Distributed receding horizon control for rotating wings unmanned aerial vehicles: a time-varying topology strategy

Giuseppe Franzè, Walter Lucia and Babak Rahami

***Abstract*— In this paper a distributed constrained control architecture is developed to address the obstacle avoidance problem for rotating wings unmanned aerial vehicles. The proposed strategy aims at achieving a flexible formation whose topology can be properly reorganized whenever narrowed corridors are accessible. This reasoning is translated into a computable procedure by resorting to model predictive control (MPC) ideas that allow to efficiently manage model uncertainties and physical constraints.**

I. Introduction

Navigation of unmanned vehicles is a classic robotics research area where single and multiple coordinated vehicles offer interesting perspectives in many applications: industrial automation, search and rescue, surveillance and inspection see [1] for a detailed literature review. More specifically, unmanned aerial vehicles (UAVs) have gained a rapid and continuous growth of interest, mainly because of their ability to effectively carry out a wide range of missions at low costs and without putting human resources at risk [2]. It is well-known that many complex factors are involved when realistic paths should be determined: obstacle occurrences, no-fly zones, uncertainty sources in both UAV dynamics and operating environments, see [3] and references therein.
By summarizing the above discussion, the design of efficient controllers for the coordination of multi-UAVs leads to methodological difficulties: 1) characterization of formal conditions ensuring closed-loop stability and satisfactorily control performance levels; 2) efficiently deal with critical obstacle scenarios, e.g. narrowed corridors; 3) coordination amongst vehicles 4) limited computational resources.

Past approaches for UAV motion planning and guidance can be found e.g. in [5]. Model-based predictive control approaches have acquired an increasing reputation when coordination problems for constrained multi-UAV systems subject to collision avoidance requirements are of interest. In [6], the objective is to stabilize a group of vehicles toward an equilibrium point in a cooperative way by means of the receding horizon control philosophy. In [7], the problem of robust flight control of unmanned rotorcrafts is addressed in various challenging conditions, aiming to capture the demanding nature of the potential requirements for their efficient and safe integration in real-life operations. In [8], a distributed linear MPC strategy is designed to solve the trajectory planning problem for rotary-wing UAVs by exploiting radio communication capacities.

Starting from this analysis, an aspect often overlooked relies on the exploitation of the time-varying environment structure, i.e. the capability to take advantage from the information (obtained by the vehicle via on-board perception units) on the nature of the obstacle occurrences. In this paper by resorting to MPC ideas, two *ad-hoc* distributed receding horizon control schemes (see [11] for a detailed review on this topic) are designed for properly dealing with obstacle free and corridor occurrences. The first algorithm refers to an UAV grid configuration and it is oriented to deal with uncertain operating conditions, while the second is formulated to comply with a standard platoon UAV configuration exploited for narrowed passages (corridors). Since the overall control architecture leads to time-varying topologies, one of the main important results is the recursive feasibility property which is guaranteed under mild set-membership requirements. Moreover, coordination constraints are always ensured and boundedness of the closed-loop trajectories is formally proved. A major merit of the proposed strategy is the capability to efficiently adapt to time-varying changing environmental conditions the UAV topology: more specifically, the vehicle formation is instructed to on-line switch from a grid configuration to a platoon one and *vicerversa* without compromising target achievement, constraints fulfilment and obstacle avoidance requirements.

Notation

Consider the following linear time-invariant (LTI) discrete-time system

$$x(t+1) = \Phi x(t) + Gu(t) + G_d d(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ denotes the state, $u(t) \in \mathcal{U} \subset \mathbb{R}^m$ the constrained input and $d(t) \in \mathbb{R}^d$ the process disturbance. It is assumed that $d(t) \in \mathcal{D} \subset \mathbb{R}^d$, $\forall t \in \mathbb{Z}_+ := \{0, 1, \ldots\}$, with $\mathcal{D}$ a compact set with $0_d \in \mathcal{D}$. Given the plant (1), the sets of states i-step controllable to $\mathcal{T}_0$ are computed via the following recursions (see [9]):

$$\mathcal{T}_i := \{x : \exists u \in \mathcal{U} : \Phi x + Gu + G_d d \in \mathcal{T}_{i-1}, \forall d \in \mathcal{D}\} \tag{2}$$

Given a set $S \subseteq \mathbb{R}^n$, $In[S] \subseteq S$ denotes its inner convex approximation. Given a discrete set D, $card(D)$ denotes its cardinality.
Given a set $S \subseteq X \times Y \subseteq \mathbb{R}^n \times \mathbb{R}^m$, the projection of S onto X is defined as $\text{Proj}_X(S) := \{x \in X \,|\, \exists y \in Y \text{ s.t. } (x, y) \in S\}$.

