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Suppressing the pressure fluctuations caused by drill string vibration by SP-EID method

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Abstract— **During drilling process, the axial vibration of the drill string always causes the fluctuations of the bottom hole pressure, which may lead to serious accidents. At the same time the bottom hole pressure control system also has the phenomenon of parameter change and the feedback delay, so that the pressure is difficult to control. In view of this situation, we develop a method combining Smith Predictor with equivalent input disturbance(SP-EID) approach to achieve the disturbance suppression and delay compensation. First, considering the changes in the model parameters, we improve the existing hydraulic model to describe the dynamic characteristics of the bottom hole pressure, at same time the mathematical form of the disturbance is given. Then we use the Smith Predictor to compensate the delay of the feedback channel and apply EID method to suppress pressure disturbances. Finally the SP-EID method is used to adjust the mud flow on the ground to control the bottom hole pressure. The simulations are conducted to compare our method with SMC method, in order to show the superiority and effectiveness of our method.**

I. INTRODUCTION

During the drilling process, the drilling fluid or mud fluid the is pumped the drill string, out of the bit, through the annulus back to the ground, and finally returned to the mud pool for purification after recycling. In the process, the drilling fluid is used to cool and clean drill bit, carry the the cuttings back to the ground [1], balance the formation pressure, maintain wellbore stability [2] and so on. In conventional drilling, the adjustment of the bottom hole pressure is usually carried out by adjusting the chemical composition of the mud or changing the flow rate of the mud pump. In the managed pressure drilling, the operator adjusts the flow rate by changing the opening of the valve to quickly control the bottom hole pressure [3].

However, abnormal fluctuation of drill string often exist in the drilling process. These fluctuations are ususlly caused by heave-induced [4] and longitudinal vibration of the drill string during drilling process [5]. If these fluctuations are not handled in time, they will prolong drilling time, reduce efficiency and even cause accidents.

But at the same time, due to changes in drilling parameters (drilling fluid rheology on the drill string vibration, bottom hole pressure and other parameters on the impact of drilling fluid rheology, and high temperature and pressure lead to bottom pressure uncertainty)[6-8], the operator is difficult to control the bottom hole pressure.

Some scholars have studied the method of suppressing bottom hole pressure disturbance. In [9], a Riemannian invariant is used to linearize and decouple the coupling model which describes the bottom flow and pressure by the nonlinear partial differential equation, and realize the good control effect. However, the model has the characteristics of linear time-invariant. In [10], the lumped parameter model is used to suppress the interference under experimental conditions; The experimental device is used to simulate the drilling environment, and the MPC method is used to suppress the disturbance of the bottom hole pressure [11]. However, the parameter change was not taken into account in the model. In [12] use output feedback and state feedback method to suppress the heave-induced pressure disturbance, and achieve good control effect. [13] also use MPC method to suppress the heave disturbance, compared with PID method, MPC method achieve a better control effect.

Most of these scholars focus on the changes in the bottom hole pressure oscillation due to the drill string fluctuations, but they do not consider the parameter are time-varying in the model. And also, there exists delay in the feedback channel, because the feedback signal is transmitted by mud pulse or a wired drill string through the annulus. All of these lead to bottom hole pressure is difficult to control.

So in this article we develop the SP-EID method to deal with the problems. The SP is used to compensate the delay of the feedback channel, and the EID control method for models with time-varying parameter to suppress disturbance in [14], which have the following characteristics: 1) Simple configuration, 2) no requirement for the differentiation of measured outputs and 3) easy to stabilize [15]. The method can suppress the disturbance for the time-varing parameters bottom hole pressure system with feedback delay. [16] also use the way combine SP and EID method, but the method can not handle delay on the feedback channel very well.

In this paper, under the inspiration of the bottom hole pressure oscillation and EID method, we propose an estimated control method based on the bottom hole pressure disturbance. In the face of the parameter uncertain system, we can effectively control the flow rate to make the disturbance minimize. The rest of this paper is organized as follows: The second part introduces the form of MPD model and disturbance; the third part gives the control method and its configuration of the system; the fourth and fifth parts give

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the simulation results and conclusion.

