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Game Theoretical Approach to Novel Reactive Power Ancillary Service Market Mechanism

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***Abstract*—In deregulated power systems, reactive power ancillary service through electricity market is becoming relevant where private generation companies participate in maintaining system wide bus voltage within the permissible limits. Marginal cost price (MCP) based real time reactive power ancillary service market faces several challenges due to the localized nature of reactive power. In this paper, a market mechanism for real time reactive power ancillary service market based on Stackelberg game model is proposed considering voltage-apparent power coupled subsystems. In the proposed Stackelberg game model, Independent System Operator (ISO) is considered as the leader and GENCOs as followers. In the formulation, each GENCO is associated with a relevance factor in the partitioned subsystem so as to consider the real time voltage support requirement in the system. The market is then formulated as Mathematical Program with Equilibrium Constraints problem (MPEC). Existence of equilibrium, incentive compatibility and individual rationality of the proposed market mechanism is then analysed in this work. The numerical examples are illustrated in PJM 5-bus system and tested on IEEE 30- bus system and Nordic 32 Bus-system. The mechanism induces truth-telling behavior of GENCOs, yields a non-negative profit and the system wide bus voltage is improved.**

***Index Terms*—Reactive power ancillary service, Wholesale electricity markets, Marginal cost price, Stackelberg game equilibrium.**

NOMENCLATURE

Indices:

i,j	Index for buses.
l	Index for lines.
L	Set of load buses.
G	Set of generator buses
N	Total number of buses.

Variables:

V_L	Vector of load bus voltage.
I_L	Vector of load bus current.
V_G	Vector of generator bus voltage.
I_G	Vector of generator bus current.
N_G	Matrix of relative electrical distance between load and generator bus.
M_L	Matrix of relative electrical distance between load buses.
V^*_{LL}	LxL diagonal matrix with diagonal entries as V^*_L(complex conjugate).
V^*_{GG}	GxG diagonal matrix with diagonal entries as V^*_G(complex conjugate).
S^*_L	Complex conjugate of apparent power at load bus $=V^*_{LL}I_L$.
S^*_G	Complex conjugate of apparent power at generator bus $=V^*_{GG}I_G$.
$V^{\|}_L$	Vector of no-load voltage at load bus coupled with other load buses.
$V^{\|\|}_L$	Vector of no-load voltage at load bus coupled with generator buses.
S_{gi}	Apparent power at i^{th} generator bus
$V^{\|\|}_j$	No-load voltage component at j^{th} load bus coupled with apparent power changes in generator bus.
$\|V_i\|$	Voltage magnitude at i^{th} bus,p.u.
$N_{G_{ji}}$	(j,i)th element of N_G.
R_i	Relevance factor of i^{th} generator bus.
P_i	Profit at i^{th} generator bus.
λ_i	Price at i^{th} generator bus ($/p.u MVAr).
Q_{gi}	Reactive power generation at i^{th} generator bus, p.u.
$C_i(Q_{gi})$	Cost function for reactive power generation at i^{th} generator bus.
Q_{vi}	Reactive power generation at i^{th} generator bus for voltage support, p.u.
C_{vi}	Cost function pertaining to Q_{vi}.
$Cres_i$	Cost function pertaining to reactive power reserve.
U_{ISO}	Utility function of independent system operator.
T_i	Utility function of pseudo independent system operator for i^{th} generation company.
π	Potential function.
t(θ)	Tariff mechanism for game participant of type θ
x(θ)	Decision variable for game participant of type θ
U^*_{ISO}	Optimal utility function value of independent system operator.
P_{gi}	Active power generation at i^{th} bus, p.u.
P_{di}	Active power demand at i^{th} bus, p.u.
θ	Angle associated with bus admittance in radians
δ	Angle of bus voltage in radians
Q_{di}	Reactive power demand at i^{th} bus, p.u.
S_l	Transmission line flow, p.u.
P_{ij}	Active power flow in line between i^{th} bus and j^{th} bus.
α_i	Dual variable of active power injection con-

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straint.

β_i	Dual variable of reactive power injection constraint.
γ_{imin}	Dual variable of minimum active power generation limit constraint.
γ_{imax}	Dual variable of maximum active power generation limit constraint.
ζ_{lmin}	Dual variable of minimum apparent power line flow limit constraint
ζ_{lmax}	] Dual variable of maximum apparent power line flow limit constraint
η_{imin}	Dual variable of minimum bus voltage magnitude limit constraint
η_{imax}	Dual variable of maximum bus voltage magnitude limit constraint
ρ_{imin}	Dual variable of minimum reactive power for voltage support limit constraint.
ρ_{imax}	Dual variable of maximum reactive power for voltage support limit constraint.
σ_i	Dual variable of stationary point KKT conditions of lower level generation company optimisation problem.
ϕ_{imin}	Dual variable of primal feasibility KKT condition on minimum reactive power generation limit constraint.
ϕ_{imax}	Dual variable of primal feasibility KKT condition on maximum reactive power generation limit constraint.
ω_{imin},ω_{imax}	Dual variable of dual feasibility KKT condition of lower level generation company optimisation problem.
ψ_{imin}	Dual variable of Complementary slackness KKT condition on minimum reactive power generation limit constraint.
ψ_{imax}	Dual variable of Complementary slackness KKT condition on maximum reactive power generation limit constraint.

Constants:

Q_{gi}^{min}	Minimum reactive power generation limit, p.u.
Q_{gi}^{max}	Maximum reactive power generation limit, p.u.
P_{gi}^{min}	Minimum active power generation limit, p.u.
P_{gi}^{max}	Maximum active power generation limit, p.u.
S_{l}^{max}	Maximum transmission line flow limit, p.u.
$\lvert V_i^{min}\rvert$	Minimum bus voltage magnitude limit, p.u.
$\lvert V_i^{max}\rvert$	Maximum bus voltage magnitude limit, p.u.
m_{1i}	Operation cost bid submitted by i^{th} generation company, \$/p.u.MVAr.
m_{2i}	Lost opportunity cost bid submitted by i^{th} generation company, \$/p.u.MVAr^2.
Y_{ij}	Element of admittance matrix,p.u.
X_{ij}	Element of reactance matrix,p.u.
V_{ref}	Reference bus voltage magnitude, p.u.

I. Introduction

MODERN Power System is moving towards a smart and competitive system, with competing generation companies (GENCOs) playing a crucial role in daily operation of the system which is being monitored by an Independent System Operator (ISO). Private generation companies participate in electricity market as well as in maintaining the system parameters within the permissible limits. It is important to control competing generation companies through transactive energy framework for safe and reliable system operation. Pricing in real time at whole sale and retail level may be considered as an efficient way for maintaining safe operation of the system.