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II. Problem formulation

Consider a group of UAVs described as discrete-time LTI systems

$$x^i(t+1) = A^i x^i(t) + B^i u^i(t) + B_d^i d^i(t),\; i = 1,\dots,L, \quad (3)$$

where the state vector $x^i(t) \in \mathbb{R}^n$ and the control input $u^i(t) \in \mathbb{R}^m$ are subject to the following constraints

$$x^i(t) \in \mathcal{X}^i,\; u^i(t) \in \mathcal{U}^i,\; i = 1,\dots,L, \quad (4)$$

with $\mathcal{X}^i$ and $\mathcal{U}^i$ compact and convex sets of $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively. Moreover, it is assumed that the exogenous signal $d^i(t) \in \mathcal{D}^i \subset \mathbb{R}^d$ is a bounded disturbance with $\mathcal{D}^i$ a compact and convex set.

Operating scenario - *The UAV formation (3) operates within an environment where the occurrence of large obstacles could almost completely obstruct the nominal path of each vehicle. Only narrow corridors are available to go beyond such barriers.* □

The following hypotheses will be exploited:

Obstacle corridor scenario - Each obstacle is a convex polygon. Then:

Definition 2.1: Let Ob_j^i be an obstacle. Then an obstacle scenario O^i is defined as

$$O^i := \{Ob_1^i, \dots, Ob_{n_i}^i\} \quad (5)$$

with n_i the number of objects involved. □

Definition 2.2: Let O^i be an obstacle scenario. Then, the non-convex obstacle-free region pertaining to O^i is defined as follows

$$O_{free}^i := \{p \in \mathbb{R}^3 : h(p) > 0\} \quad (6)$$

where $h_i : \mathbb{R}^3 \to \mathbb{R}^{n_f}$ and n_f is the number of component-wise inequalities. □

Perception capabilities - UAVs are equipped with perception units capable to detect obstacles within a pre-assigned radius R and a field of view of 360^o;

Team configuration and neighbours - A grid configuration is initially assumed, see Fig. 1. For each UAV the following definitions are used:

- *grid level*: $level(i) : \{1,\dots,L\} \to \mathbb{Z}_+$, which provides the position of the $i-th$ vehicle along the grid configuration;
- *set of neighbours*:

$$\mathcal{N}^i := \begin{array}{l}\{j \in \{1,\dots,i-1,i+1,\dots,L\} : \\ level(j) \equiv level(i)\}\end{array} \quad (7)$$

- *father*: the operator $pre(i) : \{1,\dots,L\} \to \{1,\dots,L\}$ denotes the predecessor (father) of the $i-th$ UAV;
- *set of lower level nodes*: $post(i), i = 1,\dots,L$, refers to an index subset of the UAVs belonging to the $(i+1)-th$ level.

Data exchange - Each $i-$th UAV sends to the neighbours (7) and to the $(i+1)-$th level nodes a packet containing its state trajectory $\mathbf{x}^i(\cdot)$.

It is assumed that: the obstacle scenario O is static , while the obstacle-free region O_{free} is approximated by a family of regions O_{free}^i, each one accounting for a corridor. W.l.o.g. let $O := \bigcup_i^{n_{sc}} O^i$, with O^i convexified by means of the procedure proposed in [13] which has been adapted to the 3-D environment. The UAV team acquires the information on $O^i(t)$ whenever the leader perception units detect a corridor. Then the problem to solve can be stated as follows.

Obstacle Avoidance Path planning (OAMP) Problem - *Given a grid configuration of UAVs (3), determine a distributed state-feedback control policy*

$$\begin{array}{l} u^i(t){=}g(x^i(t),\{x^j(t)\},x_f^i), j \in \mathcal{N}^i, i = 1,\dots,f; \\ u^i(t){=}g(x^i(t),x^{i-1}(t),\{x^j(t)\}), j \in \mathcal{N}^i, i{=}f{+}1,\dots,L, \end{array} \quad (8)$$

compatible with (4) such that, starting from an admissible initial condition $x(0) = [x^{1^T}(0), x^{2^T}(0), \dots, x^{L^T}(0)]^T$, the UAV team is driven to the target position $x_f = [x_f^{1^T}, x_f^{2^T}, \dots, x_f^{L^T}]^T$, regardless of any admissible obstacle scenario $O^t \in \{O^i\}_{i=1}^{n_{sc}}$ and disturbance realization $d^i(t) \in \mathcal{D}^i, i = 1,\dots,L, \forall t \geq 0$.

