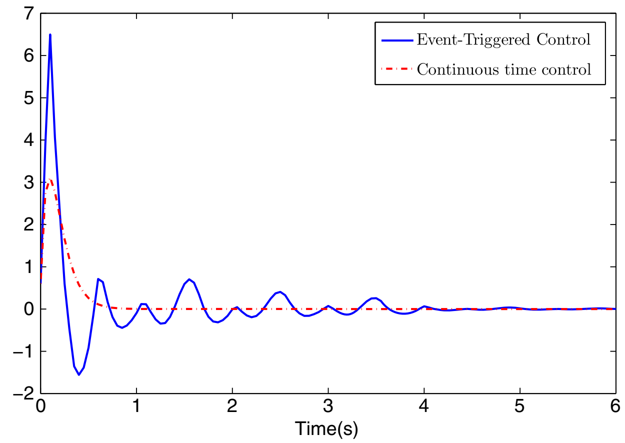

FIGURE 3 The trajectories of the state x_2 [Colour figure can be viewed at wileyonlinelibrary.com]

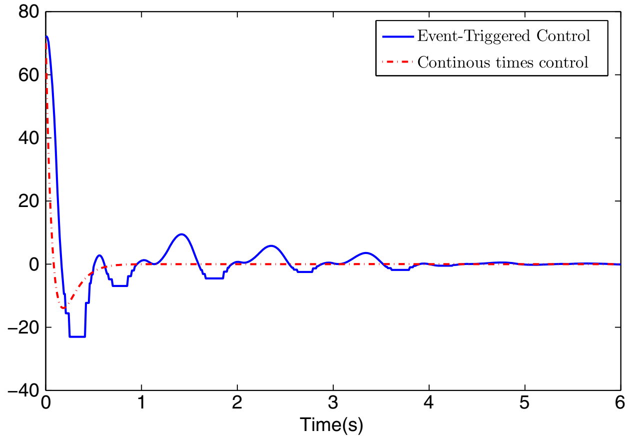

FIGURE 4 The trajectories of controller u [Colour figure can be viewed at wileyonlinelibrary.com]

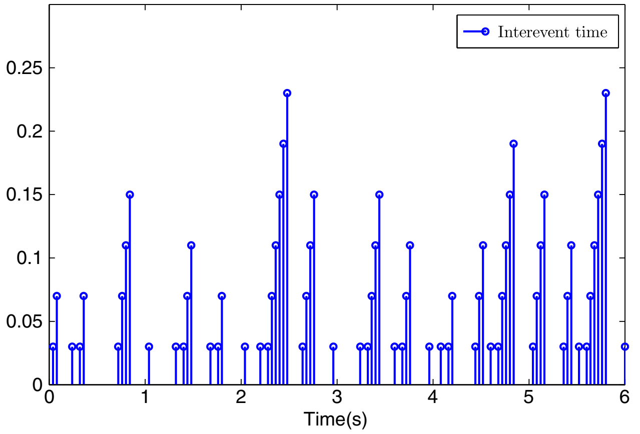

FIGURE 5 Time interval of triggering events [Colour figure can be viewed at wileyonlinelibrary.com]

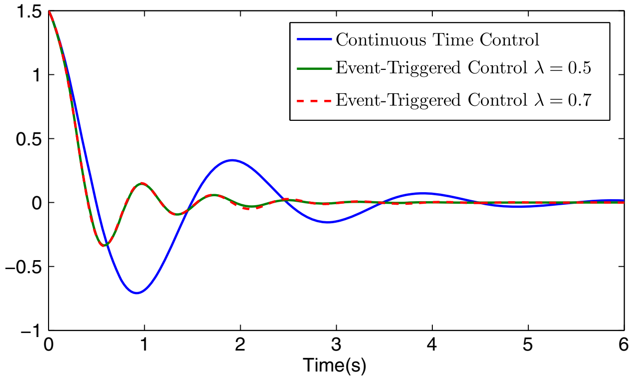

FIGURE 6 The trajectories of the state x_1 [Colour figure can be viewed at wileyonlinelibrary.com]

Example 2. Consider the networked-based single-link manipulator system with flexible joints, rotating in a vertical plane, damping ignored.[26] The dynamic model is given as follows:

$$\begin{cases} I\ddot{q}_1 + k(q_1 - q_2) + Mgl\sin(q_1) = 0 \\ J\ddot{q}_2 - k(q_1 - q_2) = u, \end{cases} \tag{37}$$

where q_1 and q_2 are the link displacement and the rotor displacement, respectively. I is the link inertia and J is the motor rotor inertia, k is the elastic constant, M is the total link mass, g is the gravity constant, l is the center of mass, and

