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Speed control of induction motors using neuro-fuzzy dynamic sliding mode control

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Abstract. This paper presents a novel approach for robust speed control of medium size induction motors using neuro-fuzzy dynamic sliding mode control. First, a simple dynamic model of induction motors is introduced. Then, a conventional sliding mode control is presented. To reduce the chattering phenomenon, dynamic sliding mode control is designed using a secondary PID-type sliding surface. Moreover, in order to eliminate the tedious trial and error procedure in choosing a proper uncertainty upper bound, uncertainties have been estimated using a neuro-fuzzy system. The online training of the neuro-fuzzy dynamic sliding mode control is based on the adaptation law derived from the stability analysis. In addition, the reconstruction error of the neuro-fuzzy system is compensated to guarantee the asymptotic convergence of the speed tracking error. Simulation results verify that the neuro-fuzzy dynamic sliding mode control is robust against various uncertainties including parametric variations, external load disturbance, unmodeled dynamics and input voltage disturbances.

Keywords: Induction motor, neuro-fuzzy systems, dynamic sliding mode control, reconstruction error, Barbalat's lemma

1. Introduction

Induction motors (IMs) are used widely in industry because of their excellent characteristics such as high robustness, easy structure, and satisfactory efficiency [1]. In the last few decades, we have witnessed widespread researches focused on the speed control of induction motors [2–7]. Due to the non-linearity and parameter variations in IMs, obtaining an exact mathematical model is a challenging task. Consequently, model-based control approaches are not suitable for high performance applications. Thus, many robust and adaptive control approaches have been proposed in the literature [8–13]. Adaptive control can overcome the parametric uncertainty [14]. In order to design an adaptive control law, the structure of the system dynamics should be available. In other words, the regressor vector should be known. Thus, conventional adaptive control laws may not be successful for complicated systems with unknown dynamics. Among robust control approaches, sliding mode control (SMC) [15] is very popular due to its simplicity and efficiency in overcoming parametric and nonparametric uncertainties. It is well known that the major advantage of SMC systems is its insensitivity to parameter variations and external disturbance once the system trajectory reaches and stays on the sliding surface [16, 17]. However, in the conventional SMC, the upper bound of uncertainties should be known in advance or estimated [18]. Overestimation of this bound will increase the chattering phenomenon which is not desirable. On the other hand, underestimation of this bound will deteriorate the system performance by increasing the tracking error [19, 20]. Usually, this bound is determined using the trial and error procedure to reach a satisfactory trade-off between accuracy and the chattering phenomenon.

In order to reduce the chattering phenomenon in conventional SMC, some solutions have been presented in

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the literature. Replacing the switching function with a saturation function is the most well-known solution [15]. However, it results in a steady state tracking error. Among various approaches presented in this field, dynamic sliding mode control (DSMC) system is more efficient [21–23]. In DSMC, the control law is obtained from integrating a function including the switching control term. This integration plays an important role in reducing the chattering phenomenon. However, the upper bound of uncertainty is required in DSMC.

Nowadays, active researches are carried out in the field of robust control based on neural networks and adaptive fuzzy systems. The characteristics of fault-tolerance, parallelism, excellent learning capabilities and especially the universal approximation property have made them very popular in nonlinear control of uncertain dynamical systems. These outstanding features motivate us to apply them for uncertainty estimation in robust dynamic sliding mode speed control of IMs. In the proposed algorithm, DSMC is designed using a secondary PID-type sliding surface to reduce the chattering phenomenon in common SMC. Moreover, in order to eliminate the tedious trial and error procedure in choosing a proper uncertainty upper bound, uncertainties have been estimated using a neuro-fuzzy system. The online training of the neuro-fuzzy DSMC is based on a stability analysis. In addition, the reconstruction error of the neuro-fuzzy system is compensated to guarantee the asymptotic convergence of the speed tracking error.

Recently, some novel control approaches based on SMC and fuzzy logic has been developed in the literature [24, 25]. In order to highlight the novelty of this paper in comparison with these approaches, it should be emphasized that feedbacks from all state variables are needed in these control laws, while measuring all state variables of induction motors such as currents and fluxes is a challenging task. Moreover, these signals are usually contaminated with noise and using them in the control law may degrade the controller performance. Thus, in this paper, we have designed a controller which does not require all state variables. In [26], a new hybrid neuro-fuzzy model that eliminates the need for predefined parameters has been developed. However, it lacks a rigorous mathematical stability analysis for guaranteeing the asymptotic convergence of the tracking error and boundedness of all state variables. In [27], an adaptive fuzzy speed controller with stability analysis has been developed. However, it requires feedbacks from all state variables. Moreover, the number of fuzzy rules in this controller is too large.

In this paper, to overcome the inadequacies of the previous methods and eliminate the need for currents and fluxes feedbacks, a robust controller using DSMC is developed based on the speed dynamic equation, which considerably simplifies the design procedure and reduces the computational load of the controller. Then, to avoid the trial and error procedure in determining the uncertainty upper bound, uncertainties are estimated using neuro-fuzzy systems. The adaptation law for adjustable parameters of the neuro-fuzzy system is derived from stability analysis. Moreover, the approximation error is compensated in the control law using a continuous robustifying term. In addition, boundedness of other state variables has been insured in this paper.

This paper is organized as follows. Section 2 describes the IM model. Section 3 presents the conventional SMC and DSMC. In Section 4, the proposed control law and stability analysis are given. Simulation results are illustrated in Section 5 and finally, Section 6 concludes the paper.