The problem will be addressed along the following lines. Under the initial configuration of Fig. 1, a distributed MPC strategy is developed in order to take advantage of all benefits deriving from the MPC philosophy: control performance optimization, constraints satisfaction, disturbance effects mitigation. When a corridor is detected, then the topology switching (from grid to platoon) allows to improve the UAV formation performance during the navigation. Therefore, a leader-follower aggregation strategy is prescribed and an *ad-hoc* MPC controller designed. Finally after the obstacle has been passed, a de-aggregation phase starts and the UAVs re-establish a grid configuration.

This *modus operandi* needs of the following ingredients: 1) a distributed MPC architecture; 2) formal conditions under which feasible topology switchings (*grid-to-platoon* and *vice-versa*) can be imposed; 3) LF aggregation and disaggregation procedures.

III. Distributed receding horizon control architecture

In this section, *ad-hoc* distributed RHC schemes will be developed to deal with different UAV formation topologies.

A. Grid topology

By referring to the grid topology, the main aspects of the proposed scheme are here summarized:

- **Receding horizon controllers -** Let $r := \max_{i=1,\dots,L} level(i)$ be the grid depth. For each element of the grid, a distributed receding horizon controller is designed as follows:
  1) *first level UAVs, namely $level(i) = 1$;* - MPC controllers with prediction horizon length $N_1 = 0$ are exploited;
  2) *lower level UAVs $level(i) > 1$;* - MPC controllers with prediction horizon lengths $N_i = level(i) - 1 > 0$. As a consequence, the maximum admissible horizon length is $r-1$.

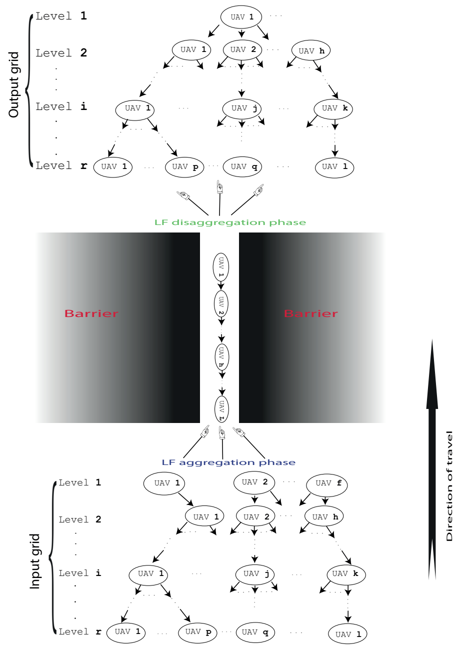

Fig. 1. Sketch of the proposed control architecture: $L = f + h + k + l$ vehicles

- **Communication facilities -** At each time instant t, data exchanges occur amongst the UAVs:
  - the first level vehicles (leaders) transmit to its followers belonging to the level 2 the state trajectory which these UAVs assume that leaders will implement over the future time instants (assumed state trajectory), i.e. $t+1$ and $t+1+N_2 = t+2$;
  - each vehicle $i : level(i) > 1$ transmits the state trajectory both to followers (UAVs belonging to the $(i+1)-th$ level) and neighbors (7), receives the assumed state trajectory from the unique predecessor belonging to the $(i-1)-th$ level.

The following notations will be used. For any vehicle $i = 1,\ldots,L$, let the $k-th$ predicted state and control moves within the horizon at time t be denoted by $x^i(k|t)$ and $u^i(k|t)$, respectively. Let $\mathbf{x}^i(t) := \{x^i(k|t)\}_{k=0}^{N_i}$ and $\mathbf{u}^i(t) := \{u^i(k|t\}_{k=0}^{N_i}$, $i = 1,\ldots,L$, be the predicted states and predicted controls within the control horizon, respectively. Moreover, over any prediction interval $[t+k,\ t+k+N_i]$ and for any $i-th$ vehicle we further define:

- the optimal state trajectory: $\mathbf{x}^{i^*}(t) := \{x^{i^*}(k|t\}_{k=0}^{N_i}$, $i = 1,\ldots,L$;
- the assumed state trajectory: $\hat{\mathbf{x}}^i(t) := \{\hat{x}^i(k|t\}_{k=0}^{N_i}$, $i = 1,\ldots,L$;

Notice that the sequence $\hat{\mathbf{x}}^i$ is transmitted to all the followers $k \in \mathcal{N}^i$.

