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# Robust impulsive reset observers of a class of switched nonlinear systems with unknown inputs

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**Abstract**

This paper is concerned with observer designs for a class of switched nonlinear systems with unknown inputs under average dwell time (ADT) switching. Firstly, for the unknown input (UI) elimination purpose, an output derivative-based method is considered. Then, in order to avoid using differentiator signals and meanwhile maintain a satisfactory estimation performance, a full-order impulsive reset switching observer is developed. The existence condition of such an observer is given via a Riccati equation. Besides, we also devote to the construction of the reduced-order observer. And furthermore, it is found that the condition under which a full-order observer exists also guarantees a reduced-order reset switching observer. Finally, a switched nonlinear electronic circuit system is used as the simulation example to verify the effectiveness of the obtained methods.


## 1. Introduction

A switched system is a hybrid system, which consists of a family of subsystems described by differential or difference equations together with a switching rule that determines the system model by shifting one subsystem to another. With these features, switched system can find its wide applications in modelling many physical systems such as power electronics, flight control systems and network control systems [1] and the applications in energy-efficient

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control or filtering in industrial process [2–4]. In the last two decades, some fundamental theories of switched systems such as stability and controllability have been extensively studied (see, [1,5–17] and references therein). Meanwhile, great efforts have also been put on the state estimations or observer designs for switched systems [18–21]. For example, in [19], an observability analysis of a switched linear system was given via geometric approach. Zhao et al. designed the multiple-mode Luenberger-type synchronous switching and asynchronous switching observers under the average dwell time (ADT) switching model [20].

The studies on state estimations of systems with unknown inputs (UIs), or unknown input observer (UIO) designs, have attracted much attention in the past decades [22–28]. Indeed, for practical systems, some inputs are unknown. For example, the cutting forces exerted in machine tools [22], the parameter perturbation [29], external disturbance and the actuator faults [30] in industrial process can all be viewed as UIs. Therefore, the discussions on UIO designs are of great significance in both theory and applications, especially in the fields of observer-based control [31,32] and observer-based fault detection and isolation (FDI) [30]. Note that the researches on unknown input observer designs for switched systems have also aroused considerable attention recently, and many results have been reported in literature (see [33–40]). In [33], Koenig et al. proposed a UIO design method for a class of switched nonlinear discrete time descriptor systems via linear matrix inequality approach under arbitrary switching signal. Bejarano et al. put forward two UIO design methods under different conditions for a class of switched linear systems, where the common Lyapunov functions were considered [34]. For a switched system with some of the subsystems being not fully observable, Gorp et al. dealt with observer design problems via high-order sliding mode techniques [35]. Then, they addressed fault detection problems when the observer matching condition is not satisfied [36]. For a discrete-time singular switched system with delays and UIs, Lin et al. developed a functional observer design method by means of unknown input decoupling way [37]. Subsequently, for the same system, they discussed the state estimation issues via unknown input decoupling way and attenuation techniques, respectively [38]. In addition, in [39], Yang et al. investigated the state estimation problems for switched systems with UIs when the current switching mode was unknown. Then, they also investigated full-order and reduced-order robust observer constructions for an ADT switched linear system [40]. More recently, by using the unknown input observer techniques, Chen et al. studied fault estimation and fault detection for switched linear systems [41,42].

The nonlinearity, as is well known, is the most common phenomenon in real world, and a large number of practical switched systems are nonlinear systems. Among the nonlinearity community, the Lipschitz nonlinear systems widely exist because most physical models satisfy a Lipschitz condition, at least locally [43]. However, to the best of our knowledge, little work for switched Lipschitz nonlinear system observation can be found in literature, and the UIO designs for switched Lipschitz nonlinear systems are still challenging open issues which deserve us to investigate deeply. On the other hand, a reduced-order observer only needs to estimate partial states which are independent of output vectors, and thus possesses a lower dimension comparing with the full-order observer. This means that the reduced-order observer can be constructed with fewer integrators and the whole system will be simpler (see [28,34,43]). However, for switched nonlinear systems, seldom did scholars consider the reduced-order observer designs.