2. Induction motor model

Induction motors are made by three stator windings and three rotor windings. In [28] a two phase equivalent machine representation with two rotor windings and two stator windings has been introduced. Their dynamics are described by

$$\begin{aligned} R_s i_{sa} + \frac{d\psi_{sa}}{dt} &= u_{sa} \\ R_s i_{sb} + \frac{d\psi_{sb}}{dt} &= u_{sb} \\ R_r i_{rd'} + \frac{d\psi_{rd'}}{dt} &= 0 \\ R_r i_{rq'} + \frac{d\psi_{rq'}}{dt} &= 0 \end{aligned} \tag{1}$$

where $R,\ i,\ \psi,\ u_s$ stands for resistance, current, flux linkage, and stator voltage input to the machine; the subscripts s and r denote stator and rotor, $(a,\ b)$ show the components of a vector regarding a fixed stator reference frame, $(d',\ q')$ show the components of a vector with respect to a frame rotating at speed $n_p\omega$; and n_p denotes the number of pole pairs of the induction machine and ω the rotor speed [29–31]. Let δ denote an angle such that

$$\frac{d\delta}{dt} = n_p\omega\,, \quad \delta(0) = 0 \tag{2}$$

Transforming the vectors $(i_{rd'},\ i_{rq'})$ and $\left(\psi_{rd'},\ \psi_{rq'}\right)$ from the rotating frame $(d',\ q')$ into vectors $(i_{ra},\ i_{rb})$ and $(\psi_{ra},\ \psi_{rb})$ in the stationary frame $(a,\ b)$ and using Equations (2), (1) becomes

$$\begin{aligned} R_s i_{sa} + \frac{d\psi_{sa}}{dt} &= u_{sa} \\ R_s i_{sb} + \frac{d\psi_{sb}}{dt} &= u_{sb} \\ R_r i_{ra} + \frac{d\psi_{ra}}{dt} + n_p\omega\psi_{rb} &= 0 \\ R_r i_{rb} + \frac{d\psi_{rb}}{dt} - n_p\omega\psi_{ra} &= 0 \end{aligned} \tag{3}$$

Under the assumptions of equal mutual inductances, linear magnetic circuits and neglected iron losses, the magnetic equations are [30]

$$\begin{aligned} \psi_{sa} &= L_s i_{sa} + M i_{ra} \\ \psi_{sb} &= L_s i_{sb} + M i_{rb} \\ \psi_{ra} &= M i_{sa} + L_r i_{ra} \\ \psi_{sb} &= M i_{sb} + L_r i_{rb} \end{aligned} \tag{4}$$

where L_s and L_r are auto inductances and M is the mutual inductance. Eliminating i_{ra}, i_{rb}, ψ_{sa} and ψ_{sb} in (3) by using (4), we obtain

$$\begin{aligned} R_s i_{sa} + \frac{M}{L_r}\frac{d\psi_{ra}}{dt} + \left(L_s - \frac{M^2}{L_r}\right)\frac{di_{sa}}{dt} &= u_{sa} \\ R_s i_{sb} + \frac{M}{L_r}\frac{d\psi_{rb}}{dt} + \left(L_s - \frac{M^2}{L_r}\right)\frac{di_{sb}}{dt} &= u_{sb} \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{R_r}{L_r}\psi_{ra} - \frac{R_r}{L_r}M i_{sa} + \frac{d\psi_{ra}}{dt} + n_p\omega\psi_{rb} &= 0 \\ \frac{R_r}{L_r}\psi_{rb} - \frac{R_r}{L_r}M i_{sb} + \frac{d\psi_{rb}}{dt} - n_p\omega\psi_{ra} &= 0 \end{aligned} \tag{6}$$

The machine's torque is expressed in terms of rotor fluxes and stator currents as

$$T = \frac{n_p M}{L_r}(\psi_{ra} i_{sb} - \psi_{rb} i_{sa}) \tag{7}$$

So that the rotor dynamics are

$$\frac{d\omega}{dt} = \frac{n_p M}{J L_r}(\psi_{ra} i_{sb} - \psi_{rb} i_{sa}) - \frac{T_L}{J} \tag{8}$$

where J is the moment of inertia of the rotor and its attachment and T_L is the load torque. By adding the rotor dynamics (9) to the electromagnetic dynamics (6) and rewriting the equations in state space form, the overall fifth order model of the induction motor is given as follows:

$$\frac{di_{sa}}{dt} = -\left(\frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2}\right) i_{sa} + \frac{M R_r}{\sigma L_s L_r^2}\psi_{ra} + \frac{n_p M}{\sigma L_s L_r}\omega\psi_{rb} + \frac{1}{\sigma L_s}u_{sa} \tag{9-a}$$

$$\frac{di_{sb}}{dt} = -\left(\frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2}\right) i_{sb} + \frac{M R_r}{\sigma L_s L_r^2}\psi_{rb} - \frac{n_p M}{\sigma L_s L_r}\omega\psi_{ra} + \frac{1}{\sigma L_s}u_{sb} \tag{9-b}$$

$$\frac{d\psi_{ra}}{dt} = -\frac{R_r}{L_r}\psi_{ra} - n_p\omega\psi_{rb} + \frac{R_r}{L_r}M i_{sa} \tag{9-c}$$

$$\frac{d\psi_{rb}}{dt} = -\frac{R_r}{L_r}\psi_{rb} + n_p\omega\psi_{ra} + \frac{R_r}{L_r}M i_{sb} \tag{9-d}$$

$$\frac{d\omega}{dt} = \frac{n_p M}{J L_r}(\psi_{ra} i_{sb} - \psi_{rb} i_{sa}) - \frac{T_L}{J} \tag{9-e}$$

where $\sigma = 1 - \left(M^2/L_s L_r\right)$. According to (9), we can express this nonlinear system using the state space representation as $\dot{X} = F(X,\ T_L,\ u_s)$ in which $X = \begin{bmatrix} i_{sa} & i_{sb} & \psi_{ra} & \psi_{rb} & \omega \end{bmatrix}^T$ is the state vector.

3. The conventional SMC and DSMC

Consider the problem of tracking control for a system of the form

$$x^{(n)} = f(x) + u \tag{10}$$

where $x \in \Re^n$ is the state vector and $u \in \Re$ is the control input. The nonlinear function $f(x)$ is uncertain and its estimations $\hat{f}(x)$ is available such that

$$f(x) = \hat{f}(x) + \Delta f(x) \quad |\Delta f(x)| \le \delta_f \tag{11}$$

in which δ_f is a known constant. Suppose that x_r is the desired trajectory. Define the tracking error as

$$e(t) = x - x_r \tag{12}$$

Consider the following sliding surface

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e(t),\ \lambda > 0 \tag{13}$$

in which λ is a design parameter. Note that on the surface $s(t) = 0$, the error dynamics is given by

$$\left(\frac{d}{dt} + \lambda\right)^{n-1} e(t) = 0 \tag{14}$$

which implies that the error will converge to zero exponentially. Thus, we should design $u(t)$ such that the sliding surface $s(t)$ converges to zero and stay there. In other words, $\dot{s}(t) = 0$ should also be satisfied.
Using (12) and (13), $\dot{s}$ is given by

$$\dot{s} = e^{(n)} + \cdots + \lambda^{n-1}\dot{e} = x^{(n)} - x_r^{(n)} + \cdots \\ +\lambda^{n-1}\dot{e} = f(x) + u - x_r^{(n)} + \cdots + \lambda^{n-1}\dot{e} \tag{15}$$