In order to formally define the optimal control problem underlying the MPC strategy, the following ingredients are required:

- Input sequence parametrization:

$$u^i(\cdot|t) \triangleq \begin{cases} u^i(k+t|t), & k = 0,\ldots,N_i - 1 \\ K^i x^i(k+t|t), & k \geq N_i \end{cases} \tag{9}$$

  with $K^i \in \mathbf{R}^{m\times n}$ a stabilizing and admissible state feedback law;
- Cost-to-go:

$$\begin{aligned} &J^i(x(t|t), x_f^i, \mathbf{u}^i(t)) \triangleq \\ &\sum_{k=t}^{t+N_i-1}\Big[\|x^i(t+k|t)\|_{R_x^i}^2 + \|u^i(t+k|t\|_{R_u^i}^2\Big] + \\ &\|x^i(t+N_i|t) - x_f^i\|_{P^i}^2 \end{aligned} \tag{10}$$

  where $R_x^i > 0$ and $R_u^i \geq 0$ are symmetric state and input weighting matrices and $P^i \geq 0$.
- Terminal constraint:

$$x^i(t+N_i|t) \in \Xi^i \subset \mathbf{R}^n \tag{11}$$

  The pair (Ξ^i, K^i) is computed such that Ξ^i is a robustly positively invariant (RPI) [9] region for the state evolutions of the closed-loop system, viz. $x^i(t+N_i|t) \in \Xi_i$ implies that $(A^i + B^i K^i)^{t+N_i+k} x^i(t+N_i|t) \in \Xi^i, \forall k \geq 0$.

Moreover, it is necessary to deal with the fact that the grid moves towards $x_f^i, i = 1,\ldots,L$: the latter prescribes that the terminal region is time-varying and the related constraint must be accordingly modified. First, the following condition must be satisfied for each vehicle:

$$x^i(t+N_i|t) \in \Xi^i(t-1) \cup \Xi^i(t) \tag{12}$$

where $\Xi^i(t)$ is computed such that

$$\bar{x}_{t-1}^i \in \Xi^i(t-1) \cap \Xi^i(t) \tag{13}$$

with $\bar{x}_{t-1}^i$ denoting an equilibrium point selected at the time instant $t-1$. Then, one has that

- If i $level(i) = 1$, then $\Xi^i(t)$ is defined as

$$\Xi^i(t) := \arg\min_{\Xi^i} dist(x_f^i, \Xi^i) \ \text{ subject to (12)-(13)} \tag{14}$$

- else

$$\Xi^i(t) := \begin{array}{l} \arg\min_{\Xi^i} dist(\Xi^i, \Xi^{pre(i)}(t-1)) \\ \text{subject to (12)-(13)} \end{array} \tag{15}$$

Finally, it is required that each UAV must maintain a safe distance from its predecessor and neighbors. This translates into grid formation constraints in terms of desired separation and the desired relative bearing between the $i-th$ vehicle, its predecessor and neighbors [10].

Hence, given the current state measurement $x^i(t|t) \in \mathcal{X}^i$ and the predecessor assumed state sequence $\hat{\mathbf{x}}^{pre(i)}(t)$ computed according to the following strategy:

$$\begin{aligned} &\hat{\mathbf{x}}^{pre(i)}(t) = \\ &\begin{cases} x^{(pre(i))^*}(t-1+k|t-1), k = 1,\ldots,N_{pre(i)}, \\ (A^{pre(i)} + B^{pre(i)}K^{i-1})^{t-1+k} x^{(pre(i))^*}(t-1+N_{pre(i)}|t-1), \\ k = N_{pre(i)} + 1, \end{cases} \end{aligned} \tag{16}$$

the optimization problem for the $i-th$ follower, hereafter denoted as **DMPC**-$\mathcal{P}_F^i(t)$, is

DMPC-$\mathcal{P}_F^i(t)$:

$$\min_{\mathbf{u^i(t)}} J^i(x(t|t), x_f^i, \mathbf{u}^i(t)) \quad (17)$$

$$x^i(t+k+1|t) = A^i x^i(t+k|t) + B^i u^i(t+k|t) \quad (18)$$

$$u^i(t+k|t) \in \mathcal{U}^i, k = 0, 1, \ldots, N_i - 1 \quad (19)$$

$$x^i(t+k|t) \in \mathcal{X}^i, k = 0, 1, \ldots, N_i - 1 \quad (20)$$

$$x^i(t+N_i|t) \in \Xi^i(t-1) \cup \Xi^i(t) \quad (21)$$

$$\begin{aligned} &\alpha_{min}^c \le \|x^i(t+k|t) - \hat{x}^j(t+k|t)\| \le \alpha_{max}^c, \\ &k = 0, 1, \ldots, N_i, \forall j \in \mathcal{N}^i, \end{aligned} \quad (22)$$

$$\alpha_{min}^c \le \|x^i(t+k|t) - \hat{x}^{pre(i)}(t+k|t)\| \le \alpha_{max}^c, k = 0, \ldots, N_i \quad (23)$$

where $\alpha_{max}^c \in \mathbf{R}^+$ and α_{min}^c are *a-priori* known bounds. Conversely, the **DMPC** optimization pertaining to the leaders, named $\mathcal{P}_L^i(t)$, does not involve the satisfaction of (23) because the grid configuration is exclusively preserved by the follower nodes.