Based on the observations above, in this paper, we devote to both the full-order and reduced-order observer designs for a class of switched Lipschitz nonlinear systems with UIs. The main contributions are summarized as

(i) A full-order impulsive reset switching UIO is constructed, and the sufficient conditions are formulated in terms of a Riccati equation together with an ADT restriction condition.
(ii) We prove that the conditions under which a full-order observer exists can also guarantee the existence of a reduced-order observer.
(iii) In order to overcome the state jump problems resulted from coordinate transformations in both full-order and reduced-order observers, two impulsive reset equations are designed such that state estimation can maintain continuous at each switching point.

The remainder of this paper is organized as follows: In Section 2, the background and necessary preliminaries are presented. A full-order and a reduced-order switching UIOs for a class of switched nonlinear systems are constructed in Section 3. In Section 4, an electronic circuit system is used as an example to illustrate the validity of our methods. Finally, the conclusions are given in Section 5.

Throughout the paper, $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space. $I_n$ refers to the identical matrix with dimension $n$. The vector norm $\|\cdot\|$ is defined as $\|\alpha\| = \sqrt{\alpha^T\alpha}$. $<\alpha, \beta>$ denotes the inner product of vectors $\alpha$ and $\beta$, i.e., $<\alpha,\beta>=\alpha^T\beta$. For any matrix $A$, $\lambda_{max}(A)(\lambda_{min}(A))$ denotes the maximum (minimum) singular value, and the $\|A\|$ represents the $2-norm$ of matrix $A$, defined as $\|A\| = \sqrt{\lambda_{max}(A^TA)}$. For matrices $X$ and $Y$, $X > Y(X < Y)$ means that matrix $X - Y$ is a symmetrical positive (negative) definite matrix, and $X \geq Y(X \leq Y)$ means that matrix $X - Y$ is a symmetrical semi-positive (semi-negative) definite matrix. $\mathcal{C}^1$ denotes the space of continuously differentiable functions, and a function $\alpha$: $[0, \infty) \rightarrow [0, \infty)$ is said to be of class of $\mathcal{K}$ if it is continuous, strictly increasing, and $\alpha(0) = 0$. Class $\mathcal{K}_\infty$ denotes the subset of $\mathcal{K}$ consisting of all the functions that are unbounded.

## 2. Background and preliminaries

Consider a class of switched Lipschitz nonlinear systems with unknown inputs described by

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + \Phi(x(t)) + D_{\sigma(t)}\omega(t), \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ and $\omega \in \mathbb{R}^q$ are state, control input, measurement output and unknown input vectors, respectively. The piecewise constant map $\sigma(t) : \mathbb{R}^+ \rightarrow \Gamma = \{1, 2, \ldots, \mathcal{N}\}$ is a switching signal function defined as $\sigma(t) = i \in \Gamma$ when $\tau_k \leq t < \tau_{k+1}$, where $\{\tau_k\}_{k=1}^{\infty}$ is the sequence of switching points, and each switching point is the time instant at which the system mode is changing from one mode to another (i.e., $\sigma(\tau_k^+) \neq \sigma(\tau_k^-)$), and $\mathcal{N}$ is the number of subsystems. $\Phi : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a nonlinear function satisfying the following Lipschitz condition.

**Assumption 1.** Nonlinear function $\Phi(x)$ satisfies the following Lipschitz condition

$$\|\Phi(x) - \Phi(\hat{x})\| \leq \gamma\|x - \hat{x}\|, \forall\ x,\ \hat{x} \in \mathbb{R}^n, \tag{2}$$

where $\gamma > 0$ is the Lipschitz constant.

**Remark 1.** In practical systems, there are many systems, such as the circuit systems given in [44] and in the simulation part of the present paper, reluctance motor (SRM) systems [45,46] and the F-18 aircraft model dynamic system [50], described by switched nonlinear systems where the switching only happens at linear parts. In this paper, we particularly pay our attention to a class of switched nonlinear systems whose linear term is switched but the nonlinear term is non-switched.

For the convenience of observer designs for system (1), an assumption, a definition and some useful lemmas are introduced in advance.

