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A reconfigurable PID fault tolerant tracking controller design for LPV systems

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ABSTRACT

This paper considers the design of a reconfigurable PID Fault Tolerant Tracking Controller (PID-FTTC) for Linear Parameter Varying (LPV) systems affected by actuator faults with the presence of disturbance. The LPV systems are represented through a polytopic LPV description with measurable gain scheduling functions. A new PID-FTTC scheme with a model reference, an adaptive PID controller and an Adaptive Polytopic Observer (APO), is developed. The main idea is to improve and to compare performances with this developed PID-FTTC versus previous similar FTC techniques especially about the settling time, the overshoot and integral error indices. By the way, this paper can reduce the conservatism of previous methods with more parameters design so as to avoid their disadvantages and to give better control loop performances especially in terms of accuracy and speed of trajectory tracking even when a fault occurs. So, in order to establish the stability of the reconfigured PID-FTTC, a new theoretical study is developed through the use of Linear Matrix Inequality (LMI). This new method is illustrated through a two-tank process where the results compared to previous ones, underline the improvements.

1. Introduction

Modern technological systems rely on sophisticated control systems to meet performance and safety requirements. A conventional feedback control design for a complex system may result in unsatisfactory performance, or even instability, in the event of malfunctions in actuators, sensors or other system components. To overcome such weaknesses, new approaches to control system design have been developed in order to tolerate component malfunctions while maintaining the required levels of stability and performances. Therefore, the demand for reliability, safety and fault tolerance is generally high. It is necessary to design control systems which are able of tolerating potential faults in these systems in order to improve the reliability and availability while providing desirable performance. More precisely, Fault Tolerant Control Systems (FTCS) are control systems that possess the ability to accommodate component failures automatically. They are capable of maintaining overall system stability and acceptable performance in the event of such failures [1].

Because of such high demand for reliability, safety and fault tolerance, the design and the reconfiguration of FTCS attracts more and more the interests of researchers and becomes an important area of research. By the way, the authors in [2] have developed a virtual actuator approach in order to tolerate actuator faults affected Linear Parameter Varying (LPV) systems. The authors have implemented a Fault Tolerant Controller (FTC) which has been considered as a state feedback controller. This control structure has also been designed in [3] so as to develop a FTC strategy which has aimed to compensate constant/time varying actuator faults affecting LPV descriptor systems. The authors in [4] have proposed a FTC approach using virtual actuators and sensors for LPV systems. The idea was to reconfigure the virtual actuator/sensor on-line by taking into account faults and operating point changes. These methodologies have allowed to ensure the stability performance with fault compensation but they can lead to inaccurate responses.

In [5], a reference model technique has been proposed for the trajectory tracking of a four-wheeled omnidirectional mobile robot. This technique has been based on designing an error feedback controller and by using a switching LPV virtual actuator; this signal has been added to the controller in order to compensate the faults and make the reconfigured system to be tolerant to it. The authors in [6] have dealt with the problem of reference tracking FTC control for LPV systems. The proposed FTC strategy has been based on the reconfiguration of the reference model by using a virtual actuator block. In [7], the authors have considered a model reference fault tolerant tracking control issue for uncertain Takagi–Sugeno (T–S) fuzzy systems with unmeasurable premise variables and under unknown inputs. These proposed reference

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model FTC strategies developed in these previous mentioned papers [5,7] and [6] have allowed to guarantee the stability and the trajectory tracking performances despite faults occurrence. However, all these techniques often lead to insufficient accuracy and slow transient responses which deteriorate the performances of the reconfigured system.

This fact has been taken into account in [8] where the authors have developed a new approach in order to deal with the topic of model reference tracking based on an active fault tolerant control for LPV systems affected by actuator faults. This proposed strategy, therein, has designed an adaptive Proportional Integral (PI) controller so as to allow the closed loop system can ensure the performances of stability and tracking trajectories with accurate responses. However, this strategy may lead sometimes to slow responses which would damage the global performances of the system.

Moreover, Proportional Integral Derivative (PID) controllers have been used for FTC as in [9] with a single-link flexible manipulator under actuator faults, in [10] for a quadrotor with actuator fault, in [11] with state and static output feedback with multi-performance indices constraints and in [12] with an active suspension system in the presence of uncertainties and actuator faults. However, the main criticism is that even if all these papers deal with PID structure, they are only made for linear systems which is very restrictive; moreover, most of them are based on robust strategies and not based on 'active' strategies which take into account the magnitude of the fault. Those previous mentioned techniques allow to ensure some performances for some small and pre-defined faults but not always for time-varying faults. So, we want here to develop a strategy which could compensate on-line the faults effects through an adaptive PID controller for LPV systems.

This new paper focuses on extending an idea presented in [8] in order to improve performances of the LPV system especially about the quick response of a PID vs a PI controller for such systems. This paper can reduce the conservatism of the previous method so as to avoid their disadvantages and to give better control loop performances especially in terms of accuracy and speed of trajectory tracking even when a fault occurs. Indeed, this paper generalizes the previous PI approach through a more complex theoretical PID development with a new FTC control law. So, in order to establish the new effective PID Fault Tolerant Tracking Controller, it is necessary to guarantee the asymptotic convergence of the faulty LPV system outputs to the reference model outputs. Thus, the stabilization of the reconfigured model is studied by a new LMI development. Moreover, the comparisons of these two methods on a two-tank process will underline the contributions brought by this paper.

So, this presented approach integrates a model reference, a new adaptive PID controller and an Adaptive Polytopic Observer (APO) for LPV systems. This approach aims to deal with the problem of fault tolerant tracking control for LPV systems affected by actuator faults in the presence of unknown inputs. The principal contribution is to synthesize an adaptive PID controller such that its reconfigured control law contains a term for faults compensation. To achieve the on-line reconfiguration of the designed controller, the APO is employed as a Fault Detection and Diagnosis (FDD) module which provides on-line information about faults estimation. Besides of the fault effects compensation and the model reference tracking, the provided control signal leads systems to be stable and to have accurate and quick responses.

The outline of the paper is organized as follows. First, an introduction for LPV systems under measurable gain scheduling functions structures are given and the problem statement of the paper is formulated in Section 2. In Section 3, the design of the PID fault tolerant tracking controller is shown. Section 4 shows a comparison of two methods on a two-tank process followed by some concluding remarks in the final section.

Notation. *In this paper, we note I as an identity matrix with appropriate dimension. For a matrix A, $A > 0$ ($A < 0$) defines a strictly positive (negative) definite matrix. In large symmetric matrix expressions, terms denoted $*$ refer to terms induced by symmetry.*

2. Preliminaries and problem statement

We introduce a continuous-time LPV system, affected by additive actuator faults and unknown inputs, whose state space representation is given as follows:

$$\begin{cases} \dot{x}(t) = A(\theta(t))\,x(t) + B(\theta(t))\left(u_f(t) + f(t)\right) + R(\theta(t))\,d(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \Re^n$, $u_f(t) \in \Re^p$ and $y(t) \in \Re^m$ are respectively the state, the control input and the measured output vectors, $f(t) \in \Re^p$ and $d(t) \in \Re^q$ denote the actuator fault and the unknown input vectors respectively. C is a Linear Time Invariant (LTI) matrix but $A(\theta(t))$, $B(\theta(t))$ and $R(\theta(t))$ are time-varying matrices which depend on a bounded parameter vector $\theta(t) \in \Re^l$ that lies into a hypercube with the following description [13]:

$$\theta(t) \in \Im = \{\theta : \theta_{min}(t) \le \theta(t) \le \theta_{max}(t);\ \forall t \ge 0\} \tag{2}$$

Under the assumption of the affine dependence of the parameter vector $\theta(t)$ [3], the system (1) may be modeled by a polytopic LPV form, where a blended representation can be obtained through a convex combination of the LTI models of $\Im$ such that:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{h} \rho_i(\theta(t))\left[A_i x(t) + B_i(u_f(t) + f(t)) + R_i d(t)\right] \\ y(t) = Cx(t) \end{cases} \tag{3}$$

where $\rho(\theta_{(t)})$ are gain scheduling functions which vary into the convex set $\wp$.

$$\wp = \left\{ \rho(\theta(t)) \in \Re^h,\ \rho(\theta(t)) = \left[\rho_1(\theta(t)), \ldots, \rho_h(\theta(t))\right]^T; \right. \\ \left. \rho_i(\theta(t)) \ge 0 \text{ and } \sum_{i=1}^{h} \rho_i(\theta(t)) = 1 \right\} \tag{4}$$

$A_i \in \Re^{n\times n}$, $B_i \in \Re^{n\times p}$, $R_i \in \Re^{n\times q}$ and $C \in \Re^{m\times n}$ are time invariant matrices for the ith linear model, which characterize the summits $\vartheta_i = \begin{bmatrix} A_i & B_i & R_i & C \end{bmatrix}$ of the polytope $\forall i \in \begin{bmatrix} 1, & \ldots, & h \end{bmatrix}$ where $h = 2^l$.

Note that the Linear Time Invariant (LTI) models which made part of the polytopic description of the LPV system, are reconstructed by on-line measurable scheduling functions that depend on several arguments such as the inputs, the system outputs or some exogenous parameters [14].

In this paper, the main idea extends the PI structure used in [8] for the tracking problem of continuous LPV systems affected by actuator faults and unknown inputs. The authors, therein, have used an adaptive polytopic observer in order to estimate faults. The provided information have been employed so as to reconfigure an active fault tolerant controller that has allowed to ensure the tracking performance with accurate responses. However, this previous method can lead to slow and inappropriate transient response which deserves the performances of the system which underlines some conservatism. In order to solve the fault tolerant tracking control problem and to reduce the mentioned conservatism of the approach proposed in [8], we propose to develop a new reconfigurable PID Fault Tolerant Tracking Controller for LPV systems whose scheme is illustrated by Fig. 1. It brings more parameter (with the derivative parameter) to be tuned and the

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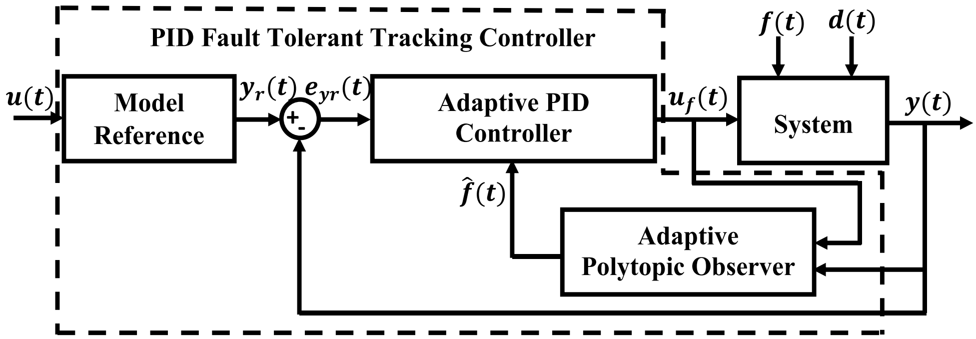

Fig. 1. PID Fault Tolerant Tracking Controller scheme.

theoretical development is bigger but it helps the engineer to perform the obtained responses for the tracking problem of such LPV systems.