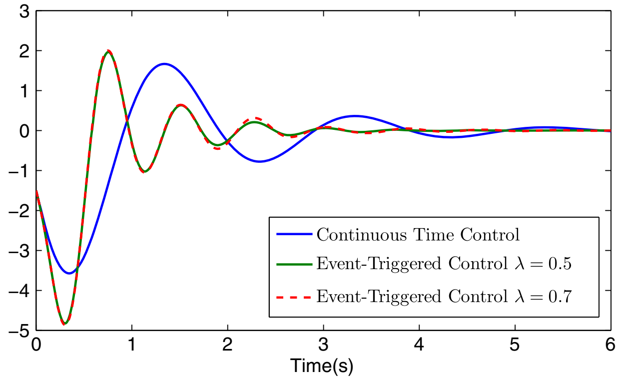

FIGURE 7 The trajectories of the state x_2 [Colour figure can be viewed at wileyonlinelibrary.com]

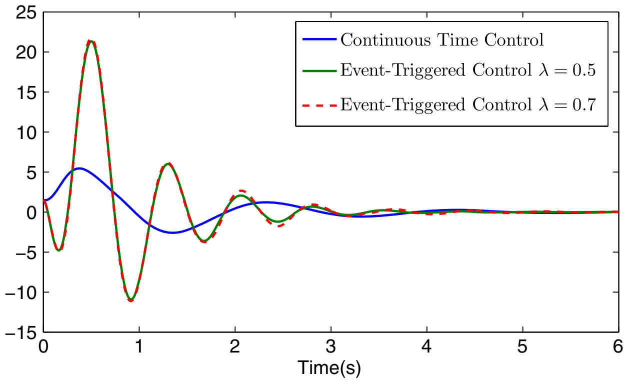

FIGURE 8 The trajectories of the state x_3 [Colour figure can be viewed at wileyonlinelibrary.com]

u is a torque. Take the coordinate transformation $x_1 = \frac{IJ}{k}q_1, \quad x_2 = \frac{IJ}{k}\dot{q}_1, \quad x_3 = Jq_2, \quad x_4 = J\dot{q}_2$, then (37) becomes

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 - \frac{k}{I}x_1 - \frac{JMgl}{k}\sin\left(\frac{k}{IJ}x_1\right) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = u + \frac{k^2}{IJ}x_1 - \frac{k}{J}x_3. \end{cases} \tag{38}$$

It can be seen from system (38) that it is neither ISS nor globally bounded, so the existing event-triggered control methods cannot be used to solve the problem. In this example, $f_1 = 0, \quad f_2 = -\frac{k}{I}x_1 - \frac{JMgl}{k}\sin(\frac{k}{IJ}x_1), \quad f_3 = 0, \quad f_4 = \frac{k^2}{IJ}x_1 - \frac{k}{J}x_3$. The coefficients are taken as $I = J = k = l = 1$, $g = 9.8$. Then, the Lipschitz constants are $L_1 = 0$, $L_2 = \max\{\frac{k}{I}, \frac{Mgl}{I}\} = 9.8 \quad, L_3 = 0, \quad L_4 = \max\{\frac{k^2}{IJ}, \quad \frac{k}{J}\} = 1$, respectively. The parameters are chosen as $c_1 = 4, c_2 = 5, c_3 = 10, c_4 = 12$ and the initial conditions are set as $x_1(0) = 1.5, x_2(0) = -1.5, x_3(0) = 1.5, x_4(0) = -1.5$. The sampling period is chosen as 0.05 seconds for discretization.

The simulation results are presented in Figures 6 to 12. In Figures 6 to 9, the blue lines are the evolution of state variables under the continuous time control, whereas the green lines and the red lines are the effects of the event-triggered control by selecting the different λ. Figures 10 and 11 represent the time interval of triggering events for selecting different parameter λ, respectively.

The amount of updates to the controllers of the event-triggered control and the continuous time sampling control are listed in Table 1.

Compared with the continuous time sampling control, it can been seen that the event-triggered control strategy reduces the amount of signal transmissions by about 60% when $\lambda = 0.5$ and it reduces about 75% when $\lambda = 0.7$.