**Assumption 2.** $rank(CD_i) = rank(D_i), \ \forall \ i \in \Gamma$.

**Remark 2.** The Assumption 2, referred as observer matching condition, is a standard matrix rank condition for unknown input observer designs [24,28,34,39,40]. This condition is somewhat conservative. Up to now, for non-switched systems, many researchers have made efforts on breaking through this restriction and significant results have been reported in literature (see the references [28,29,54]). However, for switched systems, because the high-order derivatives of output usually do not exist, the unknown input observer designs become much more challenging. Here we only concern with the case when the observer matching condition holds, and the case when the observer matching condition does not hold will be considered in the future.

**Definition 1.** [1] For a switching signal $\sigma(t)$ and any $\rho_2 > \rho_1 > 0$ , let $\Psi_{\sigma(t)}(\rho_1, \rho_2)$ be the number of the discontinuities in the open interval $(\rho_1, \rho_2)$. If there exist two positive numbers $\Psi_0$ and $\tau_a$ such that

$$\Psi_{\sigma(t)}(\rho_1, \rho_2) \le \Psi_0 + \frac{\rho_2 - \rho_1}{\tau_a}. \tag{3}$$

Then we say that $\sigma(t)$ has average dwell time (ADT) $\tau_a$, where $\Psi_0$ is called chatter bound.

**Remark 3.** It has been recognized that the average dwell time (ADT) strategy is more flexible and efficient in dealing with the stability analysis of switched systems because the conditions derived under the ADT strategy allow the occurrence of fast or slow switching in some certain finite time intervals. And thus, it has been widely used in switched system controller designs [9,10] and observer designs [20,39,40,50].

**Lemma 1.** *[1] Consider a switched system $\dot{x}(t) = f_{\sigma(t)}(x(t))$. Suppose there are a switching sequence $\{\tau_k\}_{k=1}^{\infty}$ satisfying (3), a set of $\mathcal{C}^1$ non-negative functions $V_i : \mathbb{R}^n \to \mathbb{R}, i \in \Gamma$, two $\mathcal{K}_\infty$ functions $\alpha_1$ and $\alpha_2$, and two positive numbers $\lambda$, $\mu > 0$ such that $\forall i \in \Gamma$*

$$\alpha_1(\|x(t)\|) \le V_i(x(t)) \le \alpha_2(\|x(t)\|)$$

$$\dot{V}_i(x(t)) \le -\lambda V_i(x(t))$$

*and $\forall (i, j) \in \Gamma \times \Gamma, i \ne j$*

$$V_i(x(\tau_k^+)) \le \mu V_j(x(\tau_k^-))$$

*then, the system is globally uniformly asymptotically stable for any switch signal with ADT*

$$\tau_a \ge \tau_a^* = \frac{\ln \mu}{\lambda}.$$

## 3. Full-order and reduced-order observers

In this section, we firstly use the output derivative $\dot{y}$ to offset the UI's influences, and then develop a full-order observer which actually can avoid using $\dot{y}$. Meanwhile, we also derive the sufficient conditions via a Riccati equation. Then we discuss the construction of a reduced-order observer.

### 3.1. Full-order impulsive reset observer

For observer design of system (1), it is always necessary for us to eliminate the unknown input influences. Under Assumption 2, for any $i \in \Gamma$, there is a matrix $F_i$ such that $F_iCD_i = D_i$. In fact, the generalized solution of $F_i$ can be given as

$$F_i = D_i(CD_i)^{\dagger} + Z_i\big(I_p - CD_i(CD_i)^{\dagger}\big), \tag{4}$$

where $Z_i$ is an arbitrary matrix with appropriate dimensions, and $(CD_i)^{\dagger}$ is the generalized inverse matrix of $CD_i$ which satisfies $CD_i(CD_i)^{\dagger}CD_i = CD_i$. Thus, we have

$$D_{\sigma(t)}\omega(t) = F_{\sigma(t)}CD_{\sigma(t)}\omega(t) = F_{\sigma(t)}\dot{y}(t) - F_{\sigma(t)}CA_{\sigma(t)}x(t) - F_{\sigma(t)}CB_{\sigma(t)}u(t) - F_{\sigma(t)}C\Phi(x). \tag{5}$$