II. MODEL AND DISTURBANCE TYPE

We consider the hydrodynamic model from [17], the model refer to [18] and [19], which consists of the following.

$$\frac{V_d}{\beta_d}\dot{P}_{pump}(t) = q_{pump} - q_{bit}(t) \quad (1a)$$

$$\frac{V_a}{\beta_a}\dot{P}_{choke}(t) = -q_{choke}(t) - \dot{V}_a + q_{bit}(t) + q_{res} + q_{back} \quad (1b)$$

$$M_a\dot{q}_{bit}(t) = p_{bit}(t) - p_{choke}(t) - F_a(q_{bit}(t)+q_{res})^2 - \rho_a g h_{bit} \quad (1c)$$

$$M_d\dot{q}_{bit}(t) = p_{pump}(t) - p_{bit}(t) - F_d q_{bit}^2(t) + \rho_d g h_{bit} \quad (1d)$$

$$q_{choke}(t) = K_c z_c(t) \quad (1e)$$

$$z_c(s) = F(s)u_c(s) \quad (1f)$$

Among them, P_{pump} and P_{choke} are the pressures at the pump and the chock, respectively; q_{pump}, q_{choke}, q_{back}, q_{bit} and q_{res} are flow rates through the pump, chock, back pipe, drilling bit and reservoir. u_c is choke opening command, $F(s)$ is the transfer function for the dynamic characteristics of the choke valve, z_c is the actual chock opening.

The other parameters V_d and β_d represent the volume and bulk modulus of the drill string, respectively; V_a and β_a represent the volume and bulk modulus of the annulus, respectively; ρ_a and ρ_d represent the density of the mud fluid in the annulus and the drill string, respectively; M_a and M_d respectively represent the density of the annulus and the drill string per meter; F_a and F_b represent the coefficient of friction between drill string and annulus, respectively; h_{bit} indicates the depth of the drill bit; $\dot{V}_a$ represents the volume change rate of the annulus. Some of the above parameters have some uncertainty or some constants with known conserved boundaries, which are treated as constants in this context, or only consider their slow changes, and we consider we control the p_{bit} by manipulating q_{choke} directly.

By combining the formula (1c) with (1d) we get

$$\begin{aligned}(M_a+M_d)p_{bit} &= M_a(p_{pump}(t) - F_d q_{bit}^2(t) + \rho_d g h_{bit}) \\ &+ M_d(p_{choke}(t) + F_a(q_{bit}(t)+q_{res})^2 + \rho_a g h_{bit})\end{aligned} \quad (2)$$

Taking time derivative of (2) and plugging in (1a) (1b) and (1d), we get dynamic properties of p_{bit}.

$$\dot{p}_{bit}(t) = \frac{M_a\beta_d}{(M_a+M_d)V_d}(q_{pump} - q_{bit}(t)) + \frac{M_d\beta_a}{(M_a+M_d)V_a}(-q_{choke} - \dot{V}_a + q_{bit}(t) + q_{res} + q_{back}) + \frac{2(M_dF_a(q_{bit}(t)+q_{res}) - M_aF_dq_{bit}(t))}{(M_a+M_d)M_d}(P_{pump}(t) - p_{bit}(t) - F_d q_{bit}^2(t) + \rho_d g h_{bit})$$

In this process, the $q_{bit}(t)$ can be seen as slow to change, and retain its dynamic characteristics. In the process of calculation, $q_{bit}(t)$ is treated as a constant. P_{pump} is regarded as constant and the volume of annulus is considered constant, the model is converted to

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + e \\ y(t) = Cx(t) \end{cases} \quad (3)$$

Considering the change of parameters in the system, we set the parameter variations of the input matrix and control matrix satisfy

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) \end{bmatrix} = ME(t)\begin{bmatrix} N_0 & N_1 \end{bmatrix} \quad (4)$$

In [5], the longitudinal vibration of the drill string during drilling can be expressed in the form of sine and cosine, so the model can be rewritten as

$$\begin{cases} \dot{x}(t) = (A+\Delta A(t))x(t) + (B+\Delta B(t))u(t) + e + f(t) \\ y(t) = Cx(t) \end{cases} \quad (5)$$

where e is the constant value, $f(t) = k * \sin(wt)$, N_0, N_1, and M are known constant matrices with appropriate dimensions, $E(t)$ is an unknown matrix and satisfies

$$E^T(t)E(t) \le I, \quad \forall t > 0 \quad (6)$$

III. PRESSURE CONTROL BASED ON SP-EID METHOD

First, the EID method proposes to estimate the effect of disturbance on the input channel, rather than the disturbance itself. This method enlighten us to estimate the uncertainty of the bottom hole pressure fluctuation on the input channel, thus simplifying the control configuration.