Reactive power services are considered as an important ancillary service due to its impact in maintaining the voltage stability margin. Reactive power pricing [1] can be done in real time, through the measures like payment for unit specific cost, marginal cost price (MCP) based auction etc. Due to localized nature of reactive power support required in the system, there are chances for exercising market power by generating utilities. This is reflected the most in MCP based market models [1]. Even for real power markets, locational marginal price based market is found to be less economically efficient due to market power [2]. To mitigate this, long term contracts are found to be useful [3]. To avoid exercising of market power it was suggested to estimate reactive power cost curve at transmission operator for each generator with the bid data of day ahead market [4]. Real time reactive power pricing scheme that considered maximizing the benefit of customers and minimizing real power production cost was proposed in [5]. A uniform price for reactive power by minimizing the expected payment function, was proposed in each voltage control area [4].

The reactive power requirement of the system has to be evaluated considering the reactive power generation and absorption by transmission lines depending on loading conditions. The price of reactive power service depends on the system parameters as well as the location of generation companies. Thus determination of actual reactive power requirement in the system is challenging. Modeling cost of reactive power is not as straight forward as that of real power production. Detailed cost function analysis of reactive power is presented in [6]. Sufficient models considering various aspects of reactive power cost have been found in literature, which are found to be effective [4], [7]–[10].

In this paper, a novel market mechanism for reactive power ancillary service is designed to consider the local nature of reactive power and thus mitigate market power. The proposed market model is depicted in Fig. 1

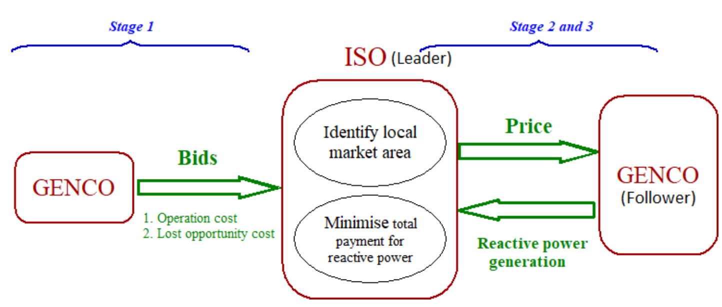

Fig. 1. Novel reactive power market model

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The market design preserves the key features of reactive power ancillary service,i.e., maintaining system wide bus voltage and sufficient reactive power reserve in the system. This is achieved by defining a suitable utility function for Independent System Operator which considers the bus voltage deviation and reactive power reserve requirement in the system. The market is modeled as a single leader multi-follower Stackelberg game wherein ISO (leader) issues price signals to each GENCO and then GENCOs (followers) respond with their reactive power generation.

The major contributions of this work are summarised below:

1) A novel mathematical model of independent system operator is defined in this work. Localized nature of reactive power is incorporated by defining a relevance factor for each GENCO in the partitioned system. Utility function is based on the bids received from participating GENCOs. The function considers the reactive power required to maintain bus voltage within permissible limits and also value of reactive power reserve in the system. With such a definition of utility function, total payment for reactive power ancillary service is reduced, system wide bus voltage is improved and sufficient reactive power reserve is maintained in the system.
2) A single leader multi-follower game based market mechanism is designed with ISO as leader and GENCOs as followers. This is different from the market mechanism discussed in literature so far as in general multi leader (GENCOs)-single follower (ISO) game based mechanisms were considered. Based on the marginal cost bids received from GENCOs, ISO declares the reactive power price (non-uniform) to each GENCO. Each GENCO then responds to the price signal received with their reactive power generation schedule. Existence of equilibrium in such a pricing mechanism is proven in this work.
3) The incentive compatibility analysis and individual rationality of the proposed single leader multi-follower game based market mechanism is then analysed. The proposed mechanism ensures that it is profit maximizing for each GENCO to reveal their actual marginal cost and also earn a non-negative profit.

The rest of this paper is organised as follows: In Section II, reactive power market mechanism based on relevance factor of GENCOs is described. Section III presents the detailed model of ISO and GENCOs and mathematical formulation of the proposed single leader multi-follower Stackelberg game model of the market mechanism. In Section IV, proof for existence of equilibrium, and discussion on conditions to ensure incentive compatibility and individual rationality are presented. Numerical results are discussed in Section V. In Section VI, we make some brief concluding remarks.

II. Reactive Power Ancillary Service Market Mechanism

Generation companies (GENCOs) that participate in reactive power ancillary service market are required to provide their bids that includes submission of their

- Operation cost (\$/MVAr)
- Lost opportunity cost (LOC, \$/MVAr^2)

Independent system operator (ISO) formulates a suitable value function which shall be detailed in Section III. ISO issues a price signal to each participant. Each GENCO then responds to the price signal with their reactive power generation schedule. It is assumed that all GENCOs are synchronised and have a two way communication with ISO as that in transactive control framework.

A. Stages of reactive power ancillary service market mechanism

Stages of the proposed reactive power market model depicted in Fig. 1 is summarized below.

Stage 1: Submission of bids in operation cost and lost opportunity cost format by GENCOs to ISO.

Stage 2: ISO partitions the system into voltage-apparent power coupled areas to determine relevance factor of each GENCO. ISO generates price signal to each GENCO so as to minimize the total payment subject to system operating constraints.

Stage 3: GENCOs respond with their optimal reactive power generation schedule that maximizes their individual profit.

Stage 1 of the proposed market is submission of operation cost and lost opportunity cost bids by GENCOs to ISO. Finding an optimal strategy of cost bids for a GENCO under an oligopoly competition market is not the scope of work discussed in this paper. It is assumed that the GENCOs submit their optimal cost bids to ISO and accordingly ISO issues price signals to the GENCOs. GENCOs then respond with their optimal generation schedule. Thus by issuing appropriate price signal at ISO for each GENCOs, A suitable reactive power generation schedule from GENCOs is achieved by issuing appropriate price signals to them by ISO considering localised nature of reactive power requirement.

B. Relevance of GENCOs in local market areas

Localisation of reactive power market helps in controlling the exercising of market power by GENCOs. The inherent coupling between real and reactive power requirement in the system affects real time pricing mechanism of reactive power as well as active power. Hence partitioning the system into local market areas based on voltage control areas may not be effective. Thus in this work the technique proposed in [11] is adopted to partition the system into voltage-apparent power coupled areas.