Substituting (5) into (1) yields

$$\dot{x}(t) = T_{\sigma(t)}A_{\sigma(t)}x(t) + T_{\sigma(t)}B_{\sigma(t)}u(t) + T_{\sigma(t)}\Phi(x) + F_{\sigma(t)}\dot{y}(t), \tag{6}$$

where

$$T_{\sigma(t)} = I_n - F_{\sigma(t)}C \tag{7}$$

which satisfies

$$T_{\sigma(t)}D_{\sigma(t)} = 0. \tag{8}$$

Since in (6) the unknown input is offset, it is convenient for us to construct a full-order observer in the form

$$\dot{\hat{x}}(t) = T_{\sigma(t)}A_{\sigma(t)}\hat{x}(t) + T_{\sigma(t)}B_{\sigma(t)}u(t) + T_{\sigma(t)}\Phi(\hat{x}(t)) + F_{\sigma(t)}\dot{y}(t) + L_{\sigma(t)}(y(t) - C\hat{x}(t)), \tag{9}$$

where matrices $L_i$, $i \in \Gamma$ are observer gain matrices.

**Remark 4.** It should be noticed that (9) contains the derivative of $y(t)$, i.e. $\dot{y}(t)$. However, on the one hand, it is usually difficult for us to measure signal $\dot{y}(t)$ directly in practical applications. On the other hand, if the output derivative information is used in observer or controller designs, it may cause high frequency noise which is actually a negative factor to designs. In order to avoid using $\dot{y}(t)$, a coordinate transformation is introduced. For switched systems, however, state jumps may occur just due to the introduction of such transformation. The impulsive reset switching observer developed in the following will address state jump issues.

Consider a full-order observer with an impulsive reset equation in the following form:

$$\dot{v}(t) = T_{\sigma(t)}A_{\sigma(t)}\hat{x}(t) + T_{\sigma(t)}B_{\sigma(t)}u(t) + T_{\sigma(t)}\Phi(\hat{x}) + L_{\sigma(t)}\big(y(t) - C\hat{x}(t)\big) \tag{10}$$

$$v(\tau_k^+) = v(\tau_k^-) + (F_{\sigma(\tau_k^-)} - F_{\sigma(\tau_k^+)})y(\tau_k) \tag{11}$$

$$\hat{x}(t) = v(t) + F_{\sigma(t)}y(t) \tag{12}$$

where $\hat{x}$ is the estimation of $x$, and $T_i$, $L_i(i \in \Gamma)$ are matrices which are determined later.

**Assumption 3.** $\forall i \in \Gamma$, there exists a positive constant $\varepsilon_i > 0$ such that the following Riccati equation

$$P_i N_i + N_i^T P_i + \frac{\gamma^2}{\varepsilon_i} P_i T_i T_i^T P_i + \varepsilon_i I_n = -Q_i \tag{13}$$

has solutions for positive definite matrices $P_i$, $Q_i > 0$, $F_i$ and $L_i$, where $T_i$ is determined by (7) and

$$N_i = T_i A_i - L_i C. \tag{14}$$

Moreover, the ADT switching signal satisfies

$$\tau_a > \tau^* = \frac{\ln \theta_2}{\theta_1}, \tag{15}$$

where $\theta_1 = \min_{i \in \Gamma} \left( \frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)} \right)$ and $\theta_2 = \max_{i,j \in \Gamma} \left( \frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)} \right)$.

**Theorem 1.** *Under Assumptions 1–3, the dynamical system (10)–(12) is a full-order asymptotic convergence observer of (1) such that* $\lim_{t\to\infty}(x(t) - \hat{x}(t)) = 0$.