A. Estimation of bottomhole pressure fluctuation as a flow input

The state space equation can be obtained from the above equation:

$$\begin{cases} \dot{x}(t) = (A+\Delta A(t))x(t) + (B+\Delta B(t))u(t) + B_d d(t) \\ y(t) = Cx(t) \end{cases} \quad (7)$$

where $d(t) = f(t) + e$. The system meet the prerequisites for using the method in [15]. A more detailed demonstration will be presented in part four.

The structure of the control system is shown in Fig. 1. The structure includes six parts: the plant, the internal model, the state observer, the state feedback controller, the disturbance estimator and the Smith Predictor.

Internal model:

$$\dot{x}_R(t) = A_R x_R(t) + B_R[r(t) - y(t)] \quad (8)$$

The internal model controller is used to track reference input accurately. Rebuilding the plant by using the Longberg full-scale observer

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu_f(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases}$$

The state feedback control law is

$$u_f(t) = K_R x_R(t) + K_p\hat{x}(t) \quad (10)$$

Transforming the plant into the following form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + [\Delta A(t)x(t) + \Delta B(t)u(t) + B_d d(t)] \\ y(t) = Cx(t) \end{cases} \quad (11)$$

There is always a signal at the control input $d_e(t)$, resulting in the same effect as $\Delta A(t)x(t) + \Delta B(t)u(t) + B_d d(t)$, so the system can be rewritten as

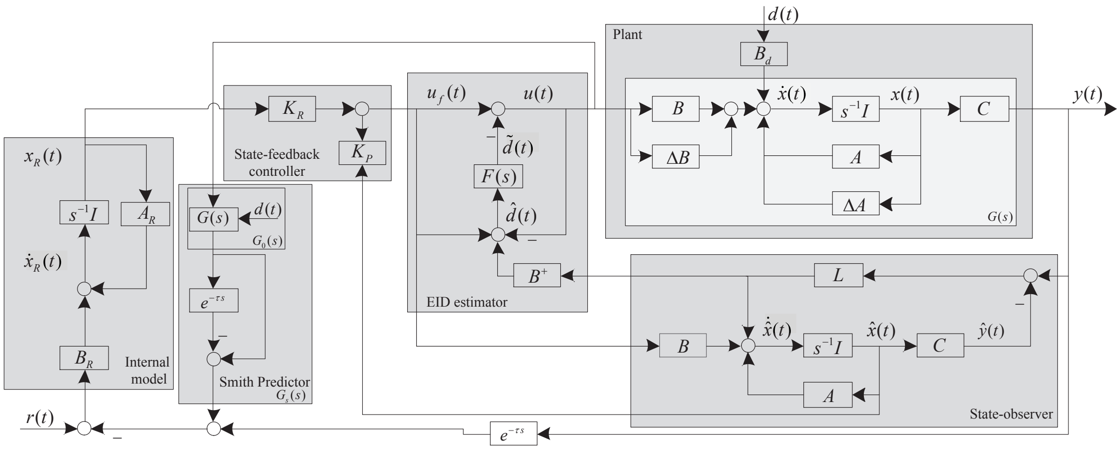

Fig. 1. The bottom hole pressure control structure of SP-EID

$$\begin{cases} \dot{x}(t) = Ax(t) + B[u(t) + d_e(t)] \\ y(t) = Cx(t) \end{cases} \tag{12}$$