Thus, the best continuous approximation of the control law $\hat{u}$ which results in $\dot{s}(t) = 0$, is given by

$$\hat{u} = x_r^{(n)} - \hat{f}(x) - \cdots - \lambda^{n-1}\dot{e} \tag{16}$$

In order to overcome uncertainties, a discontinuous term is added to $\hat{u}$:

$$u = \hat{u} - (\delta_f + \eta)\,\mathrm{sgn}(s) \tag{17}$$

where η is a positive constant and sgn(s) is the sign function. To prove convergence to the sliding mode, we show that with this control, s will converge to zero in finite time. Consider the Lyapunov candidate

$$V(t) = \frac{1}{2}s^2 \tag{18}$$

Its time derivative is given by:

$$\dot{V} = s\dot{s} \tag{19}$$

Using (15), (16) and (17), (19) is rewritten as

$$\dot{V} = s(\Delta f(x) - (\delta_f + \eta)\,\mathrm{sgn}(s)) \tag{20}$$

It follows from (20) that

$$\dot{V} \le |s|\,(|\Delta f(x)| - \delta_f) - \eta\,|s| \tag{21}$$

Since $|\Delta f(x)| - \delta_f \le 0$, it follows from (21) that

$$\frac{1}{2}\frac{d}{dt}s^2 \le -\eta\,|s| \tag{22}$$

which implies that the square of distance from the sliding surface measured by s^2 is reducing along all the system trajectories and finally we will have $s(t) = 0$.

In order to represent the system dynamics in the form of (10), one can take the time derivative of (9-e) to obtain

$$\ddot{\omega} = f(X, T_L) + G(X)u \tag{23}$$

Thus, the relative degree of the system is $n = 2$. In this paper, it is assumed that $f(X,\ T_L)$ and $G(X)$ are not available. Thus, one can design the control law based on the nominal models $f_n(X,\ T_L)$ and $G_n(X)$. We can make the design procedure simple by using model-free control approaches because they eliminate the need for nominal models. In order to accomplish this goal, we can rewrite the system dynamics (23) as

$$\ddot{\omega} = L(X,\ T_L,\ u) + u \tag{24}$$

where $L(X,\ T_L,\ u) = f(X,\ T_L) + G(X)u - u$ is the lumped uncertainty. Consider the following PID-type sliding surface

$$s(t) = \dot{e}(t) + c_1 e(t) + c_2 \int_0^t e(\tau)d\tau \tag{25}$$

in which c_1 and c_2 are positive constants, $e = \omega_r - \omega$ is the speed tracking error and ω_r is the reference speed. It is assumed that ω_r and its time derivatives up to the necessary order are bounded. Differentiating $s(t)$ with respect to time and using (24), one can obtain

$$\dot{s}(t) = \ddot{e}(t) + c_1\dot{e}(t) + c_2 e(t) = \ddot{\omega}_r(t) - \ddot{\omega}(t) + c_1\dot{e}(t) \\ +c_2 e(t) = \ddot{\omega}_r(t) - u(t) - L(X;t) + c_1\dot{e}(t) + c_2 e(t). \tag{26}$$

The control law $u(t)$ should be proposed such that $\dot{s}(t) = 0$ is satisfied for all $t \ge t_0$ where t_0 is the time when the sliding motion occurs. According to [13], the globally asymptotic stability of (26) is guaranteed if the control effort $u(t)$ is proposed as follows [13]

$$u_{SMC}(t) = \ddot{\omega}_r + c_1\dot{e}(t) + c_2 e(t) + \delta_{SMC}\,\mathrm{sgn}(s(t)) \tag{27}$$

where sgn(.) is the sign function and δ_{SMC} is the upper bound of lumped uncertainty $L(X,\ T_L,\ u)$ as $|L(X,\ T_L, u)| < \delta_{SMC}$. The most important problem with the control law (27) is the chattering phenomenon. To reduce the chattering phenomenon in conventional SMC, dynamic sliding mode control (DSMC) has been introduced [32–34]. In DSMC, a secondary sliding surface is proposed as

$$\sigma(t) = \dot{s}(t) + \lambda_1 s(t) + \lambda_2 \int_0^t s(\tau)d\tau \tag{28}$$

where λ_1 and λ_2 are positive constants. Differentiating $\sigma(t)$ with respect to time and using (24) and (26), one can obtain

$$\begin{aligned}\dot{\sigma}(t) &= \ddot{s}(t) + \lambda_1 \dot{s}(t) + \lambda_2 s(t) = \dddot{\omega}_r(t) - \dot{u}(t) \\ &- \dot{L}(X;t) + (c_1 + \lambda_1)\,\ddot{e}(t) + (c_2 + c_1\lambda_1 + \lambda_2)\,\dot{e}(t) \\ &+ (c_2\lambda_1 + c_1\lambda_2)\,e(t) + c_2\lambda_2 \int_0^t e(\tau)d\tau = \dddot{\omega}_r(t) \\ &- \dot{u}(t) + p_1 \mathrm{E}(X;t) + p_2\dot{e}(t) + p_3 e(t) \\ &+ p_4 \int_0^t e(\tau)d\tau - \left[p_1 L(X;t) + \dot{L}(X;t)\right]\end{aligned} \tag{29}$$

where $E(X;t) \equiv \ddot{\omega}_r(t) - u(t)$, $p_1 \equiv c_1 + \lambda_1$, $p_2 \equiv c_2 + c_1\lambda_1 + \lambda_2$, $p_3 \equiv c_2\lambda_1 + c_1\lambda_2$ and $p_4 \equiv c_2\lambda_2$, respectively. If $\dot{u}(t)$ is designed so that $\dot{\sigma}(t) = 0$ is satisfied, then we will have $\ddot{s}(t) + \lambda_1\dot{s}(t) + \lambda_2 s(t) = 0$. Therefore, the first sliding surface $s(t)$ converges to zero. Thus, $\dot{u}(t)$ is given by

$$\begin{aligned}\dot{u}(t) &= \dddot{\omega}_r(t) + p_1\mathrm{E}(X;t) + p_2\dot{e}(t) + p_3 e(t) \\ &+ p_4 \int_0^t e(\tau)d\tau + \delta_{DSMC}\,\mathrm{sgn}(\sigma(t))\end{aligned} \tag{30}$$

where δ_{DSMC} is uncertainty upper bound defined by

$$\left|(c_1 + \lambda_1)\,L(X;t) + \dot{L}(X;t)\right| < \delta_{DSMC} \tag{31}$$