In [11], voltage-apparent power coupling between two buses (i and j) has been derived from the bus admittance matrix of the system. Given bus admittance matrix, Y_{bus}

$$Y_{bus} = \begin{bmatrix} Y_{LL} & Y_{LG} \\ Y_{GL} & Y_{GG} \end{bmatrix} \tag{1}$$

The systems of equations governing the network is [12] as

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GM} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \tag{2}$$

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where
$\boldsymbol{Z}_{LL}$ = $\boldsymbol{Y}_{LL}^{-1}$
$\boldsymbol{F}_{LG}$ = -$\boldsymbol{Y}_{LL}^{-1}\boldsymbol{Y}_{LG}$
$\boldsymbol{K}_{GL}$ = $\boldsymbol{Y}_{GL}\boldsymbol{Y}_{LL}^{-1}$
$\boldsymbol{Y}_{GM}$ = $\boldsymbol{Y}_{GG}$ - $\boldsymbol{Y}_{GL}\boldsymbol{Y}_{LL}^{-1}\boldsymbol{Y}_{LG}$

From the bus admittance matrix, a complex matrix, which measures the relative electrical distance between load and generator bus was defined as given by

$$N_G = F_{LG}.Y_{GM}^{-1} \tag{3}$$

Also, load bus voltages are given by

$$V_L = M_L {V_{LL}^*}^{-1} S_L^* + N_G {V_{GG}^*}^{-1} S_G^* = V_L^{|} + V_L^{||} \tag{4}$$

where
$\boldsymbol{M}_L$ = $\boldsymbol{Z}_{LL}$ - $\boldsymbol{F}_{LG}\boldsymbol{Y}_{GM}^{-1}\boldsymbol{K}_{GL}$
$\boldsymbol{N}_G$ = $\boldsymbol{F}_{LG}\boldsymbol{Y}_{GM}^{-1}$
S_G^* = $V_{GG}^* I_G$
$\boldsymbol{V}_{GG}^*$ is defined as gxg diagonal matrix with elements of $\boldsymbol{V}_G^*$ as the diagonal entry.

From equation 4, it is noted that the no-load voltage at a particular load bus is influenced by the apparent power at other load buses and generator buses. The component of no-load voltage at a load bus influenced by other load buses is depicted as $\boldsymbol{V}_L^{|}$) and that by generator buses is defined as $\boldsymbol{V}_L^{||}$). Thus the voltage-apparent power relation between i^{th} generator and j^{th} load was defined as

$$|S_{gi}|/[|V_j^{||}||V_i|] = 1/|N_{G_{ji}}| \tag{5}$$

where
S_{gi} is the apparent power generation at i^{th} bus
$V_j^{||}$ is no load voltage component at j^{th} load bus influenced by apparent power changes in generator bus
V_i is voltage at i^{th} generator bus

The matrix $\boldsymbol{N}_{LG}$ was defined as $N_{L_jG_i} = \frac{1}{|N_{G_{ij}}|}$. This represents voltage-apparent power coupling between generator buses and load buses in the system. From the $\boldsymbol{N}_{LG}$ matrix, we define in this work a 'Relevance factor' (R_i) for GENCO at i^{th} bus as

$$R_i = \frac{\sum_{j \in L} N_{L_jG_i}}{\sum_{m \in G}\sum_{k \in L} N_{L_kG_m}} \tag{6}$$

Where
L is set of load bus in the partitioned subsystem with i^{th} bus as element
G is set of GENCOs in the partitioned subsystem with i^{th} bus as element.

R_i determines the voltage-apparent power coupling of i^{th} generator bus with other buses in the partitioned subsystem, i.e. local market area. Relevance factor (R_i) can be thus used to value the reactive power generation required at a bus to maintain voltage within the permissible limit. When the network configuration changes in real time due to line outage, the local market areas are redefined and accordingly relevance factor for each generation bus changes. The pricing scheme that will include relevance factor of each GENCO will help in proper valuation of reactive power generation requirement in the system to maintain system wide bus voltage within permissible limits.

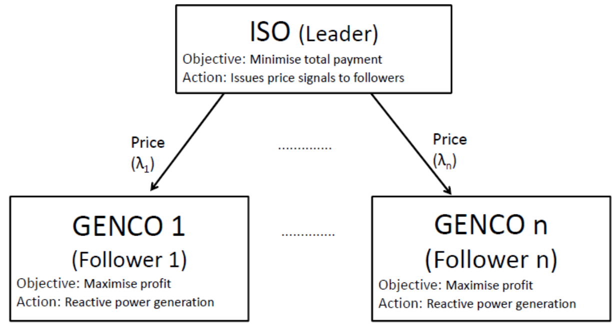

Fig. 2. Single leader multi-follower Stackelberg game

III. Mathematical Problem Formulation

In this section, mathematical model of proposed reactive power market mechanism which is depicted in Fig.2 is detailed.

As mentioned in previous sections, the market is designed as a single leader multi-follower game in which the leader (ISO) issues price signal and followers (GENCOs) respond with their optimal reactive power generation. This section details the mathematical model of leader as well as followers. The game is then solved as a Mathematical Program with Equilibrium Constraints (MPEC).

A. GENCO Model

A generation company participates in reactive power ancillary service market with an objective of maximizing their profit. Let λ_i be the price signal issued by ISO to i^{th} GENCO. Optimization problem at GENCO is to maximize profit subject to reactive power generation constraints.
Maximise

$$P_i = \lambda_i \cdot Qg_i - C_i(Qg_i) \tag{7}$$

such that:

$$Qg_i^{min} \leq Qg_i \leq Qg_i^{max} \tag{8}$$

where
Qg_i is the reactive power generation at i^{th} bus with limits Qg_i^{min} and Qg_i^{max}
$C_i(Qg_i)$ is reactive power cost function for i^{th} GENCO.

Equation (8) sets a lower and upper bound on the reactive power generation of i^{th} GENCO. Thus each GENCO responds to the price signal (λ_i) issued to it by ISO with their optimal reactive power generation that will maximize profit earned. This ensures that the actual cost function remains as a private information in the market mechanism.

B. ISO Model

Consider a market with **G** = {1, 2,....,g} set of GENCOs participating in the reactive power market model with each GENCO aiming at maximizing their profit subject to reactive

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power constraints. GENCOs submit their bids to ISO in the prescribed format which includes:

1) Operation cost (m_{1i})
2) Lost opportunity cost (m_{2i})

The objectives of ISO (leader) in reactive power ancillary service market is to issue a price signal that will:

- Minimize the total payment to GENCOs
- Maintain system wide bus voltage within limits
- Maintain sufficient reactive power reserve

Thus from the bids received in the aforementioned format, ISO determines the utility function that ensures the above objectives.