According to the literature [15], taking full advantage of the observer, we get the estimated value of EID

$$\hat{d}(t) = B^{+}LC[x(t) - \hat{x}(t)] + u_f(t) - u(t) \tag{13}$$

where

$$B^{+} = (B^T B)^{-1} B^T \tag{14}$$

As the output contains noise, so the low-pass filter is used to eliminate the noise from estimate. The filter satisfies

$$|F(j\omega)| \approx 1, \ \forall \omega \in [0, \omega_r] \tag{15}$$

where ω_r is the highest estimated angular frequency, the appropriate switching frequency of the filter is usually 10 times the frequency of ω_r, and the first order filter can well meet the needs, the state space of $F(s)$ is

$$\begin{cases} \dot{x}_F(t) = A_F x_F(t) + B_F \hat{d}(t) \\ \tilde{d}(t) = C_F x_F(t) \end{cases} \tag{16}$$

where $\tilde{d}(t)$ is the estimated value of the filtered disturbance. The $\tilde{d}(t)$ is introduced into the state feedback control rate, and we get the control law of the controlled object

$$u(t) = u_f(t) - \tilde{d}(t) \tag{17}$$

The disturbance estimator plays an important role in enhancing the effect of disturbance rejection, and it can actively compensate the various types of disturbances caused by uncertain factors.

B. Control system design

In our SP-EID method, the Smith Predictor is used to compensate delay for feedback signal. The transfer function of Smith Predictor is

$$G_s(s) = G_0(s)(1 - e^{-\tau s})$$

which can lead the controller to speed up the adjustment process and maintain system stability.

Because of the uncertainties of the system object, the controller parameters can be coupled and the separation design can not be carried out. Therefore, we use the LMI-based method for controller design and stability analysis. After the input and disturbance are set to zero, we can get

$$\begin{cases} \dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]u(t) \\ y(t) = Cx(t) \end{cases} \tag{18}$$

First we introduce three lemmas:

Lemmas 1 Schur complement [20] : Given a matrix Σ,

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix}$$

The following three conditions are equivalent:
(1)$\Sigma < 0$;
(2)$\Sigma_{11} < 0, \ \Sigma_{22} - \Sigma_{12}^T \Sigma_{11}^{-1} \Sigma_{12} < 0$;
(3)$\Sigma_{22} < 0, \ \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{12}^T < 0$.

Lemmas 2 [21] : Given a matrix $\Pi \in R^{p \times n}$, $rank(\Pi) = p$, for any matrix, exist $\bar{X} \in R^{p \times p}$ such that $\Pi X = \bar{X}\Pi$ holds for any $X \in R^{n \times n}$ if and only if X can be decomposed as

$$X = W \begin{bmatrix} \bar{X}_{11} & 0 \\ 0 & \bar{X}_{22} \end{bmatrix} W^T$$

where $W \in R^{n \times n}$ is the unitary matrix, $\bar{X}_{11} \in R^{p \times p}$, $\bar{X}_{22} \in R^{(n-p) \times (n-p)}$.

Lemmas 3 [22] : Set $\Omega_0(x)$, $\Omega_1(x)$ be a quadratic function defined on R^n, if $\Omega_1(x) < 0$, $\forall x \in R^n - \{0\}$, so the necessary and sufficient conditions for $\Omega_0(x) < 0$ are present $\varepsilon \geq 0$, so that $\Omega_0(x) - \varepsilon \cdot \Omega_1(x) < 0$ is established.

The control system includes the four states of $x(t)$, $\hat{x}(t)$, $x_F^T(t)$ and $x_R^T(t)$, we define that

$$\Delta x(t) = x(t) - \hat{x}(t) \tag{19}$$

So the closed-loop system is expressed as

$$\varphi(t) = [\hat{x}^T(t) \quad \Delta x^T(t) \quad x_F^T(t) \quad x_R^T(t)]^T \tag{20}$$