As a result,

$$u(t) = u_{DSMC}(t) = \int_0^t \dot{u}(\tau)d\tau \tag{32}$$

DSMC reduces the chattering phenomenon considerably due to the integral action in (32). However, it requires the δ_{DSMC}. This parameter should be known in advance. Usually, this bound is not available and should be determined by the tedious trial and error. To solve this problem, we can derive an adaptation law to estimate δ_{DSMC}. Nevertheless, this solution will not remove the sign function from DSMC which is the main reason for the chattering phenomenon. Thus, in this paper, the lumped uncertainty

$$y(t) = -[p_1 L(X;t) + \dot{L}(X;t)] \tag{33}$$

is estimated using adaptive neuro-fuzzy systems to remove the sign function from the control law (27) and eliminate the need for trial and error procedure to determine δ_{DSMC}.

4. The proposed control scheme

To improve the control performance, a neuro-fuzzy dynamic sliding mode control (NFDSMC) system is proposed in this study. The objective of the neuro-fuzzy system is estimating the lumped uncertainty $y(t)$ which is a function of X, T_L, u and their time derivatives. Providing the neuro-fuzzy system with these inputs is a challenging task, since some of these signals are not available or contaminated with noise. According to [35], instead of using these signals in the neuro-fuzzy system, we can use e and $\dot{e}$. This paper applies this idea to reduce the number of required feedbacks. According to the universal approximation theorem [36], there exist m, σ and $\bar{y}$ such that

$$y = y^*(t) + \Delta(t) \tag{34}$$

where $y^*(t)$ is a neuro-fuzzy system given by

$$\begin{aligned}y^*(t) &= \sum_{i=1}^{N} \bar{y}_i \xi_i(e,\dot{e}) = \Theta^{*T}\xi(e,\dot{e}) \\ \xi_i(e,\dot{e}) &= \frac{\mu_{Ai}(e)\mu_{Bi}(\dot{e})}{\sum_{i=1}^{9}\mu_{Ai}(e)\mu_{Bi}(\dot{e})}\end{aligned} \tag{35}$$

in which $\mu_{Ai}(e) \in [0, 1]$ and $\mu_{Bi}(\dot{e}) \in [0, 1]$ are the Gaussian membership functions in the *ith* rule for the inputs e and $\dot{e}$. Their means and standard deviations should be selected based on the maximum variations of e and $\dot{e}$. Also, $\Delta(t)$ is the reconstruction or approximation error of the neuro-fuzzy system. Suppose that

$$\hat{y} = \hat{\Theta}^T \xi(e,\dot{e}) \tag{36}$$

is the output of neuro-fuzzy system which should converge to $y(t)$ and $\hat{\Theta}$ is the estimation of Θ^*. In the ideal case that $\hat{\Theta} = \Theta^*$, we cannot expect $\hat{y}(t) = y(t)$, because of existence of the reconstruction error $\Delta(t)$ in (34). To compensate $\Delta(t)$, a robustifying term $u_r(t)$ is added to the control law. Therefore, the proposed NFDSMC law is defined as

$$u_{NFDSMC}(t) = \int_0^t \dot{u}_{NFDSMC}(\tau)\,d\tau \tag{37}$$

$$\begin{aligned}\dot{u}_{NFDSMC} &= \dddot{\omega}_r + p_1\mathrm{E} + p_2\dot{e} + p_3 e \\ &+ p_4 \int_0^t e(\tau)d\tau + \hat{y} + u_r\end{aligned} \tag{38}$$

In Fig. 1 the configuration of the proposed NFDSMC is shown.

Theorem 1. *Consider the induction motor represented by (24). If the control law is designed as (37) and (38), the adaptive learning algorithms of the neuro-fuzzy estimator is given by (39), and the robust compensator*

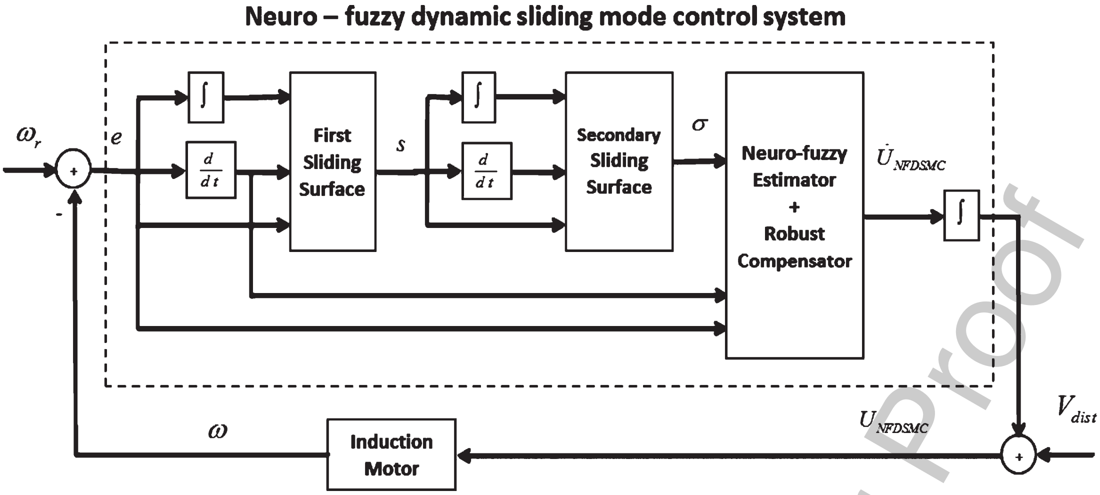

Fig. 1. The structure of proposed control system.

is designed as (40) and (41), then the stability of the proposed NFSMC control system can be guaranteed.

$$\dot{\hat{\Theta}} = \eta_\theta \sigma(t)\xi \tag{39}$$

$$u_r = \hat{h}(t) + k_2\sigma(t) \tag{40}$$

$$\dot{\hat{h}}(t) = k_1\sigma(t) \tag{41}$$

where η_w, η_α, η_β, η_γ, η_h and ξ are positive constants; and $\hat{h}(t)$ is an online estimated value of h.