This proposed method is related to the design of an adaptive PID controller which needs three types of information that are: the reference outputs, the real outputs and the faults estimation given respectively by a model reference, installed sensors into the real system and an Adaptive Polytopic Observer (APO). The implementation of this closed loop control provides stability and an asymptotical convergence of the outputs towards the reference outputs in spite of both the presence of actuator faults and unknown inputs. Moreover, the goal of this control is also to provide a good accuracy and to improve the rapidity performance whatever the time-varying faults which can occur.

Consider the following reference polytopic LPV model:

$$\begin{cases} \dot{x}_r(t) = \sum_{i=1}^{h} \rho_i(\theta(t))(A_i x_r(t) + B_i u(t)) \\ y_r(t) = C x_r(t) \end{cases} \tag{5}$$

where $x_r(t) \in \Re^n$ is state vector, $u(t) \in \Re^p$ is the control input vector and $y_r(t) \in \Re^m$ is the output vector of the model reference.

The fault estimation $\hat{f}$ is provided by an Adaptive Polytopic Observer (APO) that is given as follows [15]:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) \left[N_i z(t) + G_i u_f(t) + L_i y(t) + B_i \hat{f}(t) \right] \\ \hat{x}(t) = z(t) + T_2 y(t) \\ \hat{y}(t) = C\hat{x}(t) \\ \dot{\hat{f}}(t) = \Gamma \sum_{i=1}^{h} \rho_i(\theta(t)) \Phi_i \left(\dot{e}_y(t) + \sigma e_y(t) \right) \\ e_y(t) = y(t) - \hat{y}(t) \end{cases} \tag{6}$$

where $z(t)$ is the observer state vector, $\hat{x}(t)$ the estimated state vector, $\hat{y}(t)$ is the estimated output vector, $\hat{f}(t)$ is the estimated actuator fault and $e_y(t)$ is the output estimation error. $N_i \in \Re^{n\times n}$, $G_i \in \Re^{n\times p}$, $L_i \in \Re^{n\times m}$, $\Phi_i \in \Re^{p\times m}$ and $T_2 \in \Re^{n\times m}$ are unknown matrices with appropriate dimensions to be determined afterwards. The matrix $\Gamma \in \Re^{p\times p}$ is a positive definite learning rate and σ is a positive scalar. Throughout this paper, the following assumptions are necessary:

Assumption 1. We assume that the actuator fault distribution matrix B_i is of full column rank $\forall i = 1, \ldots, h$; i.e.:

$$rank\,(CB_i) = rank\,(B_i) = p \tag{7}$$

Assumption 2. The polytopic LPV system (5) is observable; i.e.:

$$rank \begin{pmatrix} C \\ CA_i \\ \vdots \\ CA_i^{\,n-1} \end{pmatrix} = n;\ \forall i = 1, \ldots, h \tag{8}$$

Assumption 3. The input vector $u(t)$ is bounded as $\|u(t)\| \le \alpha_1$. The fault $f(t)$ satisfies $\|f(t)\| \le \alpha_2$, its derivative is bounded such that $\|\dot{f}(t)\| \le \alpha_3$ and the unknown input vector $d(t)$ verifies $\|d(t)\| \le \alpha_4$, where $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \ge 0$.

In this paper, the main objective is to synthesize a new PID Fault Tolerant Tracking Controller which allows that the trajectories tracking performance of the faulty LPV system to the reference model is ensured. For such purpose, we propose the following structure of FTC law:

$$u_f(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) \left(K_{Pi} e_{yr}(t) + K_{Ii} \int_0^t e_{yr}(t)\,dt + K_{Di} \frac{de_{yr}(t)}{dt} - K_{fi} \hat{f}(t) \right) \tag{9}$$

where $K_{Pi} \in \Re^{p\times m}$, $K_{Ii} \in \Re^{p\times m}$, $K_{Di} \in \Re^{p\times m}$ and $K_{fi} \in \Re^{p\times p}$ are control gain matrices to be designed.

For the control problem to make sense, the polytopic LPV system (1) is assumed controllable.

Generally, the Proportional (P) action of the control algorithm provides the most part of the control signal to operate the actuators. The Integral (I) term is used for the accuracy of the control, to reduce some errors. The third part of the expression of the control law, which is Derivative (D), acts for quick transient variations (high frequencies). The term D is generally working to oppose the abrupt changes caused by the term P. However, the fourth part ($K_{fi}\hat{f}(t)$) is used to compensate changes produced by the faults occurrence. Hence, the proposed control law structure aims to ensure the stability, to track the output references with good accuracy, to improve the rapidity of the responses and to compensate the fault effects.

3. PID fault tolerant tracking controller design

3.1. System reconfiguration

Here, the purpose is to reconfigure the system through an augmented model including principally the state vector $x(t)$, the state of the tracking error $e_t(t)$, the state of the estimation error

$e_s(t)$ and the fault estimation error $e_f(t)$ such as the following augmented state vector $\tilde{x}(t)$ is defined by [16]:

$$\tilde{x}(t) = \begin{pmatrix} x(t) \\ e_t(t) \\ x_t(t) \\ e_s(t) \\ e_f(t) \end{pmatrix} = \begin{pmatrix} x(t) \\ x_r(t) - x(t) \\ \int_0^t e_t(t)\,dt \\ x(t) - \hat{x}(t) \\ f(t) - \hat{f}(t) \end{pmatrix} \tag{10}$$

According to [11], let consider the new state vector $x_t(t)$ defined in (10) such that its dynamic is given as follows:

$$\dot{x}_t(t) = e_t(t) \tag{11}$$

By taking into account the state space representations (3) and (5) and the equation of $e_t(t)$ in (10), the expression of the error $e_{yr}(t)$ will be:

$$e_{yr}(t) = y_r(t) - y(t) = Ce_t(t) + \tag{12}$$

By considering the expressions of $e_t(t)$, $x_t(t)$ and $e_f(t)$ in (10), the FTC law (9) becomes:

$$u_f(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) \big(K_{Pi}Ce_t(t) + K_{Ii}Cx_t(t) + K_{Di}C\dot{e}_t(t) + K_{fi}e_f(t) - K_{fi}f(t)\big) \tag{13}$$

Moreover, the state equation of the tracking error is:

$$\dot{e}_t(t) = \dot{x}_r(t) - \dot{x}(t) \tag{14}$$

Now, substituting the expressions of $\dot{x}_r(t)$ and $\dot{x}(t)$ respectively from (3) and (5) on the one hand and those of $u_f(t)$ (13) and $e_f(t)$ from (10) on the other hand into (14), which will be rewritten as:

$$\dot{e}_t(t) = \sum_{i,j=1}^{h} \rho_i(\theta(t)) \rho_j(\theta(t)) \big[A_{ij}e_t(t) - B_iK_{Ij}Cx_t(t) - B_iK_{Dj}C\dot{e}_t(t) - B_iK_{fj}e_f(t) + B_iu(t) + H_{ij}f(t) - R_id(t)\big] \tag{15}$$

with

$$A_{ij} = A_i - B_iK_{Pj}C \tag{16}$$

and

$$H_{ij} = B_iK_{fj} - B_i \tag{17}$$

Then, we can write the following equation:

$$\sum_{i,j=1}^{h} \rho_i(\theta(t)) \rho_j(\theta(t)) \Delta_{ij}\dot{e}_t(t) = \sum_{i,j=1}^{h} \rho_i(\theta(t)) \rho_j(\theta(t)) \big[A_{ij}e_t(t) - B_iK_{Ij}Cx_t(t) - B_iK_{fj}e_f(t) + B_iu(t) + H_{ij}f(t) - R_id(t)\big] \tag{18}$$

with $\Delta_{ij} = I + B_iK_{Dj}C$.

Under the following assumption:

Assumption 4. We assume that $\forall i, j = 1, \dots, h$, the matrices $\Delta_{ij} = I + B_iK_{Dj}C \in \Re^{n\times n}$ are invertible; i.e.:

$$\det\big(\Delta_{ij}\big) \neq 0 \tag{19}$$

$\dot{e}_t(t)$ will be rewritten by the following expression:

$$\dot{e}_t(t) = \sum_{i,j=1}^{h} \rho_i(\theta(t)) \rho_j(\theta(t)) \big[\Delta_{ij}^{-1}A_{ij}e_t(t) - \Delta_{ij}^{-1}B_iK_{Ij}Cx_t(t) - \Delta_{ij}^{-1}B_iK_{fj}e_f(t) + \Delta_{ij}^{-1}B_iu(t) + \Delta_{ij}^{-1}H_{ij}f(t) - \Delta_{ij}^{-1}R_id(t)\big] \tag{20}$$

Substituting (13) into $\dot{x}(t)$ defined in the state space (3), it may be rewritten as follows:

$$\dot{x}(t) = \sum_{i,j=1}^{h} \rho_i(\theta(t)) \rho_j(\theta(t)) \Big[A_ix(t) + B_iK_{Pj}Ce_t(t) + B_iK_{Ij}x_t(t) + B_iK_{Dj}C\dot{e}_t(t) - B_iK_{fj}\hat{f}(t) + B_if(t) + R_id(t)\Big] \tag{21}$$

After that, we substitute $\dot{e}_t(t)$ by its expression (20) and take into account the following relationship:

$$B_iK_{Dj}C\Delta_{ij}^{-1} = I - \Delta_{ij}^{-1} \tag{22}$$

$\dot{x}(t)$ (21) becomes:

$$\dot{x}(t) = \sum_{i,j=1}^{h} \rho_i(\theta(t)) \rho_j(\theta(t)) \big[A_ix(t) + \big(A_i - \Delta_{ij}^{-1}A_{ij}\big)e_t(t) + \Delta_{ij}^{-1}B_iK_{Ij}Cx_t(t) + \Delta_{ij}^{-1}B_iK_{fj}e_f(t) - \Delta_{ij}^{-1}H_{ij}f(t) + \big(I - \Delta_{ij}^{-1}\big)B_iu(t) + \Delta_{ij}^{-1}R_id(t)\big] \tag{23}$$