1) Utility Function of ISO: Sufficient reactive power generation is to be ensured by ISO that will maintain the system wide bus voltage and satisfy reactive power loads while minimizing payment and maintaining adequate reactive power reserve. The utility function of ISO thus should minimize total payment and maintain system wide bus voltage and reactive power reserve in the system. To consider the local nature of reactive power generation in the network, the network is to be partitioned into voltage-apparent power coupled areas to determine the relevance factor of each generation company in the partitioned subsystem. The partitioning scheme and the relevance factor derived in earlier section are incorporated in arriving at utility function of ISO.

The reactive power generation required at i^{th} bus for maintaining bus voltages within the permissible limit is defined as Q_{vi}. The relevance factor (Ri) derived in Section II.B determines the voltage-apparent power coupling of i^{th} generator bus with other buses in the partitioned subsystem, i.e. local market area. Thus relevance factor (Ri), is considered in arriving at the cost function of Q_{vi}. Thus from the bids received from the GENCO at i^{th} bus, a cost function (C_{vi}) pertaining to Q_{vi} is defined as follows

$$C_{vi} = (m_{1i} + m_{2i} \cdot Q_{vi}) \cdot Q_{vi} \cdot R_i \tag{9}$$

where
m_{1i} is the operation cost
m_{2i} is the lost opportunity cost
R_i is the relevance factor of i^{th} bus in a local market area

However, to quantify the reactive power generation required at a bus for maintaining bus voltage is a challenge. In this work, we consider Q_{vi} as a decision variable. Suitable upper and lower limits are derived for Q_{vi} that sets constraints on the decision variable. The derivation of limiting bounds is discussed in Appendix A.

In order to maintain sufficient reactive power reserve in the system, GENCOs are to be encouraged to keep adequate amount as reserve. This can be achieved only if the price signal issued to the GENCO by ISO compensates for the reserve not being auctioned in the market. Without loss of generality, the cost incurred by the GENCO in maintaining reserve is assumed to as the lost opportunity cost (m_{2i}) that GENCO bids. Thus cost function ($Cres_i$) pertaining to reactive power reserve at i^{th} GENCO in a local market area is defined as

$$Cres_i = m_{2i} \cdot (Qg_i^{max} - Qg_i)^2 \tag{10}$$

Hence the utility function for ISO with **G** = {1, 2,....,g} set of GENCOs

$$U_{ISO} = \sum_{i \in G} \lambda_i \cdot Qgi + C_{vi} + Cres_i \tag{11}$$

where
λ_i is the decision variable, i.e. price signal issued to i^{th} GENCO in a local market area with relevance factor R_i.

The utility function as given by Equation (11) has three terms. The first term corresponds to the total payment to GENCOs. The second term is the cost function (C_{vi}) pertaining to Q_{vi} and the third terms is the cost function ($Cres_i$) pertaining to reactive power reserve. The inclusion of C_{vi} in the utility function is to ensure that sufficient reactive power generation/absorption occurs at each generation node so as to maintain system wide bus voltage within limits. Similarly, with the inclusion of $Cres_i$ in the utility function, ISO maintains sufficient reactive power reserve in the system.

C. Single Leader Multi-Follower Game Reactive Power Market Model

With above GENCO model and utility function of ISO, the reactive power market model can be formulated as a Mathematical Program with Equilibrium Constraints (MPEC) that depicts the Stackelberg game between ISO (leader) and GENCOs (followers). The bi-level optimization problem is as follows:

Upper level:
Minimise

$$U_{ISO} = \sum_{i \in G} \lambda_i \cdot Qgi + C_{vi} + Cres_i \tag{12}$$

such that:

$$Pg_i - P_{di} = \sum_{j \in N} |V_i||V_j|Y_{ij}cos(\theta_{ij} + \delta_j - \delta_i) \quad \forall i : \alpha_i \tag{13}$$

$$Qg_i - Q_{di} = -\sum_{j \in N} |V_i||V_j|Y_{ij}sin(\theta_{ij} + \delta_j - \delta_i) \quad \forall i : \beta_i \tag{14}$$

$$Pg_i^{min} \leq Pg_i \leq Pg_i^{max} \quad \forall i \in G : \gamma_{imin}, \gamma_{imax} \tag{15}$$

$$-S_l^{max} \leq S_l \leq S_l^{max} \quad \forall l \in L : \zeta_{lmin}, \zeta_{lmax} \tag{16}$$

$$|V_i^{min}| \leq |V_i| \leq |V_i^{max}| \quad \forall i : \eta_{imin}, \eta_{imax} \tag{17}$$

$$\sum_{j \in N} \frac{X_{ij}(P_{ij})^2}{|V_j|^2} \leq Q_{vi} \leq \sum_{j \in N} \frac{(V_{ref}^2 - |V_i|^2 + 1)}{X_{ij}} \quad \forall i : \rho_{imin}, \rho_{imax} \tag{18}$$

and subject to
Lower level:
Maximise

$$P_i = \lambda_i \cdot Qg_i - C_i(Qg_i) \quad \forall i \in G \tag{19}$$

such that:

$$Qg_i^{min} \leq Qg_i \leq Qg_i^{max} \quad \forall i \in G \tag{20}$$

where
i, j are indices for buses

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l is index for line
G is the set of GENCOs
L is the set of all lines in the system
N is the set of all buses in the system
X is the bus reactance matrix
Y is the bus admittance matrix
Pg_i is the active power generation at i^{th} bus with limits Pg_i^{min} and Pg_i^{max}
Qg_i is the reactive power generation at i^{th} bus with limits Qg_i^{min} and Qg_i^{max}
V_i is the voltage magnitude at i^{th} bus bounded by V_i^{min} and V_i^{max}
θ is the angle associated with bus admittance
δ is the bus voltage angle
P_{di} is the active power demand at i^{th} bus
Q_{di} is the reactive power demand at i^{th} bus
S_l is the apparent power flow through line l

Equation (13) and (14) are the load flow constraints. Equation (15,16,17) provide the active power generation limit constraints, transmission line limit constraint and bus voltage magnitude limits respectively. Equation (18) provides the bounds on the decision variable Q_{vi}. The lower level optimization problem of GENCOs optimal reactive power generation is in Equation (19,20).

The lower level optimisation problem alone, can be further analysed as a Cournot Competition among GENCOs that play a non-cooperative game among each other. This analysis is not discussed in this paper as the focus and purpose of this work is the entire bi-level optimisation problem that results in a Single-Leader Multi-Followers Game among ISO and GENCOs. However, the equilibrium analysis of the Cournot Competition among GENCOs is straightforward with the convex nature of objective function and compactness of feasible set. Hence the Cournot competition of GENCOs is simplified and presented as single lower level problem in the above problem formulation.