**Proof.** Let $e(t) = x(t) - \hat{x}(t)$ be the estimation error, and it follows from system (6) and system (10)–(12) that when $t \neq \tau_k$,

$$\dot{e}(t) = N_{\sigma(t)} e(t) + T_{\sigma(t)} \Phi(x) - T_{\sigma(t)} \Phi(\hat{x}), \tag{16}$$

and when $t = \tau_k$,

$$\begin{aligned} e(\tau_k^+) &= x(\tau_k^+) - \hat{x}(\tau_k^+) = x(\tau_k^+) - v(\tau_k^+) - F_{\sigma(\tau_k^+)} y(\tau_k) \\ &= x(\tau_k^+) - v(\tau_k^-) - (F_{\sigma(\tau_k^-)} - F_{\sigma(\tau_k^+)}) y(\tau_k) - F_{\sigma(\tau_k^+)} y(\tau_k) \\ &= x(\tau_k^-) - v(\tau_k^-) - F_{\sigma(\tau_k^-)} y(\tau_k) = e(\tau_k^-). \end{aligned}$$

We assume $\sigma(t) = i$ for $t \in [\tau_k, \tau_{k+1})$ and $\sigma(t) = j$ for $t \in [\tau_{k-1}, \tau_k)$. Choose Lyapunov function candidate $V_1(e(t)) = e^T(t) P_{\sigma(t)} e(t)$, and then for $t \in [\tau_k, \tau_{k+1})$, the derivative of $V_1(e(t))$ alone the error dynamic system (16) is

$$\begin{aligned} \dot{V}_1(e(t)) &= e^T(t)(P_i N_i + N_i^T P_i) e(t) + 2 e^T(t) P_i T_i \big(\Phi(x) - \Phi(\hat{x})\big) \\ &\le e^T(t)(P_i N_i + N_i^T P_i) e(t) + 2\gamma \| T_i^T P_i e(t) \| \cdot \| e(t) \|. \end{aligned} \tag{17}$$

For any scalar $\varepsilon_i > 0$, we have

$$2\gamma \cdot \| T_i^T P_i e(t) \| \cdot \| e(t) \| \le \frac{\gamma^2}{\varepsilon_i} \| T_i^T P_i e(t) \|^2 + \varepsilon_i \| e(t) \|^2 = e^T(t) \left( \frac{\gamma^2}{\varepsilon_i} P_i T_i T_i^T P_i + \varepsilon_i I \right) e(t). \tag{18}$$

Then, combining (17) with (18) leads to

$$\dot{V}_1(e(t)) \le e^T(t) \left( P_i N_i + N_i^T P_i + \frac{\gamma^2}{\varepsilon_i} P_i T_i T_i^T P_i + \varepsilon_i I \right) e(t) = -e^T(t) Q_i e(t).$$

Furthermore, we have

$$\dot{V}_1(e(t)) \le -e^T(t) Q_i e(t) \le -\frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)} V_1(e(t)) \le -\min_{i \in \Gamma} \left( \frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)} \right) V_1(e(t)) = -\theta_1 \cdot V_1(e(t)). \tag{19}$$

On the other hand, at switching time instant $\tau_k$, we have

$$V_1(e(\tau_k^+)) = e^T(\tau_k^+)P_ie(\tau_k^+) = e^T(\tau_k^-)P_ie(\tau_k^-) \le \frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)}V_1(e(\tau_k^-))$$
$$\le \max_{i,j\in(\Gamma\times\Gamma)}\left(\frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)}\right)V_1(e(\tau_k^-)) = \theta_2\cdot V_1(e(\tau_k^-)). \quad (20)$$

Under the ADT condition (15), i.e., $\tau_a > \tau^* = \frac{\ln\theta_2}{\theta_1}$, and according to Lemma 1, inequality (19) together with (20) gives $\lim_{t\to\infty} e(t) = 0$. The proof is completed. □

**Remark 5.** Theorem 1 gives sufficient conditions for the existence of full-order observer (10)–(12). It should be noticed that finding solutions of parameter $\varepsilon_i > 0$, matrices $F_i$, $L_i$, $P_i$, $Q_i$ to the Riccati Eq. (13) is not a trivial work. Next, we try to find solutions by transforming it into an equivalent LMI problem.