Proof. Choose the following positive definite function.

$$V(t) = \frac{1}{2}\sigma^2(t) + \frac{1}{2\eta_\theta}\tilde{\Theta}^T\tilde{\Theta} + \frac{1}{2k_1}\tilde{h}^2(t) \tag{42}$$

where $\tilde{h}(t) = \Delta(t) - \hat{h}(t)$ and $\tilde{\Theta} = \Theta^* - \hat{\Theta}$. Differentiating $V(t)$ and using (29), (36) and (38), one can obtain

$$\dot{V} = \sigma(t)\{y - \hat{y} - u_r\} - \frac{1}{\eta_\theta}\tilde{\Theta}^T\dot{\hat{\Theta}} - \frac{1}{k_1}\tilde{h}(t)\dot{\hat{h}}(t) \tag{43}$$

It follows from (34), (35), (36) and (40) that

$$\begin{aligned}\dot{V} &= \sigma(t)\left[\tilde{\Theta}^T\xi + \tilde{h}(t) - k_2\sigma(t)\right] \\ &\quad -\frac{1}{\eta_\theta}\tilde{\Theta}^T\dot{\hat{\Theta}} - \frac{1}{k_1}\tilde{h}(t)\dot{\hat{h}}(t)\end{aligned} \tag{44}$$

Substituting $\dot{\hat{\Theta}}$ and $\dot{\hat{h}}(t)$ from (39) and (41) into (44) yield $\dot{V} = -k_2\sigma^2(t)$. Since $\dot{V}(\sigma, \tilde{\Theta}, \tilde{h}) \leq 0$, $V(\sigma, \tilde{\Theta}, \tilde{h})$ is negative semi-definite. Thus

$$V(\sigma(t), \tilde{\Theta}(t), \tilde{h}(t)) \leq V(\sigma(0), \tilde{\Theta}(0), \tilde{h}(0)) \tag{45}$$

which implies $\sigma(t)$, $\tilde{\Theta}(t)$ and $\tilde{h}(t)$ are bounded. Define the positive function

$$P(t) = k_1\sigma^2(t) = -\dot{V}(\sigma(t), \tilde{\Theta}(t), \tilde{h}(t)) \tag{46}$$

Integrating $P(t)$ with respect to time yields

$$\begin{aligned}\int_0^t P(\tau)d\tau &= V(\sigma(0), \tilde{\Theta}(0), \tilde{h}(0)) \\ &\quad - V(\sigma(t), \tilde{\Theta}(t), \tilde{h}(t))\end{aligned} \tag{47}$$

Because $V(\sigma(0), \tilde{\Theta}(0), \tilde{h}(0))$ is bounded and $V(\sigma(t), \tilde{\Theta}(t), \tilde{h}(t))$ is non-increasing and bounded, the following result is obtained:

$$\lim_{t\to\infty}\int_0^t P(\tau)d\tau < \infty \tag{48}$$

Since $\dot{P}(t)$ is bounded, therefore, $P(t)$ is uniformly continuous. By using Barbalat's lemma [13, 31], it can be shown that $\lim_{t\to\infty} P(t) \to 0$. It implies that $\sigma(t)$ will converge to zero as $t \to \infty$. As a result, the stability of the proposed control system can be guaranteed.

To ensure the boundedness of internal dynamics of IM including i_{ra}, i_{rb}, ψ_{sa} and ψ_{sb}, Equation (9) can be rewritten as

$$\dot{X} = AX + v(t) \tag{49}$$

in which $X = [i_{sa}\, i_{sb}\, \psi_{ra}\, \psi_{rb}]^T$,

$$A = \begin{bmatrix} -\left(\frac{M^2R_r+L_r^2R_s}{\sigma L_s L_r^2}\right) & 0 & \frac{MR_r}{\sigma L_s L_r^2} & 0 \\ 0 & -\left(\frac{M^2R_r+L_r^2R_s}{\sigma L_s L_r^2}\right) & 0 & \frac{MR_r}{\sigma L_s L_r^2} \\ \frac{R_r M}{L_r} & 0 & -\frac{R_r}{L_r} & 0 \\ 0 & \frac{R_r M}{L_r} & 0 & -\frac{R_r}{L_r} \end{bmatrix}$$

and

$$v(t) = \begin{bmatrix} \frac{n_p M}{\sigma L_s L_r}\omega\psi_{rb} + \frac{1}{\sigma L_s}u_{sa} \\ -\frac{n_p M}{\sigma L_s L_r}\omega\psi_{ra} + \frac{1}{\sigma L_s}u_{sb} \\ -n_p\omega\psi_{rb} \\ n_p\omega\psi_{ra} \end{bmatrix}.$$

Since the eigenvalues of A are negative, the state vector X in $\dot{X} = AX$ is exponentially stable. Moreover, the control law Equations (15, 16) and the IM speed ω are bounded. Thus, the vector $v(t)$ is bounded. Consequently, the state Equation (27) can be considered as a stable linear system with bounded inputs. Therefore, the system described in (49) has the BIBO stability.

5. Simulation results

To make the superiority of the proposed method more obvious, In Simulation 1 the proposed NFDSMC algorithm is tested and in Simulation 2 its performance is compared with the adaptive fuzzy MIMO controller designed in [27].

Simulation 1: Consider a three-phase standard IM with parameters given in Table 1 [37].

To test the control system robustness against the thermal variation of motor parameters and external load disturbance T_L, sinusoidal variations are applied to motor parameters. Thus, it is assumed that:

Table 1
Rated parameters of the first case study induction motor

Induction Motor Data		
P	Power	3 (KW)
f	frequency	60 (Hz)
V	Rated voltage	380 (V)
n_p	Number of pole pairs	2
R_s	Stator resistance	1.115 (Ω)
R_r	Rotor resistance	1.083 (Ω)
L_s	Stator inductance	0.005974 (H)
L_r	Rotor inductance	0.005974 (H)
M	Mutual inductance	0.2037 (H)
J	Total inertia	0.02 (kgm^2)

$$\begin{aligned} R_s &= R_{s0}(1 + 0.2\sin(t)) \\ R_r &= R_{r0}(1 + 0.2\cos(t)) \\ L_s &= L_{s0}(1 + 0.1\sin(t)) \\ L_r &= L_{r0}(1 + 0.1\cos(t)) \end{aligned} \tag{50}$$

$$T_L = \begin{cases} 1 & t < 17 \\ 7 & 17 \le t \le 27. \\ 2 & t > 27 \end{cases} \tag{51}$$

