3- Autonomous Underwater Vehicle "Phoenix"

The equations of motion used to simulate the dynamic behavior of the NPS Autonomous Underwater Vehicle "Phoenix" in the horizontal plane are as follows:

Sway equation

$$(m - Y_{\dot{v}})\dot{v} - (Y_{\dot{r}} - mx_G)\dot{r} = Y_r v + (Y_v - m)r + Y_{\delta s}\delta_s + Y_{\delta b}\delta_b$$

Yaw equation

$$(mx_G - N_{\dot{v}})\dot{v} - (N_{\dot{r}} - I_z)\dot{r} = N_v v + (N_r - mx_G)r + N_{\delta s}\delta_s + N_{\delta b}\delta_b$$

Turning rate

$$\dot{\psi} = r$$

Inertial position rate

$$\dot{y} = \sin\psi + v\cos\psi$$

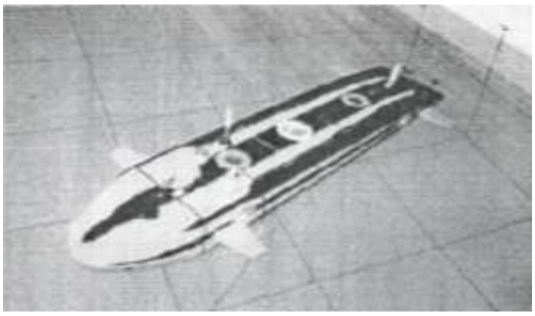
All variables in these equations are assumed to be in non-dimensional form with respect to the vehicle length (7.3 ft) and constant forward speed (approx. 3 *ft/sec).* The vehicle weighs 435 lbs and is neutrally buoyant. This is not needed in the calculations that follow, but it gives you an idea of the physical system. Time is become dimensionless so that 1 second represents the time that it takes to travel one vehicle length. In the equations of motion, the variables are defined as:

v	lateral (sway) velocity
r	turning rate (yaw)
ψ	heading angle
y	lateral deviation (cross track error)
δ_s	stern rudder deflection
δ_b	bow rudder deflection

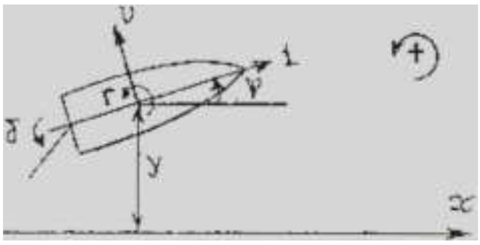
The rest are constants, m is the mass, I_z is the mass moment of inertia with respect to a vertical axis that passes through the vehicle's geometric center (amidships), x_G is the position of the vehicle's center of gravity (positive forward of amidships), and the remaining terms are the so-called hydrodynamic coefficients. Non-dimensional values for the coefficients are given in the following table:

$m = 0.0358$	$Y_{\delta s} = 0.01241$
$I_z = 0.0022$	$N_{\dot{r}} = -0.00047$
$x_G = 0.0014$	$N_{\dot{v}} = -0.00178$
$Y_{\dot{r}} = -0.00178$	$N_r = -0.00390$
$Y_{\dot{v}} = -0.03430$	$N_v = -0.00769$
$Y_r = 0.01187$	$N_{\delta s} = -0.0047$
$Y_v = -0.10700$	$N_{\delta b} = 0.0035$
$Y_{\delta b} = 0.01241$	

Figures show a picture of the vehicle and the definition of the coordinate systems used in the equations of motion.



Picture of the "Phoenix" vehicle



Geometry and axes definitions

Select state vector as $x = [\upsilon, r, \psi, y]$ *and* control vector as $u = [\delta_s , \delta_b]$. then write the state space equations, assuming small angles ψ

$$\dot{x} = Ax + Bu$$

Give the values for the *A* and *B* matrices.

1. Using matrix algebra, compute the transfer functions between lateral position y and either stern rudder deflection δ_s, or bow rudder δ_b . What are the open loop poles and zeros in each case? What is the physical significance of open loop poles that are equal to zero (if any)?
2. Draw the block diagram of the system, keeping all sine and cosine terms, and simulate:
   - 15 degrees of positive bow rudder, stern rudder at zero.
   - 15 degrees of negative stern rudder, bow rudder at zero.
   - 15 degrees of positive bow rudder, 15 degrees of negative stern rudder.

Plot a geographical *(x, y)* plot for the ship's position. For this you'll need the additional equation for the rate of change of *x* which is,

$$\dot{x} = \cos\psi + \upsilon \sin\psi$$

Allow enough time to complete a full turning circle. Comment on the effectiveness of the various rudder deflections on the turning diameter.

Consider the Phoenix equations of motion, Assume that the bow rudder is slaved to the stern rudder so that

$$\delta_b = -\delta_s$$

In other words, if δ is the deflection of the stern rudder, the bow rudder assumes a deflection of $-\delta$. In this way we get a single input system. Write this in state space form,

$$\dot{x} = Ax + Bu$$

Where x is the state variables vector $[\upsilon, r, \psi, y]$ and u is the control vector δ.

Control specifications:

1- Settling time about 10 seconds
2- Maximum overshoot be 5%