In the closed-loop system, we can get that

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu_f(t) + LC\Delta x(t) \tag{21}$$

$$\Delta\dot{x}(t) = [A+\Delta A(t) - LC]\Delta x(t) + \Delta A(t)\hat{x}(t) + \Delta B(t)u_f(t) - [B+\Delta B(t)]C_F x_F(t) \tag{22}$$

$$\dot{x}_F(t) = (A_F + B_F C_F)x_F(t) + B_F B^+ LC\Delta x(t) \tag{23}$$

$$\dot{x}_R(t) = -B_R C\hat{x}(t) - B_R C\Delta x + A_R x_R(t) \tag{24}$$

21 to 24 form the state space equation of the closed-loop system

$$\dot{\varphi}(t) = \bar{A}\varphi(t) + \bar{B}u_f(t) \tag{25}$$

The state-feedback control law is

$$u_f(t) = \bar{K}\varphi(t) \tag{26}$$

where $\bar{K} = \begin{bmatrix} K_P & 0 & 0 & K_R \end{bmatrix}$. After the certain and uncertain items are separated, the system is converted to

$$\dot{\varphi}(t) = \hat{A}\varphi(t) + \hat{B}\Gamma(t) \tag{27}$$

where

$$\Gamma(t) = E(t)\Psi\varphi(t) \tag{28}$$

$$\Psi = [N_0 + N_1 K_P \quad N_0 \quad -N_1 C_F \quad N_1 K_R] \tag{29}$$

$$\hat{A} = \begin{bmatrix} A+BK_P & LC & 0 & BK_R \\ 0 & A-LC & -BC_F & 0 \\ 0 & B_F B^+ LC & A_F + B_F C_F & 0 \\ -B_R C & -B_R C & 0 & A_R \end{bmatrix} \tag{30}$$

$$\hat{B} = \begin{bmatrix} 0 & M^T & 0 & 0 \end{bmatrix} \tag{31}$$

Assume that the output matrix is decomposed into singular values as follow

$$C = U[S,0]V^T \tag{32}$$

where U and V are unitary matrices, S is a semi-positive definite matrix. Letting $V = [V_1 \quad V_2]$ we have the theorem.

Theorem 1: The system (25) is robustly stable under the control law (26) with given parameters α and β, the following LMI is feasible if there exist symmetric positive-definite matrices X_1, X_{11}, X_{22}, X_3, and X_4, and appropriate matrices W_1, W_2, W_3

$$\begin{bmatrix} \Phi_{11} & W_2 C & 0 & \Phi_{14} & 0 & \Phi_{16} \\ * & \Phi_{22} & \Phi_{23} & -X_2 C^T B_R^T & M & X_2 N_0^T \\ * & * & \Phi_{33} & 0 & 0 & -X_3 N_1^T \\ * & * & * & \Phi_{44} & 0 & \beta W_3^T N_1^T \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{33}$$

where

$$\begin{aligned} \Phi_{11} &= \alpha(AX_1 + X_1 A^T + BW_1 + W_1^T B^T) \\ \Phi_{14} &= \beta BW_3 - \alpha X_1 C^T B_R^T \\ \Phi_{16} &= \alpha X_1 N_0^T + \alpha W_1^T N_1^T \\ \Phi_{22} &= AX_2 + X_2 A^T - W_2 C - C^T W_2^T \\ \Phi_{23} &= -BC_F X_3 + C^T W_2^T B^{+T} B_F^T \\ \Phi_{33} &= (A_F + B_F C_F)X_3 + X_3(A_F + B_F C_F)^T \\ \Phi_{44} &= \beta A_R X_4 + \beta X_4 A_R^T \end{aligned}$$

and the singular-value decomposition of X_2 is

$$X_2 = \begin{bmatrix} V_1 & V_2 \end{bmatrix} \begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$$

The gains of the state-feedback controller and observer are

$$K_P = W_1 X_1^{-1}, \quad K_R = W_3 X_4^{-1}, \quad L = W_2 USX_{11}^{-1} S^{-1} U^T \tag{34}$$

Proof: Choose a Lyapunov functional candidate to be

$$V(t) = \varphi^T(t) P \varphi(t) \tag{35}$$

where $P = diag\left\{ \frac{1}{\alpha}P_1 \quad P_2 \quad P_3 \quad \frac{1}{\beta}P_4 \right\}$, and P_1, P_2, P_3, and P_4 are undetermined positive definite matrix. The derivative of $V(t)$ along (27) as follows

$$\dot{V}(t) = \varphi^T(t)P\dot{\varphi}(t) + \dot{\varphi}^T(t)P\varphi(t) = \varphi^T(t)\begin{bmatrix} H_{11} & H_{12} & 0 & H_{14} \\ * & H_{22} & H_{23} & H_{24} \\ * & * & H_{33} & 0 \\ * & * & * & H_{44} \end{bmatrix}\varphi(t) + 2\varphi^T(t)P_2 M\Gamma(t)$$