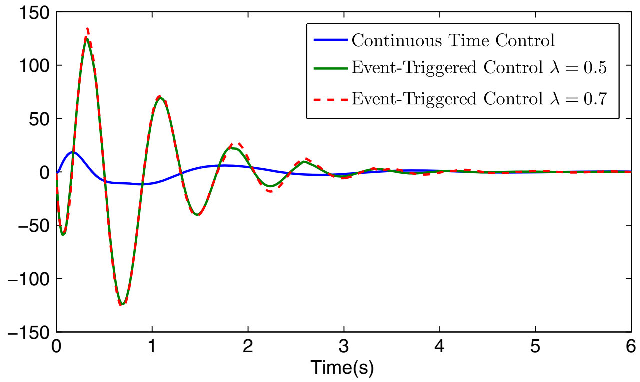

FIGURE 9 The trajectories of the state x_4 [Colour figure can be viewed at wileyonlinelibrary.com]

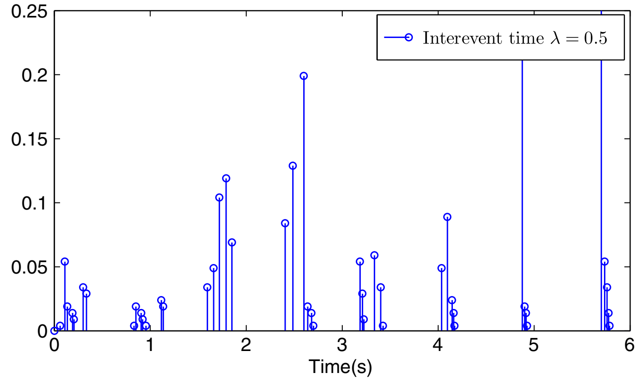

FIGURE 10 The time interval of triggering events $\lambda = 0.5$ [Colour figure can be viewed at wileyonlinelibrary.com]

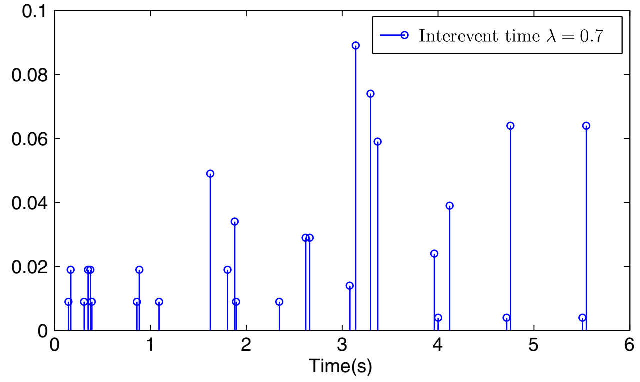

FIGURE 11 The time interval of triggering events $\lambda = 0.7$ [Colour figure can be viewed at wileyonlinelibrary.com]

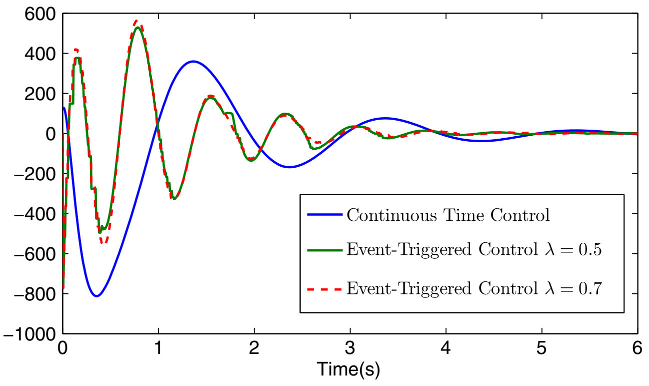

FIGURE 12 The trajectories of controller u [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Number of updates to the controller

	Event-Triggered Control	Continuous Time Sampling Control
Example 2 ($\lambda = 0.5$)	46	121
Example 2 ($\lambda = 0.7$)	27	121

6 | CONCLUSIONS

In this article, an event-triggered control strategy has been proposed for the strict feedback nonlinear systems. To determine the sampling state instants, a new user-adjustable event-triggered mechanism is developed utilizing the negative definite property of the derivatives of Lyapunov functions. Based on this, the event-triggered controller has been devised by using the sampled state vectors and backstepping methods. It has been proved that the control strategy guaranteed that all the states of the systems were globally convergent, which is an improvement of the existing boundedness results. Finally, two examples have been presented to show the validity of the proposed control approaches. In the future, the research work is focused on model uncertainty and disturbed nonlinear systems.[24,27]

ACKNOWLEDGEMENTS

This work was supported in part by the Funds of the National Natural Science Foundation of China (Grants 61621004 and 61420106016) and the Research Fund of State Key Laboratory of Synthetical Automation for Process Industries (Grant 2018ZCX03).

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How to cite this article: Zhang C-H, Yang G-H. Event-triggered control for a class of strict-feedback nonlinear systems. *Int J Robust Nonlinear Control*. 2019;1–13. https://doi.org/10.1002/rnc.4481