Before studying the state dynamic of the estimation error $e_s(t)$ defined in (10), it is assumed that there exist two matrices $T_1 \in \Re^{n\times n}$ and $T_2 \in \Re^{n\times m}$ such that the following relation holds true:

$$T_1 + T_2C = I_n \tag{24}$$

This one may be rewritten as follows:

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} \begin{bmatrix} I_n \\ C \end{bmatrix} = I_n \tag{25}$$

A particular solution for getting matrices T_1 and T_2 is to use a generalized inverse matrix such that:

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} = \begin{bmatrix} I_n \\ C \end{bmatrix}^{+} \tag{26}$$

Thus, by taking (6) and (24), the equation of $e_s(t)$ defined in (10) can be rewritten as follows:

$$e_s(t) = T_1x(t) - z(t) \tag{27}$$

So, the state dynamic of the estimation error (27) is described as follows:

$$\dot{e}_s(t) = T_1\dot{x}(t) - \dot{z}(t) \tag{28}$$

Then, we use the equations of $\dot{x}(t)$ and $\dot{z}(t)$ respectively from (3) and (5) in order to rewrite $\dot{e}_s(t)$ (28) as:

$$\dot{e}_s(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) \big[(T_1A_i + E_iC - N_i)x(t) + (T_1B_i - G_i)u(t) + N_ie_s(t) + B_ie_f(t) + M_if(t) + \bar{R}_id(t)\big] \tag{29}$$

where

$$E_i = N_iT_2 - L_i \tag{30}$$

$$M_i = T_1B_i - B_i \tag{31}$$

$$\text{and } \bar{R}_i = T_1R_i \tag{32}$$

By assuming that the following conditions are true for all $i = 1, \dots, h$:

$$T_1A_i + E_iC - N_i = 0 \tag{33}$$

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$$\text{and } T_1B_i - G_i = 0 \tag{34}$$

then the estimation error dynamic (29) is reduced to:

$$\dot{e}_s(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) \left[N_i e_s(t) + B_i e_f(t) + M_i f(t) + \bar{R}_i \bar{d}(t)\right] \tag{35}$$

The fault estimation error is described such that:

$$\dot{e}_f(t) = \dot{f}(t) - \dot{\hat{f}}(t) \tag{36}$$

3.2. Stability study

Two lemmas are used in the remainder of this paper:

Lemma 1 ([17]). *For a matrix* $P = P^T > 0$ *and a scalar* $\mu > 0$, *the following inequality holds:*

$$2x^T y \leq \frac{1}{\mu} x^T P x + \mu y^T P^{-1} y;\ x, y \in \Re^n \tag{37}$$

Lemma 2 ([14]). *Given real matrices X, Y and Z of appropriate dimensions and a matrix* Δ *satisfying* $\Delta^T \Delta \leq I$, *then:*

$$X + Y\Delta Z + Z^T \Delta^T Y^T < 0 \tag{38}$$

if and only if there exists a scalar $\psi > 0$ *verifying the inequality:*

$$X + \psi Z^T Z + \frac{1}{\psi} Y Y^T < 0 \tag{39}$$

which is equivalent to:

$$\begin{bmatrix} X & Y & \psi Z^T \\ Y^T & -\psi I & 0 \\ \psi Z & 0 & -\psi I \end{bmatrix} < 0 \tag{40}$$

In order to establish an effective PID Fault Tolerant Tracking Controller, it is necessary to guarantee the asymptotic convergence of the faulty LPV system outputs to the reference model ones. This requires the stabilization of the reconfigured model. To solve this problem, sufficient conditions are given in the next theorem.

Theorem 1. *Let assume a given polytopic LPV system (3), a PID Fault Tolerant Control (PID-FTC) (9), a reference model (5) and an Adaptive Polytopic Observer (APO) (6). For some positive scalars* σ, μ, β *and* ψ *and a positive definite matrix* Γ, *the faulty state vector* $x(t)$, *the tracking error vector* $e_t(t)$, *the estimation error vector* $e_s(t)$ *and the fault estimation error* $e_f(t)$ *are bounded if there exist positive definite matrices* $X_1 = P_1^{-1}$, $X_2 = P_2^{-1}$, $X_3 = P_3^{-1}$, Q_1, Q_2, Q_3 *and* Q_4 *and matrices* $W_{Pj} = K_{Pj}CX_2$, $W_{Ij} = K_{Ij}CX_2$, $W_{Dij} = \left(I + B_i K_{Dj} C\right)^{-1}$, $S_i = E_i C X_3$, K_{fj} *and* Φ_i *such that the following LMIs are satisfied for all* $i, j, k = 1, \ldots, h$:

$$\begin{pmatrix} \bar{X}_{ijk} & Y_i & \psi Z_k^T & P \\ * & -\psi I & 0 & 0 \\ * & * & -\psi I & 0 \\ * & * & * & -Q \end{pmatrix} < 0 \tag{41}$$

where

$$\bar{X}_{ijk} = \begin{pmatrix} \Pi_i & A_i X_2 & 0 & 0 & 0 \\ * & -\beta I & X_2 & 0 & 0 \\ * & * & -\beta I & 0 & 0 \\ * & * & * & \Theta_i & B_i \\ * & * & * & * & \Sigma_{ik} \end{pmatrix} \tag{42}$$

$$Y_i = \begin{pmatrix} W_{Dij} & 0 & 0 & 0 & 0 \\ -W_{Dij} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\Phi_i C & 0 & 0 \end{pmatrix} \tag{43}$$

$$Z_k^T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ -X_2 A_i^T + W_{Pj}^T B_i^T & 0 & 0 & 0 & 0 \\ W_{Ij}^T B_i^T & 0 & 0 & 0 & 0 \\ 0 & 0 & \Xi_k^T & 0 & 0 \\ K_{fj}^T B_i^T & 0 & 0 & 0 & 0 \end{pmatrix} \tag{44}$$

$$P = \begin{pmatrix} X_1^T & 0 & 0 & 0 & 0 \\ 0 & X_2^T & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \\ 0 & 0 & X_3^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{45}$$

$$\text{and } Q = \begin{pmatrix} \frac{3}{\mu} Q_1 & 0 & 0 & 0 & 0 \\ 0 & \frac{3}{\mu} Q_2 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{\mu} Q_3 & 0 & 0 \\ 0 & 0 & 0 & \beta I & 0 \\ 0 & 0 & 0 & 0 & \beta I \end{pmatrix} \tag{46}$$

with

$$\Pi_i = A_i X_1 + X_1 A_i^T \tag{47}$$

$$\Theta_i = T_1 A_i X_3 + X_3 A_i^T T_1^T + S_i + S_i^T \tag{48}$$

$$\Xi_k = X_3 + \frac{1}{\sigma}\left(T_1 A_k X_3 + S_k\right) \tag{49}$$

$$\Sigma_{ik} = -\frac{1}{\sigma}\left(\Phi_i C B_k + B_k^T C^T \Phi_i^T\right) + \frac{3}{\mu\sigma} Q_4 \tag{50}$$

The matrix gains of the controller (9) and the observer (6) are expressed by:

$$K_{Pj} = W_{Pj} P_2 C^{-1} \tag{51}$$

$$K_{Ij} = W_{Ij} P_2 C^{-1} \tag{52}$$

$$K_{Dj} = B_i^{-1}\left(W_{Dij}^{-1} - I\right) C^{-1} \tag{53}$$

$$N_i = T_1 A_i + E_i C \tag{54}$$

$$L_i = N_i T_2 - E_i \tag{55}$$

$$\text{and } G_i = T_1 B_i \quad \blacksquare \tag{56}$$

Proof. In the objective to stabilize the reconfigured model where its states are $x(t)$, $e_t(t)$, $x_t(t)$ $e_s(t)$ and $e_f(t)$, we will use the theory of Lyapunov. Then, the problem is turned into an optimization problem through the use of LMI so as to design the unknown matrices of both the proposed FTC controller and the observer. Thus, let us introduce a quadratic Lyapunov function defined such that:

$$\begin{aligned} V\left(\tilde{x}(t)\right) &= x^T(t) P_1 x(t) + e_t^T(t) P_2 e_t(t) + x_t^T(t) P_2 x_t(t) \\ &\quad + e_s^T(t) P_3 e_s(t) + \frac{1}{\sigma} e_f^T(t)\, \Gamma^{-1} e_f(t) \end{aligned} \tag{57}$$

where P_1, P_2, P_3 and Γ^{-1} are positive definite matrices with appropriate dimensions.

Remark 1. Despite the fact that the use of a non-quadratic Lyapunov function can enhance the system's tracking performance [18], we use a quadratic one in order to facilitate the mathematical complexity of the studied problem.

The time derivative of the quadratic Lyapunov function (57) leads to:

$$\begin{aligned} \dot{V}\left(\tilde{x}(t)\right) &= \dot{x}^T(t) P_1 x(t) + x^T(t) P_1 \dot{x}(t) + \dot{e}_t^T(t) P_2 e_t(t) \\ &\quad + e_t^T(t) P_2 \dot{e}_t(t) + \dot{x}_t^T(t) P_2 x_t(t) \\ &\quad + x_t^T(t) P_2 \dot{x}_t(t) + \dot{e}_s^T(t) P_3 e_s(t) + e_s^T(t) P_3 \dot{e}_s(t) \\ &\quad + \frac{1}{\sigma} \dot{e}_f^T(t)\, \Gamma^{-1} e_f(t) \\ &\quad + \frac{1}{\sigma} e_f^T(t)\, \Gamma^{-1} \dot{e}_f(t) \end{aligned} \tag{58}$$