$$\begin{aligned} H_{11} &= \tfrac{1}{\alpha}(P_1 A + A^T P_1 + P_1 BK_P + K_P^T B^T P_1) \\ H_{12} &= \tfrac{1}{\alpha}P_1 LC \\ H_{14} &= \tfrac{1}{\alpha}P_1 BK_R - \tfrac{1}{\beta}C^T B_R^T P_4 \\ H_{22} &= P_2 A + A^T P_2 - P_2 LC - C^T L^T P_2 \\ H_{23} &= -P_2 BC_F + B_F C^T L^T B^{+T} P_3 \\ H_{24} &= -\tfrac{1}{\beta}C^T B_R^T P_4 \\ H_{33} &= P_3(A_F + B_F C_F) + (A_F + B_F C_F)^T P_3 \\ H_{44} &= \tfrac{1}{\beta}(P_4 A_R + A_R^T P_4) \end{aligned}$$

So,

$$\dot{V}(t) - \left[\Gamma^T(t)\Gamma(t) - \varphi^T(t)\Psi^T\Psi\varphi(t)\right] = \begin{bmatrix} \varphi^T(t) & \Gamma^T(t) \end{bmatrix} \Xi \begin{bmatrix} \varphi(t) \\ \Gamma(t) \end{bmatrix} \tag{36}$$

where

$$\Xi = \begin{bmatrix} H_{11} & H_{12} & 0 & H_{14} & 0 \\ * & H_{22} & H_{23} & H_{24} & P_2 M \\ * & * & H_{33} & 0 & 0 \\ * & * & * & H_{44} & 0 \\ * & * & * & * & -I \end{bmatrix} + \begin{bmatrix} \Psi^T \\ 0 \end{bmatrix} \begin{bmatrix} \Psi & 0 \end{bmatrix} \tag{37}$$

Making full use of Lemma 1, we write $\Xi < 0$ as

$$\begin{bmatrix} H_{11} & H_{12} & 0 & H_{14} & 0 & N_0^T + K_P^T N_1^T \\ * & H_{22} & H_{23} & H_{24} & P_2 M & N_0^T \\ * & * & H_{33} & 0 & 0 & -C_F N_1^T \\ * & * & * & H_{44} & 0 & K_R^T N_1^T \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{38}$$

Pre- and post-multiplying by

$$\begin{aligned} \Pi &= diag\{\alpha P_1^{-1}, P_2^{-1}, P_3^{-1}, \beta P_4^{-1}, I, I\} \\ &= diag\{\alpha X_1, X_2, X_3, \beta X_4, I, I\} \end{aligned} \tag{39}$$

yields

$$\begin{bmatrix} \tilde{\Phi}_{11} & LCX_2 & 0 & \tilde{\Phi}_{14} & 0 & \tilde{\Phi}_{16} \\ * & \tilde{\Phi}_{22} & \tilde{\Phi}_{23} & -X_2 C^T B_R^T & M & X_2 N_0^T \\ * & * & \tilde{\Phi}_{33} & 0 & 0 & -X_3 N_1^T \\ * & * & * & \tilde{\Phi}_{44} & 0 & \beta W_3^T N_1^T \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{40}$$

where

$$\begin{aligned}
\tilde{\Phi}_{11} &= \alpha(AX_1 + X_1A^T + BK_PX_1 + X_1K_P^TB^T) \\
\tilde{\Phi}_{14} &= \beta BK_RX_4 - \alpha X_1C^TB_R^T \\
\tilde{\Phi}_{16} &= \alpha(X_1N_0^T + X_1K_P^TN_1^T) \\
\tilde{\Phi}_{22} &= AX_2 + X_2A^T - LCX_2 - X_2C^TL^T \\
\tilde{\Phi}_{23} &= -BC_FX_3 + B_FX_2C^TL^TB^{+T} \\
\tilde{\Phi}_{33} &= (A_F + B_FC_F)X_3 + X_3(A_F + B_FC_F)^T \\
\tilde{\Phi}_{44} &= \beta(A_RX_4 + X_4A_R^T)
\end{aligned}$$

The (40) is not LMI. In order to solve this matrix inequality, we first apply Lemma 2 to (32), get

$$\bar{X}_2 = USX_{11}S^{-1}U^T \tag{41}$$

such that

$$CX_2 = \bar{X}_2C \tag{42}$$

Then, letting

$$K_PX_1 = W_1, \quad K_RX_4 = W_3, \quad L\bar{X}_2 = W_2 \tag{43}$$

and substituting (42) and (43) into (40) give the LMI (33).