D. Solution using Mathematical Program with Equilibrium Constraints (MPEC)

While ISO minimises its utility function, it takes into account the equilibrium conditions of GENCOs (follower). The bilevel optimisation problem is then formulated as a single level optimization problem by incorporating the Karush-Kuhn-Tucker(KKT) conditions of the GENCOs' problem. The equilibrium is derived from the solution obtained by combining KKT conditions of MPEC formulation. Consider the GENCO optimisation problem described through Equations (25) and (8). The Lagrangian is defined as:

$$\begin{aligned}\mathcal{L}(Qg_i, \mu_{imin}, \mu_{imax}) = \lambda_i \cdot Qg_i - C_i(Qg_i) - \\ \mu_{imin} \cdot (Qg_i^{min} - Qg_i) - \\ \mu_{imax} \cdot (Qg_i - Qg_i^{max})\end{aligned} \tag{21}$$

Let dual variable set associated with Equations (13-18) be [α_i, β_i, γ_{imin}, γ_{imax}, ζ_{lmin}, ζ_{lmax}, η_{imin}, η_{imax}, ρ_{imin}, ρ_{imax}]. The dual variables associated with KKT conditions of i^{th} GENCO given by Equation (B.2) through (B.8) are [σ_i, ϕ_{imin}, ϕ_{imax}, ω_{imin}, ω_{imax}, ψ_{imin}, ψ_{imax}]. It can be observed that ϕ_{imin} and μ_{imin} are associated with the same constraint, i.e. lower bound of reactive power generation at i^{th} bus. Similarly, ϕ_{imax} and μ_{imax} are matched with the upper limit constraint on reactive power generation. This leads to a nonsquare nonlinear complementarity problem where in different dual variables are matched with same constraints. This can be solved by applying smoothing technique to complementary slackness conditions. This can also be solved to obtain the equilibria using methods in [13]. In this work, smoothing technique using Fischer–Burmeister function [14] is applied to Equations (B.7) and (B.8). The remaining constraints are well behaved and the NLP can be solved by available commercial softwares like GAMS optimisation platform.

IV. Game Theoretical Analysis

In the previous section, formulation of single leader-multi follower game model for reactive power market was discussed in detail. In this section, the equivalence of MPEC formulation with potential form game is detailed.

The optimal value of U_{ISO} in Equation (11) is:

$$U_{ISO}^* = inf \sum_{i \in G} \lambda_i \cdot Qgi + C_{vi} + Cres_i \tag{22}$$

$$\sum_{i \in G} inf[\lambda_i \cdot Qgi + C_{vi} + Cres_i] \leq U_{ISO}^* \tag{23}$$

$$\sum_{i \in G} inf \quad \mathcal{T}_i \leq U_{ISO}^* \tag{24}$$

where

$$\begin{aligned}\mathcal{T}_i = \lambda_i \cdot Qgi + (m_{1i} + m_{2i} \cdot Q_{vi}) \cdot Q_{vi} \cdot R_i \\ + m_{2i} \cdot (Qg_i^{max} - Qg_i)^2\end{aligned} \tag{25}$$

$\lambda_i \cdot Qgi$ is the optimal payment that links the decision variables of leader and followers and is a positive quantity. $\mathcal{T}_i$ can be considered as the utility function of a pseudo leader, which issues prices signal to i^{th} GENCO alone subject to constraints from Equation (13-20). This implies that each GENCO (follower) receives its price signal from a pseudo leader whose utility function is given by $\mathcal{T}_i$. The number of pseudo leaders is same as the number of followers, i.e. GENCOs participating in the network. Thus the single leader-multi follower game can be viewed as a multi leaders-multi followers game (MLMF) with shared constraints. The pseudo leaders, each with $\mathcal{T}_i$ as utility function form the multi-leaders set and GENCOs form the multi-followers set. The existence of equilibrium/equilibria of the proposed single leader-multi follower game can be thus proven using potential function game representation of MLMF game [15].

Lemma I: $\pi(x)$ is defined as a potential function for a function *f(x)*, $x \in$ X, if for all $x' \in$ X, $\pi(x)$ - $\pi(x')$ = *f(x)* -*f(x')*.

In $\mathcal{T}_i$,the utility function of the i^{th} pseudo leader that issues price signal to i^{th} GENCO in the MLMF game, the bilinear term $\lambda_i \cdot Qgi$ is approximated with first order Taylor series expansion around (λ_i,Qgi) values in the previous time slot of market operation. Let 'l' be value of λ_i and 'q' be value of Qgi in the previous market clearing. Thus on applying Taylor

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series expansion around the point (l,q) to the optimal payment term $\lambda_i \cdot Qgi$ in Equation (25),

$$\mathcal{T}_i = q \cdot \lambda_i + l \cdot Qg_i - l \cdot q + m_{2i} \cdot R_i \cdot Q_{vi}^2 + m_{2i} \cdot (Qg_i^{max})^2 + m_{2i} \cdot Qg_i^2 \tag{26}$$

It may be noted that GENCOs may either generate or absorb reactive power and thus the term $l \cdot Qg_i$ may not always be positive, thereby violating the condition of optimal payment term to be positive. This is rectified by squaring the terms and without loss of generality, approximate utility function of ith pseudo leader is defined as,

$$\mathcal{T}_i = \lambda_i^2 + m_{2i} \cdot R_i \cdot Q_{vi}^2 + m_{2i} \cdot (Qg_i^{max})^2 + [m_{2i} + 1] \cdot Qg_i^2 - l \cdot q \tag{27}$$

By Lemma I, the potential function π for the utility function $\mathcal{T}_i$ is defined as,

$$\pi = \lambda_i^2 + m_{2i} \cdot R_i \cdot Q_{vi}^2 + [m_{2i} + 1] \cdot Qg_i^2 \tag{28}$$

Theorem I (Existence of equilibrium): Suppose the set of fixed points in the feasible region of multi-leaders multi-followers game based reactive power market is non-empty. If utility function of each leader is continuous, then MLMF game with potential function π has an equilibrium if

1) The potential function π is coercive or
2) The constraint space is compact

i.e. There exists a minimiser for the potential game defined by the potential function π, which defines the equilibrium of the MLMF game.

Proof: It is to prove that for the existence of equilibrium in MLMF games with shared constraints, it is required to prove the existence of potential function π which is coercive. Thus the minimiser for potential game exists and by the definition of potential function an equilibrium for the MLMF game also thus exists [15].

The coercivity of the potential function π defined in equation (28) is now straightforward. Hence a minimiser for the potential game exists which also implies the existence of equilibrium for the proposed Stackelberg game based reactive power market model.