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By considering the relations (23) of $\dot{x}(t)$, (20) of $\dot{e}_t(t)$, (11) of $\dot{x}_t(t)$, (35) of $\dot{e}_s(t)$ and (36) of $\dot{e}_f(t)$ and by taking using the fault estimation of $\dot{\hat{f}}(t)$ in (6), Eq. (58) leads to:

$$\begin{aligned}
&\dot{V}\left(\tilde{x}(t)\right) \\
&= \sum_{i,j=1}^{h} \rho_i\left(\theta(t)\right)\rho_j\left(\theta(t)\right)\Big[x^T(t)\left(A_i^T P_1 + P_1 A_i\right)x(t) + 2x^T(t)P_1\big(A_i \\
&-\Delta_{ij}^{-1}A_{ij}\big)e_t(t) + 2x^T(t)P_1\Delta_{ij}^{-1}B_iK_{lj}Cx_t(t) + 2x^T(t)P_1\Delta_{ij}^{-1}B_iK_{fj}e_f(t) \\
&+ 2x^T(t)P_1\left(I-\Delta_{ij}^{-1}\right)B_iu(t) - 2x^T(t)P_1\Delta_{ij}^{-1}H_{ij}f(t) \\
&+ 2x^T(t)P_1\Delta_{ij}^{-1}R_id(t) + e_t^T(t)\left(A_{ij}^T\left(\Delta_{ij}^{-1}\right)^T P_2 + P_2\Delta_{ij}^{-1}A_{ij}\right)e_t(t) \\
&+ 2e_t^T(t)\left(P_2 - P_2\Delta_{ij}^{-1}B_iK_{lj}C\right)x_t(t) - 2e_t^T(t)P_2\Delta_{ij}^{-1}B_iK_{fj}e_f(t) \\
&+ 2e_t^T(t)P_2\Delta_{ij}^{-1}B_iu(t) + 2e_t^T(t)P_2\Delta_{ij}^{-1}H_{ij}f(t) - 2e_t^T(t)P_2\Delta_{ij}^{-1}R_id(t) \\
&+ e_s^T(t)\left(N_i^TP_3 + P_3N_i\right)e_s(t) + 2e_s^T(t)\left(P_3B_i - C^T\Phi_i^T\right)e_f(t) \\
&+ 2e_s^T(t)P_3M_if(t) + 2e_s^T(t)P_3\bar{R}_id(t) - \frac{1}{\sigma}\dot{e}_s^T(t)C^T\Phi_i^Te_f(t) \\
&- \frac{1}{\sigma}e_f^T(t)\Phi_iC\dot{e}_s(t) + \frac{2}{\sigma}e_f^T(t)\,\Gamma^{-1}\dot{f}(t)\Big]
\end{aligned} \tag{59}$$

Now, substituting $\dot{e}_s(t)$ by its expression (35) in Eq. (59) which will be given by:

$$\begin{aligned}
&\dot{V}\left(\tilde{x}(t)\right) \\
&= \sum_{i,j,k=1}^{h} \rho_i\left(\theta(t)\right)\rho_j\left(\theta(t)\right)\rho_k\left(\theta(t)\right)\Big[x^T(t)\left(A_i^TP_1 + P_1A_i\right)x(t) + 2x^T(t)P_1\big(A_i \\
&-\Delta_{ij}^{-1}A_{ij}\big)e_t(t) + 2x^T(t)P_1\Delta_{ij}^{-1}B_iK_{lj}Cx_t(t) + 2x^T(t)P_1\Delta_{ij}^{-1}B_iK_{fj}e_f(t) \\
&+ 2x^T(t)P_1\left(I-\Delta_{ij}^{-1}\right)B_iu(t) - 2x^T(t)P_1\Delta_{ij}^{-1}H_{ij}f(t) + 2x^T(t)P_1\Delta_{ij}^{-1}R_id(t) \\
&+ e_t^T(t)\left(A_{ij}^T\left(\Delta_{ij}^{-1}\right)^TP_2 + P_2\Delta_{ij}^{-1}A_{ij}\right)e_t(t) \\
&+ 2e_t^T(t)\left(P_2 - P_2\Delta_{ij}^{-1}B_iK_{lj}C\right)x_t(t) \\
&- 2e_t^T(t)P_2\Delta_{ij}^{-1}B_iK_{fj}e_f(t) + 2e_t^T(t)P_2\Delta_{ij}^{-1}B_iu(t) + 2e_t^T(t)P_2\Delta_{ij}^{-1}H_{ij}f(t) \\
&- 2e_t^T(t)P_2\Delta_{ij}^{-1}R_id(t) + e_s^T(t)\left(N_i^TP_3 + P_3N_i\right)e_s(t) + 2e_s^T(t)\Big(P_3B_i - C^T\Phi_i^T \\
&-\frac{1}{\sigma}N_k^TC^T\Phi_i^T\Big)e_f(t) + 2e_s^T(t)P_3M_if(t) + 2e_s^T(t)P_3\bar{R}_id(t) - \frac{1}{\sigma}e_f^T(t)\big(\Phi_iCB_k \\
&+B_k^TC^T\Phi_i^T\big)e_f(t) - \frac{2}{\sigma}e_f^T(t)\Phi_iCM_kf(t) + \frac{2}{\sigma}e_f^T(t)\Phi_iC\bar{R}_kd(t) \\
&+\frac{2}{\sigma}e_f^T(t)\,\Gamma^{-1}\dot{f}(t)\Big]
\end{aligned} \tag{60}$$

Under the Assumption 3, we apply the Lemma 1 on the following term from Eq. (60) which can be bounded such as:

$$\begin{aligned}
&2x^T(t)P_1\left(I-\Delta_{ij}^{-1}\right)B_iu(t) \\
&\quad\le \frac{1}{\mu}x^T(t)Q_1x(t) + \mu u^T(t)B_i^T\left(I-\Delta_{ij}^{-1}\right)^T \\
&\quad\quad P_1Q_1^{-1}P_1\left(I-\Delta_{ij}^{-1}\right)B_iu(t) \\
&\quad\le \frac{1}{\mu}x^T(t)Q_1x(t) + \eta_{1ij}
\end{aligned} \tag{61}$$

In the same way, the following terms can be also bounded:

$$-2x^T(t)P_1\Delta_{ij}^{-1}H_{ij}f(t) \le \frac{1}{\mu}x^T(t)Q_1x(t) + \eta_{2ij} \tag{62}$$

$$2x^T(t)P_1\Delta_{ij}^{-1}R_id(t) \le \frac{1}{\mu}x^T(t)Q_1x(t) + \eta_{3ij} \tag{63}$$

$$2e_t^T(t)P_2\Delta_{ij}^{-1}B_iu(t) \le \frac{1}{\mu}e_t^T(t)Q_2e_t(t) + \eta_{4ij} \tag{64}$$

$$2e_t^T(t)P_2\Delta_{ij}^{-1}H_{ij}f(t) \le \frac{1}{\mu}e_t^T(t)Q_2e_t(t) + \eta_{5ij} \tag{65}$$

$$-2e_t^T(t)P_2\Delta_{ij}^{-1}R_id(t) \le \frac{1}{\mu}e_t^T(t)Q_2e_t(t) + \eta_{6ij} \tag{66}$$

$$2e_s^T(t)P_3M_if(t) \le \frac{1}{\mu}e_s^T(t)Q_3e_s(t) + \eta_{7i} \tag{67}$$

$$2e_s^T(t)P_3\bar{R}_id(t) \le \frac{1}{\mu}e_s^T(t)Q_3e_s(t) + \eta_{8i} \tag{68}$$

$$-\frac{2}{\sigma}e_f^T(t)\Phi_iCM_kf(t) \le \frac{1}{\mu}e_f^T(t)Q_4e_f(t) + \eta_{9ik} \tag{69}$$

$$\frac{2}{\sigma}e_f^T(t)\Phi_iC\bar{R}_kd(t) \le \frac{1}{\mu}e_f^T(t)Q_4e_f(t) + \eta_{10ik} \tag{70}$$

$$\frac{2}{\sigma}e_f^T(t)\,\Gamma^{-1}\dot{f}(t) \le \frac{1}{\mu}e_f^T(t)Q_4e_f(t) + \eta_{11} \tag{71}$$

with

$$\eta_{1ij} = \mu\alpha_1^2\lambda_{\max}\left(B_i^T\left(I-\Delta_{ij}^{-1}\right)^TP_1Q_1^{-1}P_1\left(I-\Delta_{ij}^{-1}\right)B_i\right) \tag{72}$$

$$\eta_{2ij} = \mu\alpha_2^2\lambda_{\max}\left(H_{ij}^T\left(\Delta_{ij}^{-1}\right)^TP_1Q_1^{-1}P_1\Delta_{ij}^{-1}H_{ij}\right) \tag{73}$$

$$\eta_{3ij} = \mu\alpha_4^2\lambda_{\max}\left(R_i^T\left(\Delta_{ij}^{-1}\right)^TP_1Q_1^{-1}P_1\Delta_{ij}^{-1}R_i\right) \tag{74}$$

$$\eta_{4ij} = \mu\alpha_1^2\lambda_{\max}\left(B_i^T\left(\Delta_{ij}^{-1}\right)^TP_2Q_2^{-1}P_2\Delta_{ij}^{-1}B_i\right) \tag{75}$$

$$\eta_{5ij} = \mu\alpha_2^2\lambda_{\max}\left(H_{ij}^T\left(\Delta_{ij}^{-1}\right)^TP_2Q_2^{-1}P_2\Delta_{ij}^{-1}H_{ij}\right) \tag{76}$$

$$\eta_{6ij} = \mu\alpha_4^2\lambda_{\max}\left(R_i^T\left(\Delta_{ij}^{-1}\right)^TP_2Q_2^{-1}P_2\Delta_{ij}^{-1}R_i\right) \tag{77}$$

$$\eta_{7i} = \mu\alpha_2^2\lambda_{\max}\left(M_i^TP_3Q_3^{-1}P_3M_i\right) \tag{78}$$

$$\eta_{8i} = \mu\alpha_4^2\lambda_{\max}\left(\bar{R}_i^TP_3Q_3^{-1}P_3\bar{R}_i\right) \tag{79}$$

$$\eta_{9ik} = \mu\alpha_2^2\lambda_{\max}\left(M_k^TC^T\Phi_i^TQ_4^{-1}\Phi_iCM_k\right) \tag{80}$$

$$\eta_{10ik} = \mu\alpha_4^2\lambda_{\max}\left(\bar{R}_k^TC^T\Phi_i^TQ_4^{-1}\Phi_iC\bar{R}_k\right) \tag{81}$$

$$\eta_{11} = \mu\alpha_3^2\lambda_{\max}\left(\Gamma^{-1^T}Q_4^{-1}\Gamma^{-1}\right) \tag{82}$$