In order to examine the speed regulation capability in response to sudden variations of the speed command, ω_c is defined the summation of the constant value 155 and the square wave with altitude 10 and frequency 0.1 Hz. Also, the reference model

$$\frac{\omega_d(s)}{\omega_c(s)} = \frac{16}{(s+2)^4} \tag{52}$$

is chosen to regulate the transient response of the speed control system. Finally, in order to verify the ability of the proposed control law in rejecting input voltage disturbances, the following voltage disturbance has been inserted to u_{sb}.

$$v_{dist} = \begin{cases} 15 & 38 \le t \le 39 \\ 0 & otherwise \end{cases} \tag{53}$$

The initial values of θ in the neuro-fuzzy approximator are selected zero. As discussed before, if $\dot{\sigma}(t) = 0$ is satisfied, then $s(t)$ will converge to zeros. Thus, the design parameters c_1, c_2, λ_1 and λ_2 in $s(t)$ and $\sigma(t)$ should be selected so that the convergence speed of $\dot{\sigma}(t)$ is faster than that of $\dot{s}(t)$. Therefore, we define $s(t)$ and $\sigma(t)$ as

$$\begin{aligned} s(t) &= \dot{e}(t) + 0.02e(t) + 0.0001\int_0^t e(\tau)d\tau \\ \sigma(t) &= \dot{s}(t) + 10s(t) + 25\int_0^t s(\tau)d\tau \end{aligned} \tag{54}$$

The parameters k_1 and k_2 in u_r are chosen as 0.001 and 0.01, respectively. In Fig. 2 the control effort is presented which verifies its smoothness. Moreover, as illustrated in this figure, the motor voltage is under the maximum permitted voltage. The tracking performance of the proposed NFDSMC algorithm is illustrated in Fig. 3. As shown in this figure, the asymptotic convergence of the motor speed to the command signal is satisfying in terms of fast external load disturbance

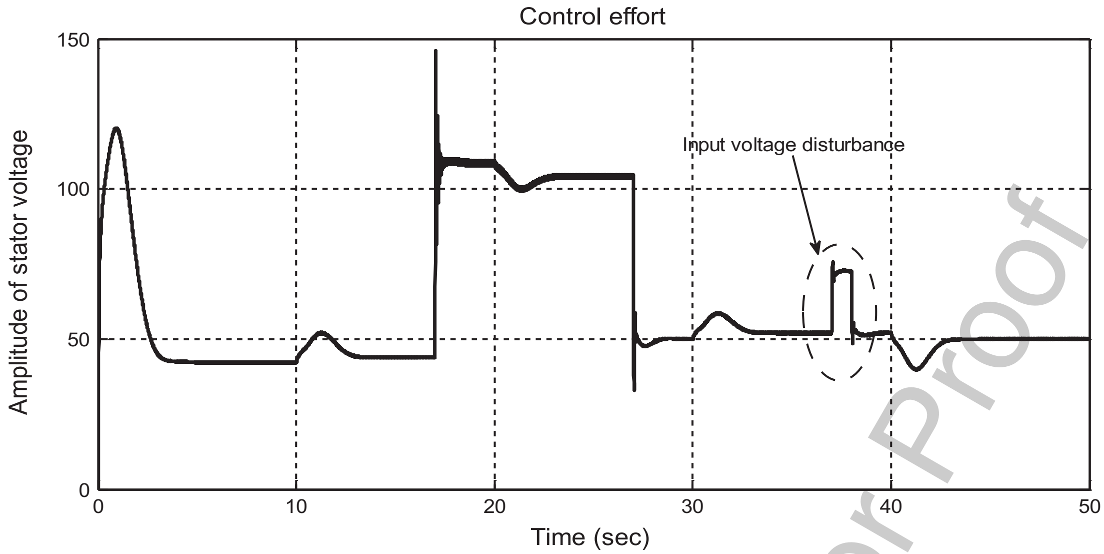

Fig. 2. The control effort.

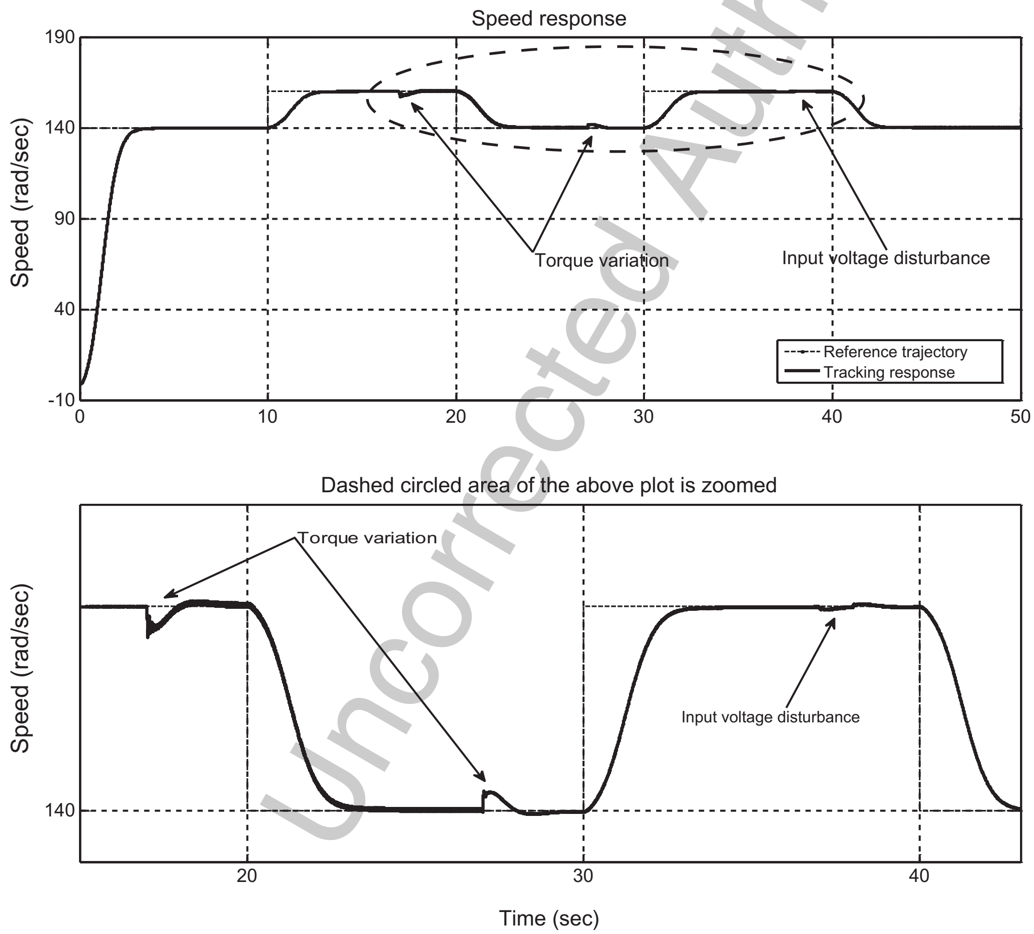

Fig. 3. The tracking performance of the proposed control scheme.