Regarding uniqueness of the equilibrium point, for a potential game to achieve a global minimiser, the following conditions are to be satisfied

1) π is convex
2) The set of fixed points within the feasible region of the MLMF game is also convex

Convexity of π is direct from the definition in equation (28). However the non-convexity in the ISO system constraints can guarantee only a local minima. The non convexity can be handled using suitable convexification techniques,and with feasible set compactness it is possible to achieve global minimiser for the potential game, thereby arriving at a unique equilibrium in the MLMF game. Hence, the existence of equilibrium is proven however uniqueness cannot be guaranteed due to non-convexity. Convexification techniques help in arriving at approximate solutions.

A. Incentive Compatibility and Individual Rationality

In proposed reactive power market, price signal issued by ISO to each GENCO depends on marginal cost function bids submitted by GENCOs. Consider a market with tarif mechanism as $t(\theta)$ and $x(\theta)$ as the decision variable where θ is the type of the participant,. Then allocation $y = (x(\theta), t(\theta))$ is said to be direct revelation mechanism if agents in the market simultaneously and truthfully announce their types. Thus such a mechanism that induces truth-telling behavior among the agents (GENCOs) to declare their actual type (marginal cost function) is said to be incentive compatible.

Theorem II (Incentive Compatibility): Let λ_{ij} be the price signal issued to i^{th} GENCO with marginal cost function θ under the single leader multi-followers Stackelberg game based reactive power market model. The mechanism is said to be incentive compatible if

1) $\lambda_{ij} = a.\theta + b$
2) $a > 1$
3) $b > (1\text{-}a).\theta$

which implies that the decision variable of the followers, i.e. reactive power generation is implementable.

Proof: We first discuss the following Lemma to derive at the conditions for the proposed mechanism to be incentive compatible.

Lemma II: A decision function $x(\theta)$ is implementable if and only if it is monotone (nondecreasing). Also, if $x(\theta)$ is implementable there exists a tariff mechanism $t(\theta)$ such that allocation $y = (x(\theta), t(\theta))$ is incentive compatible [16].

An equivalent to monotonicity condition on the decision function are [16]:

- Sorting condition or Spence-Mirrlees condition, which is expressed as

$$\frac{\partial}{\partial \theta}\left(\frac{\partial u/\partial x}{\partial u/\partial t}\right) \geq 0 \tag{29}$$

where u is the utility function of the agent with decision function x.

- There exists a p and q such that,

$$\left|\frac{\partial u/\partial x}{\partial u/\partial t}\right| \leq p + q\,|t| \tag{30}$$

uniformly over x, t, θ

By Lemma II, a monotone non decreasing decision function is implementable. The equivalence to monotonicity of a function is given by Equations (29) and (30). Hence decision function ($x(\theta)$) that satisfies Equations (29) and (30) is implementable. Thus Lemma II implies that, the allocation ($x(\theta)$, $t(\theta)$) is incentive compatible.

In the proposed mechanism, the price signal issued to a GENCO with marginal cost function θ is tariff and corresponding optimal reactive power generation is the decision function. Then the utility function for the GENCO with marginal cost function θ is defined as

$$u = t(\theta).x(\theta) - \int_0^{x(\theta)} \theta(x)dx \tag{31}$$

Thus $x(\theta)$ be implementable by satisfying the conditions provided in Equation (29) and (30) which is achieved with $t(\theta) = a.\theta + b$ and the following:

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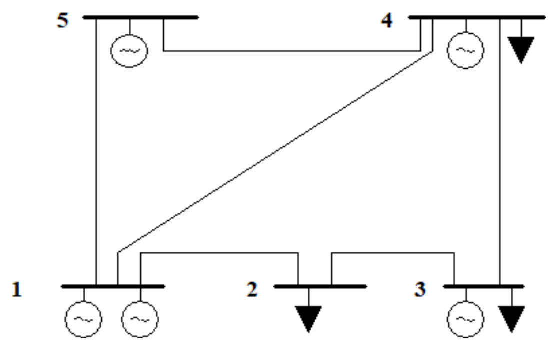

Fig. 3. PJM 5-bus system

- $a > 1$
- $b >$ (1-a).θ

By imposing the above conditions on the price signal issued by ISO to each GENCO, under equilibrium, incentive compatibility is achieved in the proposed reactive power market mechanism.

From the conditions derived in *Theorem II*, under equilibrium it can be noticed that the price received

$$\lambda = a.\theta + b > a.\theta + (1-a).\theta > \theta \quad (32)$$

The mechanism induces a truth-telling behavior among GENCOs under equilibrium. And at the same time the GENCOs are assured a non-negative profit. Thus individual rationality is preserved. This thus helps in mitigating the exercise of market power in reactive power market.

V. Simulation Results

In Section V-A, PJM 5-bus system [17] is considered to illustrate economic efficiency of the proposed method by comparing with locational marginal price (LMP) formulation. In Section V-B, IEEE 30-bus test system is considered for testing proposed localized reactive power market model A practical system like Nordic 32-bus system is presented in Section V-C.

A. PJM 5-bus System Illustrative Example

Consider PJM 5-bus system as shown in Fig. 3 with cost function provided in Table I. Proposed market model is compared with locational marginal price based model. LMP model considered in this work is based on AC-OPF formulation with ISO utility function as objective function. To demonstrate economic efficiency of the proposed system, we consider the case when generator at bus 1 reports a cost function different from actual (magnifies their actual cost of production by factors ranging from 1 to 6). Remaining GENCOs report their actual cost.

The actual operation cost of reactive power generation at Bus 1 is 0.86$/100MVAr. When it bids its actual operation cost and lost opportunity cost, the price received through LMP mechanism is 0.234$/100MVAr. Which is much lesser than its actual operation cost. Thus GENCOs tend to magnify their actual cost while bidding. When GENCO at bus 1 bids an operation cost of 5.16$/100MVAr which is six times

TABLE I
Reactive Power Cost for Generators in PJM 5-bus System

Bus	Operation cost ($/100MVAr)	Lost opportunity cost ($/100MVAr^2)
1	0.86	0.46
1	0.86	0.46
3	0.68	0.39
4	0.75	0.43
5	0.6	0.5