B. Platoon topology

By resorting to the ideas proposed in [12], let restrict the attention to the corridor of Fig. 1 and denote with x_{in} and x_{fin} (x_{fin} an equilibrium) the entrance point and the exit point along the corridor, respectively. Then, the corridor crossing problem will be addressed by means of a dual-mode receding horizon control approach which prescribes the computation of: 1) a robust stabilizing state-feedback control law and the corresponding RPI region; 2) a sequence of one-step state ahead controllable sets.

To this end, the state constraints are re-defined by taking into account the corridor-free region, namely O_{free}^i,

$$\mathcal{X}_c^i \triangleq \mathcal{X}^i \cap O_{free}^i \; i = 1, \ldots, L. \quad (24)$$

1) Robustly positively invariant regions: The terminal constraint sets must designed such the Cartesian structure $\mathcal{T}_0 := \prod_{i=1}^{L} \mathcal{T}_0^i$ is preserved. Note that $\mathcal{T}_0 \subset \mathbf{R}^{nL}$ is the terminal set of the centralized system achievable by collecting all the L system models (3).

Since such sets are independent each other, the pairs $(\mathcal{T}_0^i, F^i)$, $i = 1, \ldots, L$, are achievable by resorting to standard worst-case approaches, see e.g. [9].

2) One-step state ahead controllable sets: Here for each $i-th$ UAV, a family of one-step controllable sets $\mathcal{T}_j^i$ to the target sets $\mathcal{T}_0^i, i = 1, \ldots, L$, is computed by carefully taking care that the one-step state predictions are evaluated along interacting subsystem models.

Then, the one-step controllable sets sequence $\{\mathcal{T}_j^i\}$, $i = 1, \ldots, L$, are obtained by means of the following recursions:

$$\mathcal{T}_j^1 = \{x^1 \in \mathcal{X}_c^1 : \exists u \in \mathcal{U}^1, A^1x^1 + B^1u^1 \in \tilde{\mathcal{T}}_{j-1}^1\}, \quad (25)$$

$$\mathcal{T}_j^i = \{x^i \in \mathcal{X}_c^i : \exists u \in \mathcal{U}^i, A^ix^i + B^iu^i \in \tilde{\mathcal{T}}_{j-1}^i, \quad (26)$$

$$\alpha_{min}^c \le dist(x^i, \mathcal{T}_j^{i-1}) \le \alpha_{max}^c, \quad (27)$$

$$\alpha_{min}^c \le dist(A^ix^i + B^iu^i, \mathcal{T}_{j-1}^{i-1}) \le \alpha_{max}^c\}, i = 2, \ldots, L;$$

$$\mathcal{T}_{j-1}^i \subset \mathcal{T}_j^i, \forall j, i = 1, \ldots, L, \quad (28)$$

with $\tilde{\mathcal{T}}_{j-1}^i := \mathcal{T}_{j-1}^i \sim B_d^i \mathcal{D}^i$, $i = 1, \ldots, L$, and $\sim$ *the Pontryagin-Minkowski set difference*.

The reasoning behind (25)-(28) can be summarized as follows. At each step, the leader one-step controllable region is computed by complying with the corridor constraints (24) and irrespective of any LF configuration requirement. Conversely, the UAV followers must take care of the coordination constraints (23) that here should have to be

$$\alpha_{min}^c \le \|x^i(t+k|t) - \hat{x}^{i-1}(t+k|t)\| \le \alpha_{max}^c, k = 0, 1. \quad (29)$$

Since an off-line computation is performed (no time dependency) and, by construction, the one-step ahead state evolution should have to be driven in a finite number of steps within the terminal region $\mathcal{T}_0^i$, the inequalities (29) can be recast in terms of the following set-containment conditions:

- $\alpha_{min}^c \le dist(x^i, \mathcal{T}_j^{i-1}) \le \alpha_{max}^c$: the set of state x^i is selected such that the distance from the same-level controllable region of the predecessor UAV along the LF chain is bounded by α_{min}^c and α_{max}^c, i.e $\mathcal{T}_j^i \cap \mathcal{T}_j^{i-1} = \emptyset$;
- $\alpha_{min}^c \le dist(A^ix^i + B^iu^i, \mathcal{T}_{j-1}^{i-1}) \le \alpha_{max}^c$: the set of one-step ahead state prediction $X^{i^+} := A^ix^i + B^iu^i$ is such that $X^{i^+} \cap \mathcal{T}_{j-1}^{i-1} = \emptyset$.