By considering the inequalities (61)–(71), $\dot{V}\left(\tilde{x}(t)\right)$ will be bounded as follows:

$$\begin{aligned}
&\dot{V}\left(\tilde{x}(t)\right) \\
&\le \sum_{i,j,k=1}^{h}\rho_i\left(\theta(t)\right)\rho_j\left(\theta(t)\right)\rho_k\left(\theta(t)\right) \\
&\times\Big[x^T(t)\left(A_i^TP_1 + P_1A_i + \frac{3}{\mu}Q_1\right)x(t) + 2x^T(t)P_1\big(A_i \\
&-\Delta_{ij}^{-1}A_{ij}\big)e_t(t) + 2x^T(t)P_1\Delta_{ij}^{-1}B_iK_{lj}Cx_t(t) + 2x^T(t)P_1\Delta_{ij}^{-1}B_iK_{fj}e_f(t) \\
&+ e_t^T(t)\left(A_{ij}^T\left(\Delta_{ij}^{-1}\right)^TP_2 + P_2\Delta_{ij}^{-1}A_{ij} + \frac{3}{\mu}Q_2\right)e_t(t) + 2e_t^T(t)\big(P_2 \\
&-P_2\Delta_{ij}^{-1}B_iK_{lj}C\big)x_t(t) - 2e_t^T(t)P_2\Delta_{ij}^{-1}B_iK_{fj}e_f(t) + e_s^T(t)\Big(N_i^TP_3 \\
&+P_3N_i + \frac{2}{\mu}Q_3\Big)e_s(t) + 2e_s^T(t)\left(P_3B_i - C^T\Phi_i^T - \frac{1}{\sigma}N_k^TC^T\Phi_i^T\right)e_f(t) \\
&-\frac{1}{\sigma}e_f^T(t)\left(\Phi_iCB_k + B_k^TC^T\Phi_i^T + \frac{3}{\mu\sigma}Q_4\right)e_f(t)\Big] + \delta
\end{aligned} \tag{83}$$

where the scalar δ is the maximum value over i, j and k such that:

$$\begin{aligned}
\delta &= \max_{i,j,k}\big(\eta_{1ij} + \eta_{2ij} + \eta_{3ij} + \eta_{4ij} + \eta_{5ij} \\
&\quad+\eta_{6ij} + \eta_{7i} + \eta_{8i} + \eta_{9ik} + \eta_{10ik} + \eta_{11}\big)
\end{aligned} \tag{84}$$

Then, the above inequality (83) will be rewritten as follows:

$$\dot{V}\left(\tilde{x}(t)\right) \leq \tilde{x}^T(t)\left(\sum_{i,j,k=1}^{h}\rho_i(\theta(t))\rho_j(\theta(t))\rho_k(\theta(t))\Lambda_{ijk}\right) \times \tilde{x}(t) + \delta \tag{85}$$

where Λ_{ijk} is a matrix defined as follows:

$$\Lambda_{ijk} = \begin{pmatrix} \tilde{\Pi}_i & P_1\left(A_i - \Delta_{ij}{}^{-1}A_{ij}\right) & P_1\Delta_{ij}{}^{-1}B_iK_{lj}C & 0 & P_1\Delta_{ij}{}^{-1}B_iK_{fj} \\ * & \tilde{\Omega}_{ij} & P_2{}^T - P_2\Delta_{ij}{}^{-1}B_iK_{lj}C & 0 & -P_2\Delta_{ij}{}^{-1}B_iK_{fj} \\ * & * & 0 & 0 & 0 \\ * & * & * & \tilde{\Theta}_i & \tilde{\Xi}_{ik} \\ * & * & * & * & \Sigma_{ik} \end{pmatrix} \tag{86}$$

with

$$\tilde{\Pi}_i = A_i{}^TP_1 + P_1A_i + \frac{3}{\mu}Q_1 \tag{87}$$

$$\tilde{\Omega}_{ij} = A_{ij}{}^T\left(\Delta_{ij}{}^{-1}\right)^TP_2 + P_2\Delta_{ij}{}^{-1}A_{ij} + \frac{3}{\mu}Q_2 \tag{88}$$

$$\tilde{\Theta}_i = N_i{}^TP_3 + P_3N_i + \frac{2}{\mu}Q_3 \tag{89}$$

$$\tilde{\Xi}_{ik} = P_3B_i - C^T\Phi_i{}^T - \frac{1}{\sigma}N_k{}^TC^T\Phi_i{}^T \tag{90}$$

$$\text{and } \Sigma_{ik} = -\frac{1}{\sigma}\left(\Phi_iCB_k + B_k{}^TC^T\Phi_i{}^T\right) + \frac{3}{\mu\sigma}Q_4 \tag{91}$$

Under the constraint:

$$\sum_{i,j,k=1}^{h}\rho_i(\theta(t))\rho_j(\theta(t))\rho_k(\theta(t))\Lambda_{ijk} < 0 \tag{92}$$

If there exist a positive scalar ε defined by:

$$\varepsilon = \min\lambda_{\min}\left(-\sum_{i,j,k=1}^{h}\rho_i(\theta(t))\rho_j(\theta(t))\rho_k(\theta(t))\Lambda_{ijk}\right) \tag{93}$$

and verifying:

$$\varepsilon\left\|\tilde{x}(t)\right\|^2 > \delta;\ \forall t \geq 0\ (\text{or } -\varepsilon\left\|\tilde{x}(t)\right\|^2 + \delta < 0;\ \forall t \geq 0) \tag{94}$$

The time derivative of the Lyapunov function $\dot{V}\left(\tilde{x}(t)\right)$ (85) will be bounded as follows:

$$\dot{V}\left(\tilde{x}(t)\right) \leq -\varepsilon\left\|\tilde{x}(t)\right\|^2 + \delta \tag{95}$$

This proves that:

$$\dot{V}\left(\tilde{x}(t)\right) < 0 \tag{96}$$

Now, it can be concluded that the augmented system, with state vector $\tilde{x}(t)$, is stable. Thus, we can deduce that all the components of state vector $\tilde{x}(t)$ such that: the faulty state vector $x(t)$, the state vector $x_t(t)$, the tracking error vector $e_t(t)$, the estimation error vector $e_s(t)$ and the fault estimation error vector $e_f(t)$ are bounded.

Thus, by taking into account the constraint (92), we can define the following matrix:

$$X = \begin{pmatrix} P_1{}^{-1} & 0 & 0 & 0 & 0 \\ * & P_2{}^{-1} & 0 & 0 & 0 \\ * & * & P_2{}^{-1} & 0 & 0 \\ * & * & * & P_3{}^{-1} & 0 \\ * & * & * & * & I \end{pmatrix} > 0 \tag{97}$$

such that $\Psi_{ijk} = X\Lambda_{ijk}X < 0$.

Then, using the below change of variables:

$$X_1 = P_1{}^{-1} \tag{98}$$

$$X_2 = P_2{}^{-1} \tag{99}$$

$$X_3 = P_3{}^{-1} \tag{100}$$

so as to compute $\Psi_{ijk} < 0$ that gives:

$$\Psi_{ijk} = \begin{pmatrix} X_1\tilde{\Pi}_iX_1 & \left(A_i - \Delta_{ij}{}^{-1}A_{ij}\right)X_2 & \Delta_{ij}{}^{-1}B_iK_{lj}CX_2 & 0 & \Delta_{ij}{}^{-1}B_iK_{fj} \\ * & X_2\tilde{\Omega}_{ij}X_2 & X_2 - \Delta_{ij}{}^{-1}B_iK_{lj}CX_2 & 0 & -\Delta_{ij}{}^{-1}B_iK_{fj} \\ * & * & 0 & 0 & 0 \\ * & * & * & X_3\tilde{\Theta}_iX_3 & X_3\tilde{\Xi}_{ik} \\ * & * & * & * & \Sigma_{ik} \end{pmatrix} < 0 \tag{101}$$

where

$$X_1\tilde{\Pi}_iX_1 = \Pi_i + \frac{3}{\mu}X_1Q_1X_1 \tag{102}$$

$$X_2\tilde{\Omega}_{ij}X_2 = \Omega_{ij} + \frac{3}{\mu}X_2Q_2X_2 \tag{103}$$

$$X_3\tilde{\Theta}_iX_3 = \Theta_i + \frac{2}{\mu}X_3Q_3X_3 \tag{104}$$

$$\text{and } X_3\tilde{\Xi}_{ik} = B_i - X_3C^T\Phi_i{}^T - \frac{1}{\sigma}X_3N_k{}^TC^T\Phi_i{}^T \tag{105}$$

with

$$\Pi_i = X_1A_i{}^T + A_iX_1 \tag{106}$$

$$\Omega_{ij} = X_2A_{ij}{}^T\left(\Delta_{ij}{}^{-1}\right)^T + \Delta_{ij}{}^{-1}A_{ij}X_2 \tag{107}$$

$$\text{and } \Theta_i = X_3N_i{}^T + N_iX_3 \tag{108}$$

Using the expressions (103) and (105) respectively of $X_2\tilde{\Omega}_{ij}X_2$ and $X_3\tilde{\Xi}_{ik}$ so as to rewrite Ψ_{ijk} (101) as this way:

$$\Psi_{ijk} = X_{ijk} + Y_{ij}IZ_{ijk} + Z_{ijk}{}^TIY_{ij}{}^T < 0 \tag{109}$$

where

$$X_{ijk} = \begin{pmatrix} X_1\tilde{\Pi}_iX_1 & A_iX_2 & 0 & 0 & 0 \\ * & \frac{3}{\mu}X_2Q_2X_2 & X_2 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & X_3\tilde{\Theta}_iX_3 & B_i \\ * & * & * & * & \Sigma_{ik} \end{pmatrix} \tag{110}$$

$$Y_{ij} = \begin{pmatrix} \Delta_{ij}{}^{-1} & 0 & 0 & 0 & 0 \\ -\Delta_{ij}{}^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\Phi_iC & 0 & 0 \end{pmatrix} \tag{111}$$

$$\text{and } Z_{ijk} = \begin{pmatrix} 0 & -A_{ij}X_2 & B_iK_{lj}CX_2 & 0 & B_iK_{fj} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Xi_k & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{112}$$

with

$$\Xi_k = X_3 + \frac{1}{\sigma}N_kX_3 \tag{113}$$

By applying the Lemma 2, Eq. (109) is verified if and only if there exists a scalar $\psi > 0$ satisfying:

$$X_{ijk} + \psi Z_{ijk}{}^TZ_{ijk} + \frac{1}{\psi}Y_{ij}Y_{ij}{}^T < 0 \tag{114}$$

that may be equivalent to:

$$\begin{pmatrix} X_{ijk} & Y_{ij} & \psi Z_{ijk}{}^T \\ * & -\psi I & 0 \\ * & * & -\psi I \end{pmatrix} < 0 \tag{115}$$