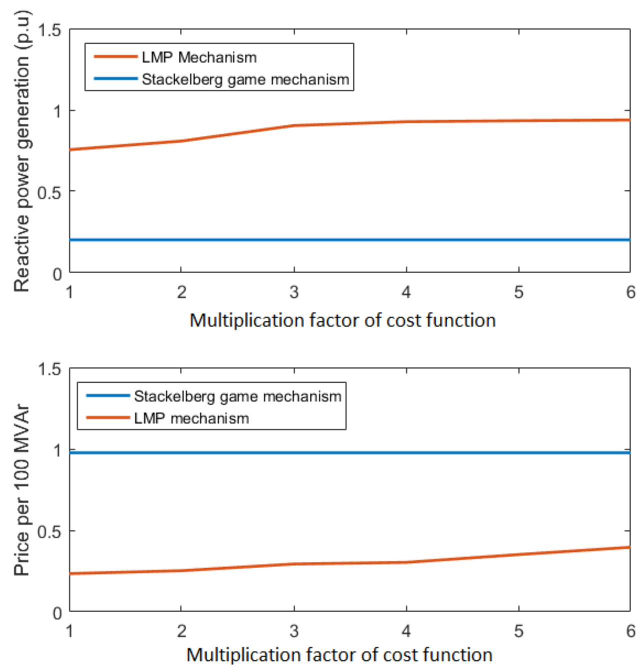

Fig. 4. Price per 100MVAr and p.u reactive power generation at bus 1 when cost function was magnified during bid.

the actual operation cost, under LMP mechanism the price received is only 0.396$/100MVAr,thereby GENCO further magnifying their actual cost. Where as in the proposed Stackelberg game model, the price received by GENCO bus 1 is 0.976$/100MVAr. The same price is received even if GENCO magnifies its actual cost while bidding. Hence generation company does not again additional benefit by bidding false cost functions. The price received under Stackelberg game model is 13.48% higher than the operation cost. Thus by bidding actual operation cost, GENCOs receive a non-negative profit thereby preserving individual rationality. The price received per 100 MVAr and the reactive power generation under both mechanism is compared in Fig. 4.

The total cost of reactive power generation(quadratic cost function) at bus 1 under LMP mechanism is compared with Stackelberg game model. The cost of reactive power generation when GENCO bids its actual cost under LMP based market is 6.84 times the cost under Stackelberg game based market. It is observed that when generation at Bus 1 magnifies their actual cost of production by factors ranging from 1 to

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TABLE II
REACTIVE POWER COST OF GENERATORS IN IEEE 30-BUS SYSTEM

Bus	Operation cost ($/100MVAr)	Lost opportunity cost ($/100MVAr^2)
1	0.86	0.46
2	0.68	0.39
13	0.75	0.43
22	0.6	0.5
23	0.75	0.90
27	0.73	0.38

6, cost of reactive power generation increases under LMP mechanism, where as in Stackelberg game mechanism cost is much less.

While GENCOs exhibit a truth telling behavior, from Fig. 4 it can be observed that per unit price is much higher in Stacklberg game mechanism than that received in LMP mechanism. Thus individual rationality is achieved. Under Stackelberg game mechanism, GENCO does not get any additional benefit by magnifying its cost function during bidding as it receives same profit. This demonstrates incentive compatibility of the proposed pricing mechanism.

B. IEEE 30-bus Test System

Consider IEEE 30-bus system with 6 generator buses [18]. Each generator bus is considered as a generation company. Thus the reactive power market has 6 GENCOs participating in the market. The actual operation and lost opportunity costs of these 6 GENCOs are provided in Table. II.

The proposed market mechanism is implemented in IEEE 30- bus system using GAMS optimisation platform. Three cases were considered for study in the test system

- Base case
- Outage of Line 12-15
- Outage of Line 21-22

In base case, line 12-15 was a moderately loaded where are line 21-22 was heavily loaded. Hence these two lines were considered for investigating contingency cases.

System is partitioned into voltage-apparent power coupled areas using isoperimetric partitioning technique [11]. The advantage of using this partitioning technique is that it incorporates inherent coupling between active and reactive power, thereby making it suitable for real time market operations. The GENCO participants in each market area for all three cases considered, is tabulated in Table. III with the corresponding relevance factor.

The total voltage deviation (TVD) given by [19] is evaluated in all three cases of the system and is tabulated in Table. IV.

$$TVD = \sqrt{\sum_{i \in N} (|V_i| - V_{ref})^2} \tag{33}$$

where
V_{ref} is the reference voltage at a bus

TABLE III
LOCAL REACTIVE POWER MARKET AREAS IN IEEE 30-BUS SYSTEM

Gen. bus	Base case		Line 12-15 outage		Line 21-22 outage	
	Area	Relevance factor	Area	Relevance factor	Area	Relevance factor
1	1	0.253	1	0.254	1	0.3367
2	1	0.253	1	0.254	1	0.3366
13	1	0.246	1	0.245	1	0.3267
22	1	0.247	1	0.246	2	0.3294
23	2	0.488	2	0.495	2	0.3302
27	2	0.512	2	0.504	2	0.3405

TABLE IV
SYSTEM PARAMETERS IN IEEE 30-BUS SYSTEM

Parameter	**Base case**	**Line 12-15 outage**	**Line 21-22 outage**
TVD	0.0618	0.0659	0.089
Net MVAr generation	116.609	116.786	121.647
Total MVAr reserve	289.291	289.114	284.253

TABLE V
REACTIVE POWER PRICE PER P.U (A) AND MVAR GENERATION IN P.U (B) IN IEEE 30-BUS SYSTEM

Gen. bus	Base case		Line 12-15 outage		Line 21-22 outage	
	A	B	A	B	A	B
1	0.932	0.15277	0.938	0.15952	0.884	0.10158
2	0.905	0.31879	0.899	0.31038	0.963	0.39239
13	0.813	0.13134	0.808	0.12535	0.821	0.14189
22	1.022	0.34736	1.022	0.34692	1.005	0.32755
23	0.932	0.12111	0.947	0.12945	1.001	0.15967
27	0.670	0.09472	0.672	0.09624	0.670	0.09339
Payment/p.u		1.069	1.071		1.135	
Average price/p.u		0.879	0.881		0.891	

Table. IV also provides total reactive power generation and reactive power reserve available for all three cases considered in the system. The price per unit and reactive power generation at each GENCO in three cases is tabulated in Table V.

Observations and inference

It can be observed that, the system wide-bus voltage is maintained within the limits and voltage deviation is minimised with the proposed utility function. This is achieved by ensuring sufficient reactive power generation in the system. However, for safe operation of system it is necessary to maintain reactive power reserve as well. From Table IV it is observed that adequate reserve is available even under Line 21-22 (heavy loaded line) outage condition. The price volatility is controlled with the proposed market mechanism under system contingencies as observed from Table V. The per unit price varies with system conditions which is determined by defining local market areas and relevance factor. When system configuration changes due to contingeny like line outage, local market areas also re-configure with respect to that in base case. Accordingly, relevance factors of generator buses change which reflect in prices per unit of GENCOs.