As the on-line phase is concerned, the *i*-th UAV selects its current command u^i by resorting to the current state measurement $x^i(t)$ according to the following convex optimization problem:

$$u^i(t) = \arg\min_{u^i} J_{j(t)}(x^i(t), u^i) \quad (30)$$

subject to

$$A^ix^i(t) + B^iu^i \in In[\mathcal{T}_{j(t)-1}^i], u^i \in \mathcal{U}^i \quad (31)$$

Here, the running cost $J_{j(t)}(x^i(t), u^i)$ is chosen without loss of generality as follows:

$$J_{j(t)}(x^i(t), u^i) \triangleq \|A^ix^i(t) + B^iu^i\|^2_{(P_{j(t)-1}^i)^{-1}} \quad (32)$$

where $P_{j(t)-1}^i$ is the shaping matrix of the ellipsoidal region $In[\mathcal{T}_{j(t)-1}^i] \triangleq \{x^i \in \mathbf{R}^{n_i} \mid (x^i - x_c^i)^T (P_{j(t)-1}^i)^{-1}(x^i - x_c^i) \le 1\}$, with $x_c^i \in \bigcup_j \mathcal{T}_j^i$ an admissible equilibrium along the corridor.

3) Distributed leader-follower scheme under corridor path constraints: An important question concerns with the existence of a state trajectories tube which covers the distance between the entrance and exit of the given corridor, see Fig. 2: an admissible set of paths can be computed by straightforwardly adapt the **OCSP** procedure presented in [13] to the proposed distributed framework.

Then, the resulting algorithm is as follows.

Distributed Receding Horizon Control Leader-Follower Algorithm (DRHC-LF)

INITIALIZATION:

1: Given x_{in}, x_{fin} and $\mathcal{X}_c^i$, $i = 1, \ldots, L$;

2: **compute** $\mathcal{T}_0^i \subset \mathbf{R}^{n_i}$, $i = 1, \ldots, L$, ▷ the RPI regions

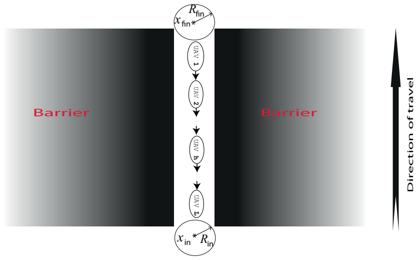

Fig. 2. Corridor crossing

3: **compute** F^i, $i = 1, \dots, L$, ▷ the stabilizing state feedback gains such that (4) are fulfilled;
4: **generate** the sequences $\{\mathcal{T}_j^i\}_{j=1}^{N_i}$, $i = 1, \dots, L$, via recursions (25)-(27) such that $x(0) \in \bigcup_j \{\mathcal{T}_j^1 \times \cdots \times \mathcal{T}_j^L\}$
5: **store** $\{\mathcal{T}_j^i\}_{j=1}^{N_i}$ and $F^i, i = 1, \dots, L$.

DRHC-LF - On-line Phase - UAV i-th

1: **Find** $j(t) = min\{j : x^i(t) \in \mathcal{T}_j^i\}$
2: **if** $i > 1$ **then receive** $j(t-1)$ from the $(i-1)$-th UAV and assign $j_{pre} := j(t-1)$;
3: **if** $j(t) < j_{pre}$ **then** $j(t) \leftarrow j_{pre}$
4: **end if**
5: **end if**
6: **if** $j(t) == 1$ **then** $u^{i^*}(t|t) = F^i x^i(t)$
7: **else**
Solve optimization (30)-(31);
8: **end if**
9: **if** $i < L$ **then transmit** $j(t)$ to the $(i+1)$-th UAV;
10: **end if**
11: **Apply** $u^{i^*}(t|t)$;
12: $t \leftarrow t+1$ and goto **Step 1**.

IV. Time-varying topology: LF aggregation and disaggregation strategies

For the sake of simplicity and without loss of generality, it is assumed that only two UAV configurations are allowed: platoon and one only grid (i.e. the neighbours sets $\mathcal{N}^i, i = 1, \dots, L$ are fixed).Let q^s, $s = 1, \dots, r$, be the cardinality of the $i-th$ grid level. Let $\mathcal{H}^i$, $i = 1, \dots, L$, be the set of integers ranging from $\sum_{k=1}^{i-1} q^{k-1} + 1$ to q^i.