Combing (6) and (28) yields

$$\Gamma^T(t)\Gamma(t) \leq \varphi^T(t)\Psi^T\Psi\varphi(t) \tag{44}$$

Applying Lemma 3 and $\varepsilon = 1$ to (36), we get to konw $\Xi < 0$ if and only if $\dot{V}(t) < 0$. So, (33) guarantees the asymptotically stability of the system (25) with uncertainties. Equation (34) is obtained from (41) and (43).The control algorithm for the control system parameters is shown below.

Step 1: Select the internal model parameters based on the reference input.

Step 2: Select the low-pass filter parameter A_F, B_F, and C_F according to (15).

Step 3: Choose positive parameters α, β and calculate the singular value decomposition of matrix C to get a feasible solution to the LMI and get the value of K_P, K_R and L.

IV. SIMULATION RESULTS

The value of these parameters for the hydrodynamic model from [16], they are given as follows, $\beta_a = \beta_d = 14000$, $V_d = 28.3$, $V_a = 96.1$, $M_a = 1700$, $M_d = 5700$, $F_a = 20800$, $F_d = 165000$, $\rho_a = \rho_d = 0.0125$, $h_{bit} = 2000$, $g = 9.8$, $P_{pump} = 1$, $q_{res} = 0.001$, $\dot{V}_a = 0$, $q_{pump} = 0.01$, $q_{back} = 0.003$, $q_{bit} = 0.01$, $p_{bit}(0) = 320$. We put these parameters in (2) and (3), so the dynamic characteristics of the bottom hole pressure from the hydrodynamic model can be identified as

$$A = [0.071], \quad C = [1], \quad B = [-112.214], \quad B_d = [1]$$

$$e = [-14.7235], d(t) = \begin{cases} e & 0 < t < 20 \\ e + k * \sin(3.14t) & t \geq 20 \end{cases}$$

From the system matrix we know that the plant is a non-minimum phase system. Considering the uncertainty of hydraulic parameters during the drilling process, we set $M = 1$, $N_0 = 0.005$, $N_1 = 5$, $E(t) = \sin(0.5\pi t)$.

Note that the plant has no zero on the imaginary axis, and satisfy rank$(B,\ AB,\ A^2B,\ \dots,\ A^{n-1}) = n$, rank$(C^T,\ A^TC^T,\ \dots,(A^T)^{n-1}C^T) = n$, so the plant is observable and controllable, which means the system meets the prerequisites for using the method in [15]. The reference input $r(t)$ is set to 340, and select the internal model parameters $A_R = -0.03$, $B_R = 5$. The maximum frequency of the disturbance is 3.14, so we set the ω_r as 3.14, the form of filter as follows

$$F(s) = \frac{b}{s+a}$$

where $a = 101$, $b = 100$, satisfy the conditions (15), so $A_F = -101$, $B_F = 100$, $C_F = 1$. Since the output matrix is a first order matrix, no decomposition is required. But if we do not find suitable α and β, the system will be unstable or even divergence. So in order to track the reference input and make the system to a steady state, we set the initial values of α and β to 1 and 0.1, respectively, and we specify that K_P and K_R satisfy the following conditions

$$\left|\frac{K_R}{K_P}\right| > 4$$

Then we use the LMI toolbox in MATLAB to solve (33), we get $\alpha = 10.2072$, $\beta = 0.9475$ and

$$K_P = 0.0079, \quad K_R = -0.0323, \quad L = 44.3078$$

In order to show the superiority of our method in the feedback channel delay compensation, we make comparison among EID and SP-EID with same parameters and we set the delay $\tau = 0.03$. The result show in Fig. 2. From the figure we know that our method has shorter regulation time and better disturbance suppression effect.