Dissociating the term $\frac{3}{\mu}X_1Q_1X_1$ from the inequality (115) in order to reformulate it as follows:

$$\begin{pmatrix} \tilde{X}_{ijk} & Y_{ij} & \psi Z_{ijk}{}^T \\ * & -\psi I & 0 \\ * & * & -\psi I \end{pmatrix} - \bar{X}_1^T\left(-\frac{3}{\mu}Q_1\right)\bar{X}_1 < 0 \tag{116}$$

where

$$\tilde{X}_{ijk} = \begin{pmatrix} \Pi_i & A_iX_2 & 0 & 0 & 0 \\ * & \frac{3}{\mu}X_2Q_2X_2 & X_2 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & X_3\tilde{\Theta}_iX_3 & B_i \\ * & * & * & * & \Sigma_{ik} \end{pmatrix} \tag{117}$$

and

$$\bar{X}_1 = \begin{pmatrix} X_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{118}$$

Now, we apply the modified Schur Lemma on the inequality (116) and then we repeat the same steps successively for $\frac{3}{\mu}X_2Q_2X_2$ and $\frac{2}{\mu}X_3Q_3X_3$ in order to get:

$$\begin{pmatrix} \widehat{X}_{ijk} & Y_{ij} & \psi Z_{ijk}{}^T & \widehat{P} \\ * & -\psi I & 0 & 0 \\ * & * & -\psi I & 0 \\ * & * & * & -\widehat{Q} \end{pmatrix} < 0 \tag{119}$$

where

$$\widehat{X}_{ijk} = \begin{pmatrix} \Pi_i & A_iX_2 & 0 & 0 & 0 \\ * & 0 & X_2 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & \Theta_i & B_i \\ * & * & * & * & \Sigma_{ik} \end{pmatrix} \tag{120}$$

$$\widehat{P} = \begin{pmatrix} X_1{}^T & 0 & 0 \\ 0 & X_2{}^T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & X_3{}^T \\ 0 & 0 & 0 \end{pmatrix} \tag{121}$$

$$\text{and } \widehat{Q} = \begin{pmatrix} \frac{3}{\mu}Q_1 & 0 & 0 \\ * & \frac{3}{\mu}Q_2 & 0 \\ * & * & \frac{2}{\mu}Q_3 \end{pmatrix} \tag{122}$$

In the following, we replace the two zero in the diagonal of the matrix $\widehat{X}_{ijk}$ (120) by the term $(\beta I - \beta I = 0)$ for all positive scalar $\beta > 0$. After that, we dissociate the terms $\beta I = I(\beta I)I$ successively from the inequality (119) in a similar way to (116) and we use the modified Schur Lemma so as to rewrite (119) as follows:

$$\begin{pmatrix} \bar{X}_{ijk} & Y_{ij} & \psi Z_{ijk}{}^T & P \\ * & -\psi I & 0 & 0 \\ * & * & -\psi I & 0 \\ * & * & * & -Q \end{pmatrix} < 0 \tag{123}$$

where

$$\bar{X}_{ijk} = \begin{pmatrix} \Pi_i & A_iX_2 & 0 & 0 & 0 \\ * & -\beta I & X_2 & 0 & 0 \\ * & * & -\beta I & 0 & 0 \\ * & * & * & \Theta_i & B_i \\ * & * & * & * & \Sigma_{ik} \end{pmatrix} \tag{124}$$

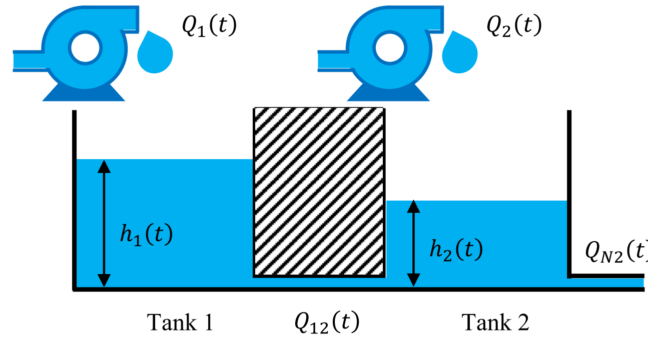

Fig. 2. The Two-tank process scheme.

$$P = \begin{pmatrix} X_1{}^T & 0 & 0 & 0 & 0 \\ 0 & X_2{}^T & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \\ 0 & 0 & X_3{}^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{125}$$

$$\text{and } Q = \begin{pmatrix} \frac{3}{\mu}Q_1 & 0 & 0 & 0 & 0 \\ * & \frac{3}{\mu}Q_2 & 0 & 0 & 0 \\ * & * & \frac{2}{\mu}Q_3 & 0 & 0 \\ * & * & * & -\beta I & 0 \\ * & * & * & * & -\beta I \end{pmatrix} \tag{126}$$

Using the expressions (16) and (33) respectively of A_{ij} and N_i and taking into account the following change of variables:

$$W_{Pj} = K_{pj}CX_2 \tag{127}$$

$$W_{Ij} = K_{Ij}CX_2 \tag{128}$$

$$W_{Dij} = \Delta_{ij}{}^{-1} \tag{129}$$

$$\text{and } S_i = E_iCX_2 \tag{130}$$

the inequality (123) becomes reformulated as a Linear Matricial Inequality (LMI) which can be rewritten as in the Theorem 1. This one ends the proof of the theorem. □

4. Numerical example: a two-tank process

4.1. Two-tank process presentation

Hydraulic systems are used in many industrial plants of water treatment or storing liquids. This type of systems has been considered by many researchers in order to illustrate FTC approaches for instance the two-tank process [4,19] and the three-tank process [19–21].

Thereafter, we consider a two-tank process presented in Fig. 2. The considered two-tank process is mainly composed by two liquid tanks which may be filled through two identical and independent pumps. These ones provide the liquid flows $Q_1(t)$ and $Q_2(t)$. Note that these two tanks are interconnected to each other through a pipe whose flow is defined by $Q_{12}(t)$, while the outflow of the liquid from the process is made by a second pipe providing a flow $Q_{N2}(t)$ located at the second tank.

By using both the mass balance and the Torricelli's law, and by assuming that the process operates under the constraint $h_1(t) > h_2(t)$, the two-tank process is then described by a non-linear representation such that [4]:

$$\begin{cases} \frac{dh_1(t)}{dt} = \frac{1}{A}\left(Q_1(t) - Q_{12}(t)\right) \\ \frac{dh_2(t)}{dt} = \frac{1}{A}\left(Q_2(t) + Q_{12}(t) - Q_{N2}(t)\right) \end{cases} \tag{131}$$

with

$$Q_{12}\left(t\right)=c_{12}\sqrt{h_{1}\left(t\right)-h_{2}\left(t\right)} \tag{132}$$

$$\text{and } Q_{N2}\left(t\right)=c_{2}\sqrt{h_{2}\left(t\right)} \tag{133}$$

where the liquid levels of the tank 1 and the tank 2 are denoted by $h_1(t)$ and $h_2(t)$ respectively. Moreover, these two variables are also used as state variables. $A = 1.54 \times 10^{-2}$ m^2 represents the area of the cylindric tanks. $c_{12} = 6 \times 10^{-4}$ m$^{5/2}$/s and $c_2 = 13 \times 10^{-4}$ m$^{5/2}$/s denote the flow constants respectively of the interconnecting pipe and of the outflow pipe.

By considering the state vector $x(t) = \begin{pmatrix} h_1(t) & h_2(t) \end{pmatrix}^T$, the control input vector $u(t) = \begin{pmatrix} Q_1(t) & Q_2(t) \end{pmatrix}^T$ and the measured output vector $y(t)$, the non-linear representation (131) of the two-tank process is then approximated by LPV structure such that:

$$\begin{cases} \dot{x}\left(t\right)=A\left(\theta\left(t\right)\right)x\left(t\right)+Bu\left(t\right) \\ y\left(t\right)=Cx\left(t\right) \end{cases} \tag{134}$$

with the matrices $A(\theta(t))$, B and C are defined such that:

$$A\left(\theta\left(t\right)\right)=\begin{pmatrix} -a_{11}\theta_{1}\left(t\right) & 0 \\ a_{11}\theta_{1}\left(t\right) & -a_{22}\theta_{2}\left(t\right) \end{pmatrix}; \quad B=\begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix} \text{ and } C=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{135}$$

with $a_{11} = \frac{c_{12}}{A}$; $a_{22} = \frac{c_2}{A}$; $b_{11} = \frac{1}{A}$ and $b_{22} = \frac{1}{A}$

Note that the matrix $A(\theta(t))$ is a time-varying matrix whose parameters vector: $\theta(t) = \begin{pmatrix} \theta_1(t) & \theta_2(t) \end{pmatrix}^T$, is then expressed as follows:

$$\theta_{1}\left(t\right)=\frac{\sqrt{h_{1}\left(t\right)-h_{2}\left(t\right)}}{h_{1}\left(t\right)} \tag{136}$$

$$\theta_{2}\left(t\right)=\frac{\sqrt{h_{2}\left(t\right)}}{h_{2}\left(t\right)} \tag{137}$$

In the remainder of this section, we assume that the two-tank process operates between these operating ranges: $h_1(t) \in \begin{bmatrix} 0.6\text{ m}; & 1.8\text{ m} \end{bmatrix}$ and $h_2(t) \in \begin{bmatrix} 0.1\text{ m}; & 0.3\text{ m} \end{bmatrix}$. Hence, the two time-varying parameters will be bounded as follows:

$$\begin{aligned} &0.6804\ \text{m}^{-1/2} \le \theta_1(t) \le 1.1785\ \text{m}^{-1/2} \text{ and} \\ &1.8257\ \text{m}^{-1/2} \le \theta_2(t) \le 3.1623\ \text{m}^{-1/2} \end{aligned} \tag{138}$$