LF-Aggregation (LF-Agg) - i-th UAV

Input: $level(i)$, $\mathcal{N}^i$, $\mathcal{H}^{level(i)}$
Output: i^{agg}, $\mathcal{N}^j, \forall j \in \mathcal{N}^i$, $\mathcal{H}^{level(i)}$ $count^{level(i)}$

1: **sort** *the index set* $I := \{i, \{j\}_{j=1}^{card(\mathcal{N}^i)}\}$ *according to the ascending order of the numerical values* $\|x^j(t-1|t-1) - x_{in}\|, j \in I$;
2: **map** *the integer* i *into* $\mathcal{H}^{level(i)}$, *i.e. the position* i^{agg};
3: $\mathcal{N}^j \leftarrow \mathcal{N}^j \setminus \{i\}, \forall j \in \mathcal{N}^i$;
4: $\mathcal{H}^i \leftarrow \mathcal{H}^i \setminus \{i^{agg}\}$;
5: $count^{level(i)} \leftarrow count^{level(i)} + 1$;

LF-Disaggregation (LF-Dis) - i-th UAV

Input: $count^s, s = 1, \dots, r$;
Output: $level(i)$, $\mathcal{N}^j$, $count^s, s = 1, \dots, r$;

1: **find** $j = \arg\min_s count^s > 0$
2: **assign** $level(i) := j$;
3: $\mathcal{N}^j \leftarrow \mathcal{N}^j \cup \{i\}$
4: $count^j \leftarrow count^j - 1$;

Notice that the activation of the **LF-Agg** procedure reduces the number of elements belonging to the grid until an entire level is reset; at the same time, the platoon configuration is going to built up within the corridor starting from a single UAV (the trivial case). Conversely, the **LF-Dis** procedure empties the platoon by jointly rebuilding the nominal grid. Finally, feasibility issues are below summarized:

Proposition *1:* Given the optimal solution $\mathbf{u}^{i^*}(t)$ of the optimization **DMPC**-$\mathcal{P}_F^i(t)$. Then at $t+1$, there always exists an admissible solution of **DMPC**-$\mathcal{P}_F^i(t+1)$ for any obstacle occurrence $O^t \in \{O^i\}_{i=1}^{n_{sc}}$.

Proposition *2:* Let the sequences of sets $\mathcal{T}_j^i$ be non-empty and $x(0) \in \bigcup_j \{\mathcal{T}_j^1 \times \cdots \times \mathcal{T}_j^L\}$. Then, the **DRHC-LF** algorithm always satisfies the constraints for any admissible obstacle scenario.

Proofs - Omitted for the sake of space. □

V. The distributed MPC algorithm

Under the hypothesis that $x^i(0), x_f^i, i = 1, \dots, L$ are known and a feasible input sequence $\{u^i(k|0)\}_{k=0}^{N_i-1}$ has been off-line determined a distributed MPC algorithm, hereafter denoted as **DMPC**, can be formally stated.

DMPC-Algorithm - UAV $i-th$

Input: α_{min}^c, α_{max}^c, β, γ, $x^i(0)$, x_f^i
Output: $u^{i^*}(t/t)$
Initialization: N_i, $\mathcal{N}^i$

1: **if** detected == false **then**
2: **Determine** the update terminal region $\Xi^i(t)$ via (14) (respectively (15));
3: **Solve** optimization (17)-(23);
4: **Compute** the assumed state sequence $\hat{\mathbf{x}}^i(t)$ according to (16);
5: **Transmit** $\hat{\mathbf{x}}^i(t)$ to all $post(i)$ vehicles;
6: **Receive** $\hat{\mathbf{x}}^j(t)$ from the neighbours $j \in \mathcal{N}^i$ and $\hat{\mathbf{x}}^{pre(i)}(t), i \geq 2$, from the UAV $pre(i)$;
7: **Apply** $u^{i^*}(t|t)$;
8: $t \leftarrow t+1$ and goto **Step 2**
9: **else**
10: **Acquire** the obstacle scenario sc;
11: **if** $x^i(t) \in \mathcal{B}(x_{in}^{sc}, R_{in}^{sc})$ **then Activate** the **LF-Agg** procedure;
12: **Transmit** $\mathcal{N}^j$, $\mathcal{H}^{level(i)}$ to all $j \in \mathcal{N}^i$;
13: $i \leftarrow i^{agg}$;
14: **Activate** the **DRHC-LF** algorithm;

```
15:         Goto Step 7
16:     else if x^i(t) ∈ B(x^{sc}_{fin}, R^{sc}_{fin}) then Activate the LF-Dis procedure;
17:         Transmit count^s, s = 1,...,r, to the (i+1)-th UAV;
18:         detected:= false;
19:         Activate the DRHC-LF algorithm;
20:         Goto Step 7
21:     else if x^i(t) ∈ ⋃_j T^i_j then Activate the DRHC-LF algorithm;
22:         Goto Step 7
23:     else
24:         Solve optimization (17)-(23) subject to the convexified additional constraint (6);
25:         Goto Step 4
26: end if
```