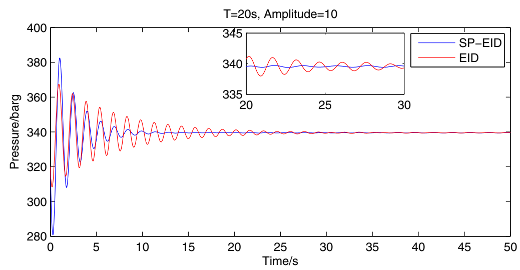

Fig. 2. Disturbance suppression by EID and SP-EID(k=10)

The core of the method is to use the disturbance estimator to suppress the disturbance. If the estimator is removed, we find that the disturbance will not be suppressed. In order to show the superior performance of SP-EID on disturbance suppression we also compare SP-SMC with the slide mode control method to show the accuracy and superiority of our method. We use the slide mode controller instead of the disturbance estimator. We add SP to SMC method to compensate the delay of the feedback channel. In the slide mode controller, we choose the switching function as

$$S(t) = -\begin{bmatrix} K_P & K_R \end{bmatrix}\begin{bmatrix} \hat{x}^T(t) & x_R^T(t) \end{bmatrix}^T$$

the slide mode control law as

$$u_S(t) = -K_S \times \text{sgn}[S(t)]$$

where K_S is positive matrix, and its value is set to 0.8.

We use the same parameters K_P, K_R and L to make comparison among SP-EID method and SP-SMC. The result show in the Fig. 3. and Fig. 4. In contrast to other methods, our method is able to track the steady state well, and when the disturbance is applied, our method can suppress the disturbance well. When the amplitude of the disturbance is 10, the maximum fluctuation of the EID method is 0.6, and the maximum fluctuation of the SMC method is 4.8. When the disturbance is further adjusted to 50, the maximum fluctuation of the EID method is 1.04, and the fluctuation of the SMC method increase to reach 5.14.

Through the above comparison we find that the SP-EID method can suppress the disturbance for the parameter uncertain system with feedback delay, and SP-EID method do not need to quickly switch the state of the controller, which means the SP-EID method can meet the requirements of industrial site, and achieve good control effect.

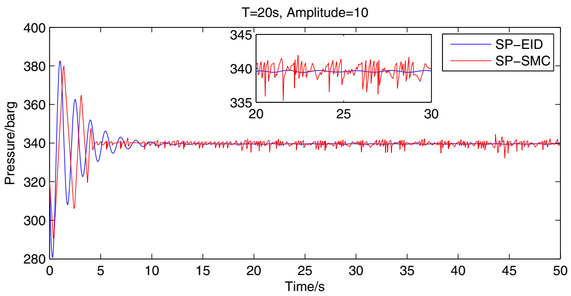

Fig. 3. Disturbance suppression by EID and SMC(k=10)

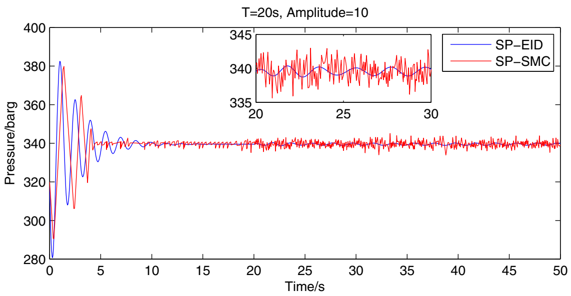

Fig. 4. Disturbance suppression by EID and SMC(k=50)

V. CONCLUSION

In this paper, we propose SP-EID method to suppress the pressure fluctuation caused by longitudinal vibration drill string, compensate delay in feedback channel and give the configuration and design process of the whole control system. Compared with the SMC method and EID method, our method does not need to switch the state of the controller frequently, and compensate delay in feedback channel very well. At the same time, after the disturbance is applied, our method has superior performance in terms of disturbance suppression. Our method has the following advantages:

- The configuration is simple and the control system is easy to design.
- The method can be applied to the minimum phase system and non-minimum phase system.
- The method has a certain robustness to the system with uncertain system structure, can suppress the disturbance well and meet the actual needs of industry.

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