Thereafter, we consider that the two pumps of the studied process are affected by additive faults $f_1(t)$ and $f_2(t)$ where the actuator fault vector will be given by $f(t) = \begin{pmatrix} f_1(t) & f_2(t) \end{pmatrix}^T$. Besides, we consider that the second tank is disturbed by an unknown input $d(t)$ such that its distribution matrix is represented by $R = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$. Consequently, the whole process will be described by the following polytopic LPV representation:

$$\begin{cases} \dot{x}\left(t\right)=\sum_{i=1}^{4}\rho_{i}\left(\theta\left(t\right)\right)\left(A_{i}x\left(t\right)+B\left(u\left(t\right)+f\left(t\right)\right)+Rd\left(t\right)\right) \\ y\left(t\right)=Cx\left(t\right) \end{cases} \tag{139}$$

where the gain scheduling functions are given by the following expressions:

$$\rho_{1}\left(\theta\left(t\right)\right)=\frac{\theta_{1}\left(t\right)-\underline{\theta_{1}}}{\overline{\theta_{1}}-\underline{\theta_{1}}}\frac{\theta_{2}\left(t\right)-\underline{\theta_{2}}}{\overline{\theta_{2}}-\underline{\theta_{2}}} \tag{140}$$

$$\rho_{2}\left(\theta\left(t\right)\right)=\frac{\theta_{1}\left(t\right)-\underline{\theta_{1}}}{\overline{\theta_{1}}-\underline{\theta_{1}}}\frac{\overline{\theta_{2}}-\theta_{2}\left(t\right)}{\overline{\theta_{2}}-\underline{\theta_{2}}} \tag{141}$$

$$\rho_{3}\left(\theta\left(t\right)\right)=\frac{\overline{\theta_{1}}-\theta_{1}\left(t\right)}{\overline{\theta_{1}}-\underline{\theta_{1}}}\frac{\theta_{2}\left(t\right)-\underline{\theta_{2}}}{\overline{\theta_{2}}-\underline{\theta_{2}}} \tag{142}$$

$$\text{and } \rho_{4}\left(\theta\left(t\right)\right)=\frac{\overline{\theta_{1}}-\theta_{1}\left(t\right)}{\overline{\theta_{1}}-\underline{\theta_{1}}}\frac{\overline{\theta_{2}}-\theta_{2}\left(t\right)}{\overline{\theta_{2}}-\underline{\theta_{2}}} \tag{143}$$

and the matrices A_1, A_2, A_3 and A_4 are expressed as follows:

$$A_{1}=\begin{pmatrix} -a_{11}\underline{\theta_{1}} & 0 \\ a_{11}\underline{\theta_{1}} & -a_{22}\underline{\theta_{2}} \end{pmatrix} \tag{144}$$

$$A_{2}=\begin{pmatrix} -a_{11}\underline{\theta_{1}} & 0 \\ a_{11}\underline{\theta_{1}} & -a_{22}\overline{\theta_{2}} \end{pmatrix} \tag{145}$$

$$A_{3}=\begin{pmatrix} -a_{11}\overline{\theta_{1}} & 0 \\ a_{11}\overline{\theta_{1}} & -a_{22}\underline{\theta_{2}} \end{pmatrix} \tag{146}$$

$$\text{and } A_{4}=\begin{pmatrix} -a_{11}\overline{\theta_{1}} & 0 \\ a_{11}\overline{\theta_{1}} & -a_{22}\overline{\theta_{2}} \end{pmatrix} \tag{147}$$

with $\underline{\theta_i}$ is the minimum value of the time-varying parameter θ_i for all $i = 1, 2$ and $\overline{\theta_i}$ is the maximum one.

4.2. Simulation results

The proposed PID-FTC controller is synthesized by solving numerically an optimization problem defined into the Theorem 1. For this goal, we employ the MATLAB® software (Release 2013a) of MathWorks, Inc. under which we develop a program proceeding the following algorithm:

- Step 1: Compute the matrices T_1 and T_2 from (26).
- Step 2: Make a choice of positive values for the scalars σ, μ, β and ψ. Define also the values of a symmetric and positive definite matrix Γ. The parameter values $\mu = 1$, $\sigma = 1$, $\psi = 2$, $\beta = 2$ and $\Gamma = I$ are chosen in this case.
- Step 3: Solve the LMIs (41) through the use of the MATLAB YALMIP toolbox and then find the solutions for the unknown matrices: X_1, X_2, X_3, Q_1, Q_2, Q_3, Q_4, W_{Pj}, W_{Ij}, W_{Dij}, S_i, K_{fj} and Φ_i.
- Step 4: Compute the matrices E_i and then the matrices K_{Pj}, K_{Ij}, K_{Dj}, N_i, L_i and G_i by using the expressions (51)–(56).

$\forall i, j = 1, \ldots, h$ For the simulation, we create a model under the Simulink environment of MathWorks, Inc. This model is represented by the scheme given in Fig. 1. In which, each block is modeled by a polytopic form wherein four sub-systems are interpolated through the gain scheduling functions (140)–(143). Each sub-system uses a S-function block for which a M-file code including the matrices provided by the LMIs solving, is implemented in the MATLAB environment.

In order to simulate the created model, we consider that the liquid levels in the reference model are given as follows:

$$h_{r1}(t)=\begin{cases} 0.9\text{ m for } 0\text{ s} \le t \le 20\text{ s} \\ 1.2\text{ m for } 20\text{ s} < t \le 40\text{ s} \end{cases} \tag{148}$$

and

$$h_{r2}(t)=\begin{cases} 0.16\text{ m for } 0\text{ s} \le t \le 20\text{ s} \\ 0.20\text{ m for } 20\text{ s} < t \le 40\text{ s} \end{cases} \tag{149}$$

the unknown input $d(t)$ which disturbs the second pump is assumed to be represented by an abrupt signal such that:

$$d(t)=\begin{cases} 1\times10^{-3}\text{ m}^{3}/\text{s for } 25\text{ s} \le t \le 30\text{ s} \\ 0\text{ m}^{3}/\text{s elsewhere} \end{cases} \tag{150}$$

The above references $h_{r1}(t)$ and $h_{r2}(t)$ and the unknown input $d(t)$ are used into the closed loop of the two-tank process, which is simulated for the following faults scenario. Indeed, we consider that the system works without faults until the moment $t = 10$ s when an additive fault $f_1(t)$ occurs in the first pump. At the moment $t = 15$ s, the second pump is also affected by an

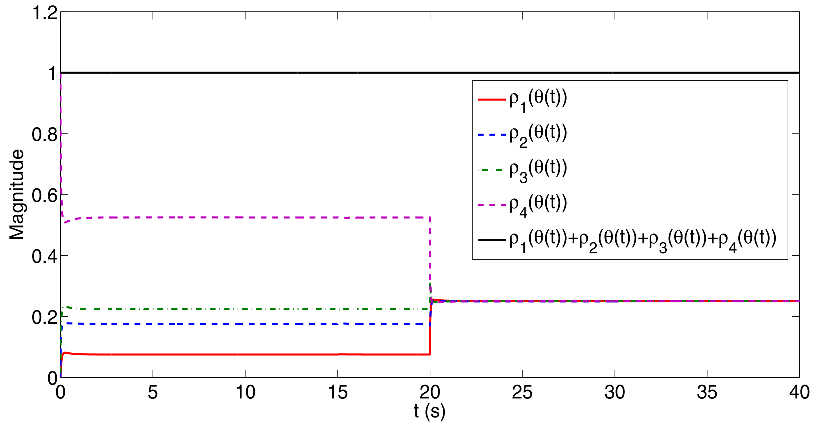

Fig. 3. Gain scheduling functions.

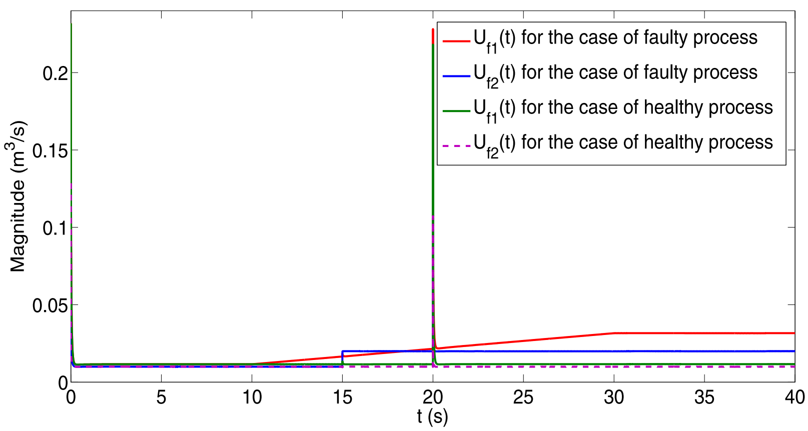

Fig. 4. The FTC signals $u_{f1}(t)$ and $u_{f2}(t)$ provided by the proposed PID-FTTC.

additive fault $f_2(t)$. After these moments, the two faults represent respectively as an incipient and an abrupt faults such that their expressions can be given by:

$$f_1(t) = \begin{cases} 0 \text{ m}^3/\text{s for } 0 \text{ s} \le t < 10 \text{ s} \\ -1 \times 10^{-3}\,(t-10) \text{ m}^3/\text{s for } 10 \text{ s} \le \text{t} \le 30 \text{ s} \\ -2 \times 10^{-2} \text{ m}^3/\text{s for t} > 30 \text{ s} \end{cases} \tag{151}$$

and

$$f_2(t) = \begin{cases} 0 \text{ m}^3/\text{s for } 0 \text{ s} \le t < 15 \text{ s} \\ -1 \times 10^{-2} \text{ m}^3/\text{s for t} \ge 15 \text{ s} \end{cases} \tag{152}$$

The results of the simulation are illustrated by the following figures. The Fig. 3 presents the gain scheduling functions:

The control signals synthesized by the FTC controller, are plotted into Fig. 4.

The Fig. 4 compares the FTC signals for the cases of faulty system and of the healthy system: it shows that the faults effects are avoided through a compensation brings by this control law at the time instants of the faults appearance. Indeed, at $t = 10$ s $u_{f1}(t)$ starts to increase progressively until $t = 30$ when it stabilizes at 0.03163 m^3s^{-1} while $u_{f2}(t)$ increases abruptly at $t = 15$ s and keeps its magnitude almost constant (0.01995 m^3s^{-1}). Noting that, at the time instants $t = 0$ s and $t = 20$ s, the liquid level references $h_{r1}(t)$ and $h_{r2}(t)$ are changed suddenly which result great tracking errors. To deal with this situation, the proposed PID-FTTC controller reacts immediately and provides intense commands that appear as pulses in the control signals.

In the following, the outputs of the closed loop process handled by the PID-FTTC are mainly compared to those given by an old approach given in [8]. This one has dealt with a model reference tracking problem despite actuator faults and unknown inputs occurrence by synthesizing an Active Fault Tolerant Controller (AFTC) with a PI structure.

The (Figs. 5 and 6) show on the one hand the comparisons of the real liquid levels $h_1(t)$ and $h_2(t)$ between: our new PID-FTTC, the old approach AFTC and a classical PID controller and their references $h_{r1}(t)$ and $h_{r2}(t)$ on the other hand:

The Figs. 5 and 6 underline that the real liquid levels of the two tanks handled by the developed PID-FTTC and the AFTC track similarly their references with good accuracy despite actuator faults and disturbance that act on the system. In contrast, the proposed approach gives more quick responses than their for the case using the AFTC and for the case using conventional PID.