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TABLE VI
SIMULATION RESULTS IN NORDIC 32-BUS SYSTEM

Parameter	**Base case**	**Double circuit 4022-4031 outage**	**Single circuit 4031-4041 outage**
TVD	0.3036	0.3059	0.3103
Net MVAr generation	3621.763	3770.696	3714.41
Total MVAr reserve	3553.237	3404.304	3460.59
Average price/p.u	1.07	1.14	1.15

C. Nordic 32-bus Test System

A practical system like Nordic 32-bus system [20] is considered for studying. Three cases studied in the system are. Case 1: Base case of Nordic 32-bus system. Case 2: Outage of double circuit line between buses 4022 and 4031. Case 3: Outage of single circuit tie-line between buses 4031 and 4041.

Table. VI provides total voltage deviation, total reactive power generation and reactive power reserve available for all three cases considered in the system. The average price in each case is also tabulated.

It can be observed that the system parameters are within permissible limits and sufficient reserves are maintained in the system, during line outages. The average price does not increase drastically with contingencies in the system. Thus, the market mechanism can aid the existing local voltage regulation schemes for an improved voltage profile in the system.

Thus Stackelberg game based reactive power market mechanism based on proposed ISO utility function incorporates relevance factor for each GENCO according to system operating conditions and has the following advantages:

1) Improvement in system-wide bus voltage, i.e. Total voltage deviation is reduced.
2) Maintenance of sufficient reactive power reserve during contingency as well.
3) Price per unit of reactive power does not increase drastically under contingencies by suitable system partitioning and considering relevance factor of each GENCO based on partitioned system.
4) Incentive compatibility and individual rationality is preserved.

VI. CONCLUSION AND FUTURE WORK

In this work, a novel reactive power ancillary service market mechanism is modeled as a single leader multi-followers Stackelberg game in which ISO is considered as leader and GENCOs participating in market as followers. Localised nature of reactive power market is incorporated by determining the relevance factor for each GENCO. Existence and uniqueness of equilibrium in the proposed market mechanism is discussed. The market mechanism is proven to be incentive compatible and ensures individual rationality.

The mechanism induces truth telling behavior of GENCOs, which will yield them a non-negative profit. Economic efficiency achieved through incentive compatibility and individual rationality is illustrated on PJM 5-bus system. Improvement in system parameters and curtailment of price volatility is detailed using IEEE 30-bus system. Thus it can be concluded that, the proposed market mechanism is capable of improving system -wide bus voltage magnitude and maintaining sufficient reactive reserve in the system under contingency as well without issuing highly volatile price signals. Incentive compatibility and individual rationality features make the mechanism economically efficient. The future work in progress includes finding optimal bidding strategy on this market mechanism. The work can also be extended to a simultaneous active-reactive power market model.

APPENDIX A
REACTIVE POWER REQUIREMENT FOR VOLTAGE SUPPORT

Consider two buses i and j in an N bus system.

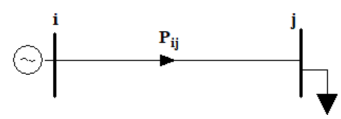

Fig. 5. Generator -load bus system.

The reactive power generation (Q_i) required at i^{th} bus when there is no reactive power generation at j^{th} bus is given by

$$Q_i = \frac{V_i^2 - V_j^2}{X_{ij}} \tag{A.1}$$

The minimum reactive power (Q_{vi}^{min}) required for shipment of active power at i^{th} bus [21] is given by

$$Q_{vi}^{min} = \sum_{j \in N} \frac{X_{ij}(P_{ij})^2}{V_j^2} \tag{A.2}$$

where
P_{ij} is the active power flow between i^{th} bus and j^{th} bus
X_{ij} is the $(i,j)^{th}$ element of bus reactance matrix
V_j is the voltage at j^{th} bus
Equation A.2 is considered as the lower bound for reactive power required for voltage support. Let $Vref_i$ be the desired voltage at i^{th} bus. Then reactive power required to maintain desired voltage at i^{th} bus is

$$Qref_i = \frac{Vref_i^2 - Vref_j^2}{X_{ij}} \tag{A.3}$$

Thus the additional reactive power generation required at i^{th} bus, if all buses connected to it are load bus with no reactive power support is given by

$$\Delta Q_i = \sum_{j \in N} \left[\frac{Vref_i^2 - |V_i|^2}{X_{ij}} + \frac{|V_j|^2 - Vref_j^2}{X_{ij}} \right] \tag{A.4}$$

However to determine the actual reactive power generation required for voltage support alone at the bus in real time is challenging. Hence in this work an upper bound is set by considering the worst case for value of ($V_j^2 - Vref_j^2$). Thus upper bound for reactive power generation (Q_{vi}^{max}) required at i^{th} bus is set as

$$Q_{vi}^{max} = \sum_{j \in N} \frac{Vref_i^2 - |V_i|^2 + 1}{X_{ij}} \tag{A.5}$$

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Thus reactive power generation required at i^{th} bus in a localised market area, for maintaining bus voltages, defined as Q_{vi} is a decision variable for ISO model with lower and upper limits set by Q_{vi}^{min} and Q_{vi}^{max} respectively.

Appendix B
KKT conditions of GENCO's optimisation problem

Consider the GENCO optimisation problem described through Equations (25) and (8). The Lagrangian is defined as:

$$\mathcal{L}(Qg_i, \mu_{imin}, \mu_{imax}) = \lambda_i \cdot Qg_i - C_i(Qg_i) - \mu_{imin}.(Qg_i^{min} - Qg_i) - \mu_{imax}.(Qg_i - Qg_i^{max}) \quad \text{(B.1)}$$

The Karush-Kuhn Tucker (KKT) conditions at optimal value of reactive power generation (Qg_i^*) are

1) Stationary point

$$\lambda_i - C_i'(Qg_i^*) + \mu_{imin} - \mu_{imax} = 0 : \sigma_i \quad \text{(B.2)}$$

where
$C'_i(Qg_i^*)$ is the first derivative of the cost function evaluated at Qg_i^*.

2) Primal feasibility condition

$$Qg_i^{min} - Qg_i^* \leq 0 : \phi_{imin} \quad \text{(B.3)}$$

$$Qg_i^* - Qg_i^{max} \leq 0 : \phi_{imax} \quad \text{(B.4)}$$

3) Dual feasibility condition

$$\mu_{imin} \geq 0 : \omega_{imin} \quad \text{(B.5)}$$

$$\mu_{imax} \geq 0 : \omega_{imax} \quad \text{(B.6)}$$

4) Complementary slackness

$$\mu_{imin}.(Qg_i^{min} - Qg_i^*) = 0 : \psi_{imin} \quad \text{(B.7)}$$

$$\mu_{imax}.(Qg_i^* - Qg_i^{max}) = 0 : \psi_{imax} \quad \text{(B.8)}$$

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