Table 2
RMSE for various learning rates

η_θ	15	20	25	30	35	40	50
RMSE	0.69	0.47	0.54	0.58	0.70	0.88	1.181

rejection and robustness against motor parameter variation and undesirable disturbances of the input voltage.

In some control systems, increasing the learning rate reduces the tracking error and improves the controller performance [19, 20, 35, 38]. In this simulation, the learning rate η_θ has been set to 20. To study its effect and make quantitative comparisons, consider the following performance criterion:

$$RMSE = \sqrt{\frac{1}{50}\int_0^{50} e^2(t)dt} \tag{55}$$

Table 2 presents the value of RMSE for various learning rates. This table verifies the aforementioned claim. Also, we are not permitted to increase the learning rate arbitrarily, since it will make the control system unstable.

In order to investigate the influence of the neuro-fuzzy estimator in improving the controller performance, a comparison between NFDSMC and DSMC is performed in this simulation. The control law defined in (30–32) is applied to the same induction motor. Suppose that the uncertainty upper bound δ_{DSMC} is set to 600. This bound has been set by trial and error. Nevertheless, optimization algorithms such as particle swarm optimization (PSO) can be used to find the optimal value of this bound. For example, in [39], PSO tunes the upper bound of uncertainty in robust control of a robot manipulator. In practical implementations, the exact amount of this bound is unknown. This is the reason, why uncertainties are estimated using a neuro-fuzzy system in this paper. The tracking performance of DSMC is presented in Fig. 4. A comparison between this figure and Fig. 3 shows that NFDSMC is superior, since when the external load torque is applied, speed variation in NFDSMC is negligible, while in DSMC, it is considerable.

To investigate the proposed controller performance on other IMs, it has been applied to the speed control of another motor described in Table 3 and the tracking performance is illustrated in Fig. 5. As shown in this

Table 3
Rated parameters of the second case study induction motor

Induction Motor Data		
P	Power	149.2 (KW)
f	frequency	60 (Hz)
V	Rated voltage	460 (V)
n_p	Number of pole pairs	2
R_s	Stator resistance	0.01818 (Ω)
R_r	Rotor resistance	0.09956 (Ω)
L_s	Stator inductance	0.00019 (H)
L_r	Rotor inductance	0.00019 (H)
M	Mutual inductance	0.009415 (H)
J	Total inertia	2.6 (kgm^2)
T_L	load torque	$T_L = \begin{cases} 20 & t < 17 \\ 350 & 17 \le t \le 27 \\ 100 & t > 27 \end{cases}$

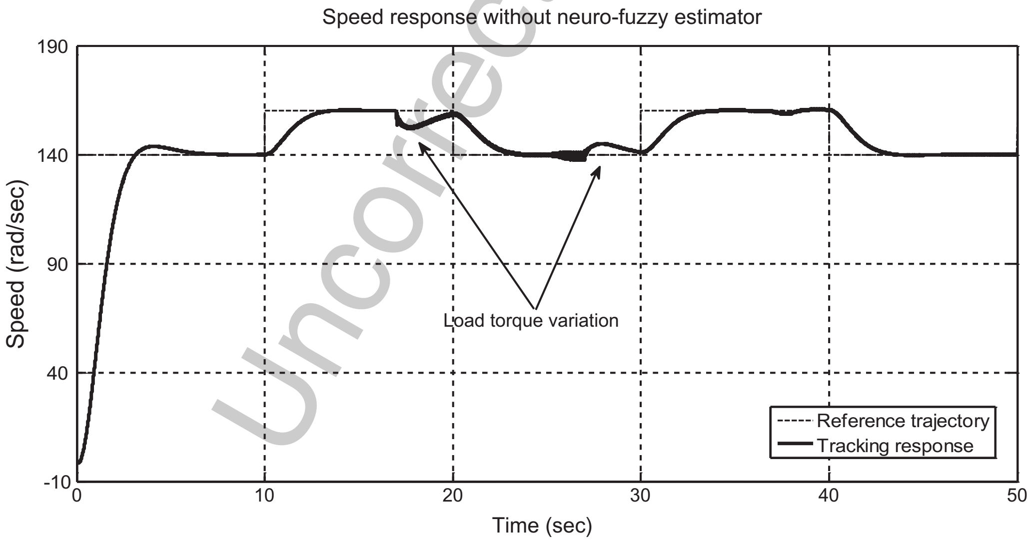

Fig. 4. The output of the control system without neuro-fuzzy estimator.

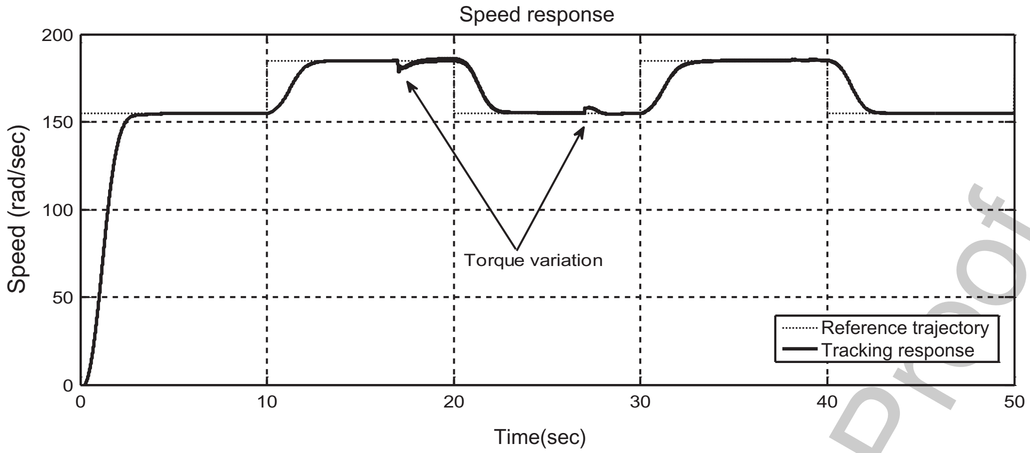

Fig. 5. The tracking performance of the proposed control scheme for the IM model described in Table 3.