Theorem 1: Let the initial $x(0) = [x^{1^T}(0), x^{2^T}(0), \ldots, x^{L^T}(0)]^T$ and the target $x_f = [x_f^{1^T}, x_f^{2^T}, \ldots, x_f^{L^T}]^T$ conditions be given. Then, the **DMPC** Algorithm always satisfies constraints and ensures Uniformly Ultimately Bounded (UUB) for all occurrences of $O^t \in \{O^i\}_{i=1}^{n_{sc}}$.

Proof - Omitted for the sake of space. □

VI. Simulations

In this section, the effectiveness of the **DMPC** algorithm is investigated by considering a formation of 5 rotorcraft UAV operating within the obstacle scenario of Fig. 3.

According to [15], each $i-th$ rotorcraft UAV is modeled as a double integrator

$$x^i(t+1) = \begin{bmatrix} I_2 & \Delta T_s I_2 \\ 0_2 & I_2 \end{bmatrix} x^i(t) + \begin{bmatrix} \frac{(\Delta T_s)^2 I_2}{2} \\ \Delta T_s I_2 \end{bmatrix} u^i(t) \quad (33)$$

with $T_s = 0.5\,s$ the sampling time, $x^i = [p^i_x, p^i_y, v^i_x, v^i_y]^T \in \mathbf{R}^4$ and $u^i = [a^i_x, a^i_y]^T \in \mathbf{R}^2$ the state space and acceleration vectors, respectively. Moreover, the following component-wise constraints are prescribed:

$$\begin{array}{c} |u^i_j(t)| \leq 0.5[m/s^2], \;\; j = 1,2 \\ -25 \leq p^i_x \leq 17[m], \;\; -45 \leq p^i_y \leq 50[m], i = 1,\ldots,5. \end{array} \quad (34)$$

The obstacle scenario consists of the two narrow corridors of Fig. 3, namely **C1** and **C2**. Starting from the nominal grid the UAVs aim is that the leader **UAV 1** is capable to reach the goal location $x^1_f = [25, 15, 0, 0]^T$ despite of any obstructions. The following knobs have been exploited: $R = 6.5\,m$ $\alpha^c_{min} = 0.5\,[m]$, $\alpha^c_{max} = 30\,m$, $R^{sc}_{in} = R^{sc}_{fin} = 1\,m, sc = 1,2$; $x^1_{in} = [-4.3\;-21.5\;0\;0]^T$, $x^1_{fin} = [-4.3\;0\;0\;0]^T$, $x^2_{in} = [4.5\;6.75\;0\;0]^T$ and $x^2_{fin} = [4.5\;28\;0\;0]^T$. While the off-line phase of the **DRHC-LF** algorithm gave rise to the families of one-step state ahead controllable sets $\{\mathcal{T}_j\}_{j=1}^{20}$ and $\{\mathcal{T}_j\}_{j=1}^{25}$, in charge to cover the corridor regions **C1** and **C2**, respectively.

Notice that the team-leader accomplishes the prescribed mission Fig. 3 by always fulfilling input, environmental and formation constraints.

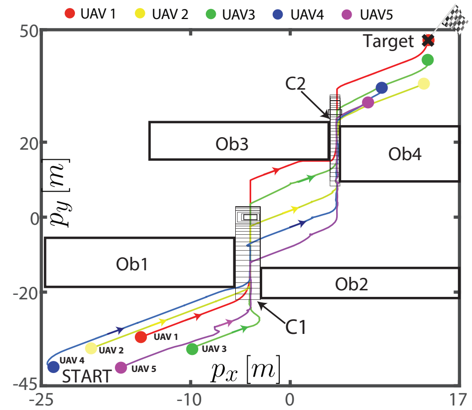

Fig. 3. Working environment and UAV trajectories

VII. Conclusions

In this paper, an *ad-hoc* MPC-based control architecture has been presented for UAV formations. By considering operating scenarios characterized by obstacle free regions described as narrowed corridors, the obstacle avoidance and path planning problem has been faced by means of two robust MPC schemes capable to properly take advantage from the topology peculiarities.

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