It is evident that the proposed PID-FTTC is able to immediately overpower the effects of the actuator fault $f_1(t)$ occurring at $t = 10$ s and it continues tracking with negligible error. A same result is obtained in the case using the AFTC approach. This is justified by the fact that $f_1(t)$ is incipient whose magnitude gradually increases. In contrast, the use of a conventional PID controller show significant overshoots in the output tracking error for the first tank from the moment $t = 10$ s, while it reaches an acceptable steady output after 21.04 s.

Our proposed controller leads to totally compensate the effects of the constant fault $f_2(t)$ which occurs at $t = 15$ s. It allows to track the desired trajectory with a negligible error after 0.54 s. In contrast, the AFTC allows to recover the tracking performance

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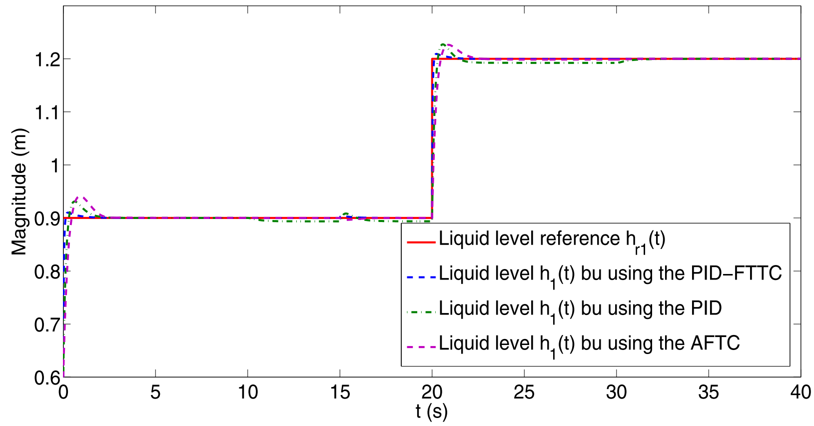

Fig. 5. Different comparisons of reference tracking $h_{r1}(t)$ with the real liquid level $h_1(t)$ by using the PID-FTTC, an AFTC and also a PID controller.

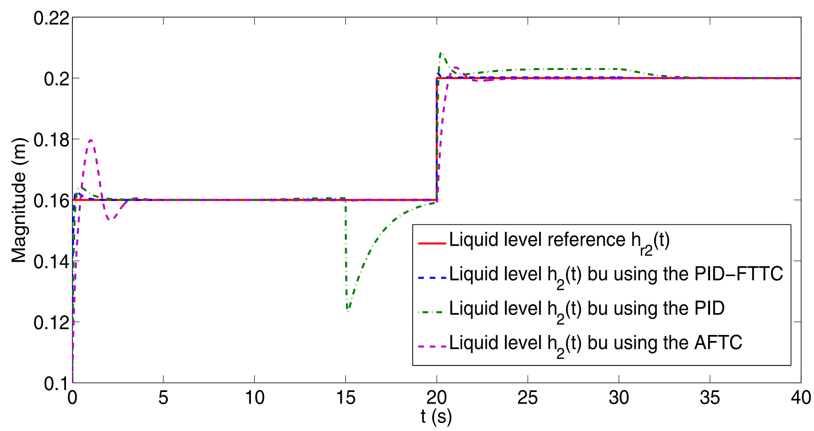

Fig. 6. Different comparisons of reference tracking $h_{r2}(t)$ with the real liquid level $h_2(t)$ by using the PID-FTTC, an AFTC and also a PID controller.

Table 1
Performance indices of the 1st tank's closed loop control.

Index	For 0 s ≤ t < 20 s			For 20 s ≤ t ≤ 40 s		
	PID	AFTC	PID-FTTC	PID	AFTC	PID-FTTC
Rise time (s)	0.31	0.45	0.12	0.31	0.48	0.12
Settling time (s)	2.06	2.29	1.28	11.04	2.38	1.38
Peak time (s)	0.60	0.91	0.24	0.58	0.87	0.23
Peak overshoot (%)	3.44	4.56	1.12	2.25	2.16	0.75

Table 2
Performance indices of the 2nd tank's closed loop control.

Index	For 0 s ≤ t < 20 s			For 20 s ≤ t ≤ 40 s		
	PID	AFTC	PID-FTTC	PID	AFTC	PID-FTTC
Rise time (s)	0.20	0.45	0.12	0.10	0.69	0.03
Settling time (s)	2.90	4.22	1.33	14.05	3.78	0.40
Peak time (s)	0.45	1.01	0.23	0.26	1.04	0.09
Peak overshoot (%)	2.62	11.23	1.75	4.30	1.75	0.80

after 0.92 s. This can be justified by the fault behavior which suddenly happens. It is clear from Fig. 6 that the PID controller leads to unstable response with a big tracking error which reaches 23% after 0.14 s. This undesirable situation continues during 19.05 s then it comes back to a steady output.

In order to quantify the comparison study, we use some performance indices in the two following Tables 1 and 2 where we compare the rise time, the settling time, the peak time and the peak overshoot (%) indices in the cases using the PID, the AFTC and the PID-FTTC methods.

In addition, to evaluate the fitness of the developed approach, we compare it to the methods using the PID and the AFTC controllers. This comparison is based on the Integral Absolute Error (IAE) and the Integral Square Error (ISE) performance indices which are expressed as follows [22]:

$$IAE = \int_0^{\infty} \left|e_{yr}(t)\right| dt \tag{153}$$

$$ISE = \int_0^{\infty} {e_{yr}}^2(t) dt \tag{154}$$

The following Table 3 presents the result of this comparison:

These indices prove that the developed PID-FTTC accommodates effectively the time-varying and constant actuator faults and exhibits better tracking performance with rapid and accurate responses compared to the AFTC and the PID. Furthermore, we conclude that the outputs of the model reference and their of the faulty process handled by the PID-FTTC show a very good agreement which verifies that the performances of the proposed FTC strategy are very satisfactory. Moreover, the developed controller drives to better performances than the AFTC proposed in [8] and also to the conventional PID control.

Finally, the developed APO observer (6) is used to estimate the magnitudes of the multiple actuator faults so as to reconfigure the designed controller. The figures (Figs. 7 and 8) compare the real faults to their estimations provided by the designed observer in order to demonstrate its effectiveness.

Since Figs. 7 and 8, it is shown that the designed APO gives perfect estimations for the original incipient and abrupt faults.

Table 3
Performance indices: IAE and ISE.

Index	Closed loop of the 1st tank			Closed loop of the 2nd tank		
	PID	AFTC	PID-FTTC	PID	AFTC	PID-FTTC
IAE ($\times 10^{-2}$)	22.81	17.91	3.61	10.02	4.63	0.57
ISE ($\times 10^{-4}$)	121.40	182.00	23.71	14.20	8.56	0.35

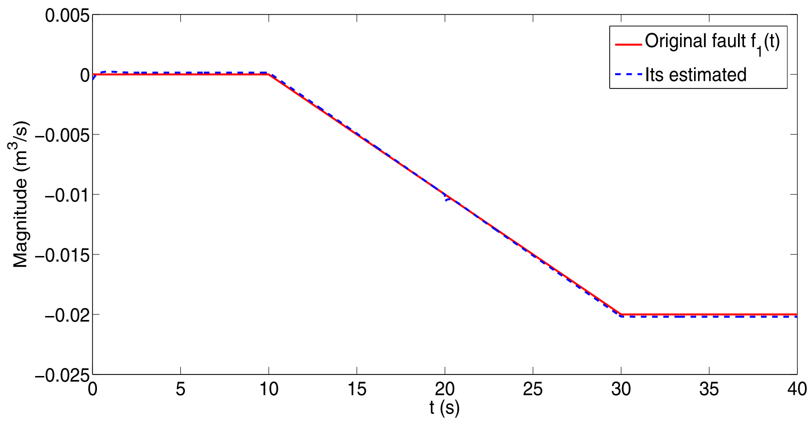

Fig. 7. The real actuator fault $f_1(t)$ and its estimate given by the developed APO.

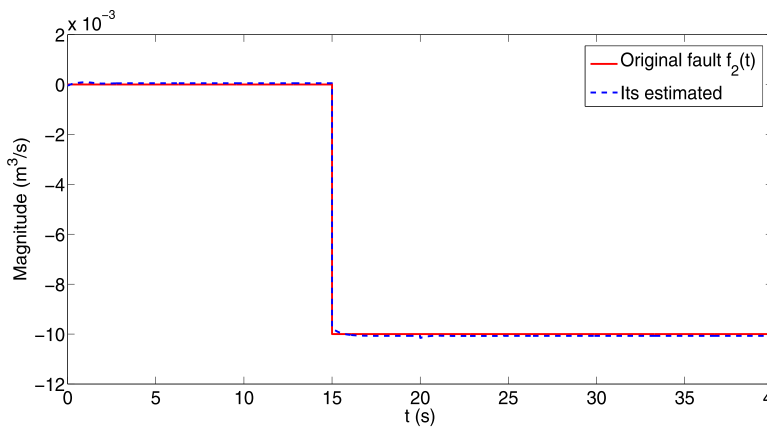

Fig. 8. The real actuator fault $f_2(t)$ and its estimate given by the developed APO.

5. Conclusion

This paper deals with the topic of fault tolerant tracking control for polytopic LPV systems with measurable gain scheduling functions. The main contribution is to synthesize a new PID Fault Tolerant Tracking Controller (PID-FTTC) in order to compensate actuator faults and ensure some performances such as stability, rapidity, accuracy and model reference tracking in spite of the presence of unknown inputs.

This new approach leads to better results and also to higher performances compared to previous methods: the PID structure brings more parameters to be tuned and adds a degree of freedom to reduce the conservatism and to enhance the tracking control problem. The stability of the reconfigured system is ensured to a new LMI study which helps the engineer to tune the gains so as to obtain acceptable tracking performances for LPV systems. A two-tank process is employed to show the effectiveness and the relevance of the proposed PID-FTC approach. The advantages of the proposed PID-FTTC scheme are shown via a comparison with an PI structure of the AFTC [8] and also with the conventional PID controller on this two tank process. The comparison of these three controllers reveals that the PID-FTTC performs well the system especially in terms of rapidity and accuracy of the trajectories tracking's responses.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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