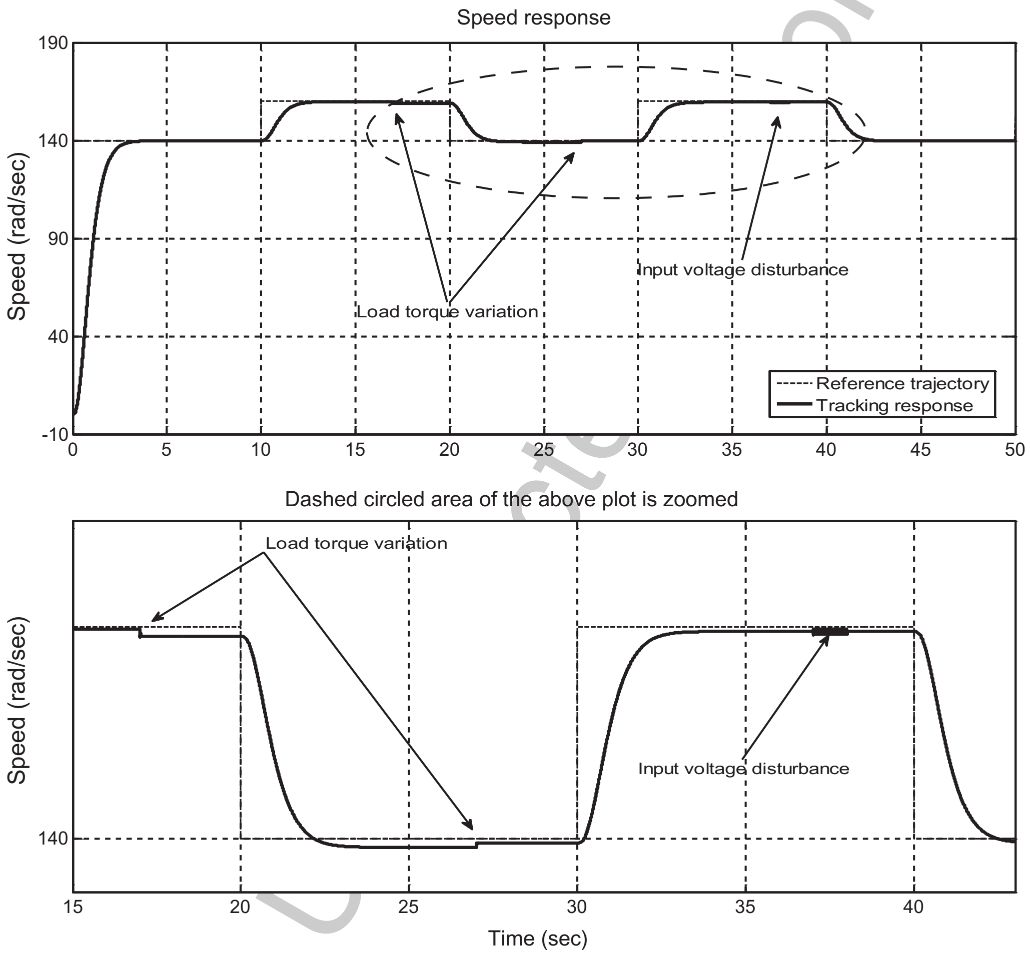

Fig. 6. The tracking performance of the adaptive fuzzy MIMO method [28].

figure, the proposed method is successful in the speed control of another induction motor with its relevant load torque variation.

Simulation 2: In this simulation, the adaptive fuzzy MIMO control presented in [27] is used for the speed control of the IM model described in Simulation 1. The reference model, external load torque, and input voltage disturbances are the same as Simulation 1. The tracking performance of this control scheme is illustrated in Fig. 6. As shown in the zoomed Figure, this control approach fails in rejecting the external load torque and input voltage disturbances. However, as shown in Fig. 3, the proposed method completely eliminates the effect of these disturbances.

It should be noted that the adaptive fuzzy MIMO method [27], requires feedbacks from all state variables, while in the proposed method, providing the controller with currents and fluxes feedbacks is not necessary. Moreover, there are 6 uncertain functions which should be estimated in the adaptive fuzzy MIMO method, while in the proposed method just the lumped uncertainty is estimated by neuro-fuzzy systems. As a result, the neuro-fuzzy approach presented in this paper is much simpler and less computational. In addition, the non-singularity of the estimated input gain matrix in [27] is a critical condition which can be violated easily and make the control system unstable. Another superiority of the proposed controller is its maximum tolerable external load torque. The NFDSMC in this paper is robust against 20 Nm external load torque, while the MIMO approach in [27] cannot tolerate external load torques more than 7 Nm.

6. Conclusion

In this paper, a speed controller for IMs is presented using a neuro-fuzzy dynamic sliding mode control. In Section 2, the IM nonlinear model is described. Section 3 explains SMC and DSMC. Uncertainties including un-modeled dynamics, motor parameter variations, load torque, and input voltage disturbances are estimated using a simple neuro-fuzzy system in Section 4. Moreover, the reconstruction error of the neuro-fuzzy system is compensated in the control law. The adaptation laws for online tuning of the parameters in the neuro-fuzzy system and approximation error compensator are derived based on the stability analysis. To guarantee the asymptotic convergence of the speed tracking error, Barbalat's lemma has been applied. In addition, the boundedness of all state variables is ensured in the stability analysis, which makes the proposed method superior than previous related works. In Section 5, satisfactory performance of the proposed method is verified using computer simulations. It has been shown that dynamic sliding mode fails in compensation of large external load torque disturbance, while the proposed neuro-fuzzy dynamic sliding mode control is robust against it. Also, a comparison with an adaptive fuzzy method has been performed which obviously indicates that the proposed controller is much simpler, less computational and more efficient in rejecting external disturbances.

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