Denote $R_i = D_i(CD_i)^\dagger$ and $H_i = I_p - CD_i(CD_i)^\dagger$, then $F_i = R_i + Z_iH_i$. By substituting $F_i = R_i + Z_iH_i$, we have

$$T_i = I_n - R_iC - Z_iH_iC := \Delta_i - Z_iH_iC, \quad (21)$$

where $\Delta_i = I_n - R_iC$. Then, rewrite Riccati Eq. (13) as a matrix inequality

$$P_iN_i + N_i^TP_i + \frac{\gamma^2}{\varepsilon_i}P_iT_iT_i^TP_i + \varepsilon_iI_n < 0. \quad (22)$$

Substituting (14) and (21) into (22), and according to the Schur complement lemma, we obtain

$$\begin{bmatrix} P_i\Delta_iA_i+(\Delta_iA_i)^TP_i-P_iZ_iH_iCA_i-(H_iCA_i)^T(P_iZ_i)^T-P_iL_iC-C^T(P_iL_i)^T+\varepsilon_iI_n & \gamma P_i(\Delta_i-Z_iH_iC) \\ * & -\varepsilon_iI_n \end{bmatrix} < 0,$$

Let $\bar{Z}_i = P_iZ_i$ and $\Pi_i = P_iL_i$, and then the computation of matrices $F_i$, $L_i$, $P_i$, $Q_i$ can be completed by solving the LMI:

$$\begin{bmatrix} P_i\Delta_iA_i+(\Delta_iA_i)^TP_i-\bar{Z}_iH_iCA_i-(H_iCA_i)^T\bar{Z}_i^T-\Pi_iC-C^T\Pi_i^T+\varepsilon_iI_n & \gamma P_i\Delta_i-\gamma\bar{Z}_iH_iC \\ * & -\varepsilon_iI_n \end{bmatrix} < 0, \quad (23)$$

Now the solutions can be obtained by following Algorithm 1.

**Algorithm 1.**

**(1)** Solve the LMI (23) to get matrices $P_i, \Pi_i, \bar{Z}_i$ and parameter $\varepsilon_i$.
**(2)** Compute matrices $Z_i = P_i^{-1}\bar{Z}_i$, $L_i = P_i^{-1}\Pi_i$, $F_i = R_i + Z_iH_i$, $T_i = I_n - F_iC$, $N_i = T_iA_i - L_iC$ and $Q_i = -(P_iN_i + N_i^TP_i + \frac{\gamma^2}{\varepsilon_i}P_iT_iT_i^TP_i + \varepsilon_iI_n)$.
**(3)** Compute the ADT $\tau^*$ according to (15).

### 3.2. Reduced-order impulsive reset observer

Now, based on the full-order observer (10)–(12) and the Assumptions 1–3, we try to synthesize a reduced-order observer step by step. By using Smith orthogonal procedure, there are matrices $S \in \mathbb{R}^{p\times p}$ and $\hat{C} \in \mathbb{R}^{p\times n}$ such that $C = S\hat{C}$ and $\hat{C}\hat{C}^T = I_p$ (see Appendix A). We extend $\hat{C}$ to an orthogonal matrix $W = \begin{bmatrix} \hat{C}^T & M^T \end{bmatrix}^T$ satisfying $W^{-1} = W^T$, and thus we have $\hat{C}\hat{C}^T = I_p$ and $MM^T = I_{n-p}$. For system (1), taking a coordinate transformation $\bar{x} = Wx$ leads to

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}_{\sigma(t)}\bar{x}(t) + \bar{B}_{\sigma(t)}u(t) + W\Phi(x(t)) + \bar{D}_{\sigma(t)}\omega(t), \\ y(t) = \bar{C}x(t) \end{cases} \tag{24}$$

where $\bar{A}_{\sigma(t)} = WA_{\sigma(t)}W^T, \bar{B}_{\sigma(t)} = WB_{\sigma(t)}, \bar{C} = CW^T = [S \;\; 0]$ and $\bar{D}_{\sigma(t)} = WD_{\sigma(t)}$.

**Lemma 2.** *There exist matrices $F_i$, $T_i$, $L_i$, $N_i$, $i \in \Gamma$, such that (13) holds, if and only if*

$$-\bar{Q}_i := \bar{P}_i(\bar{A}_i - \bar{F}_i\bar{C}\bar{A}_i - \bar{L}_i\bar{C}) + (\bar{A}_i - \bar{F}_i\bar{C}\bar{A}_i - \bar{L}_i\bar{C})^T\bar{P}_i + \frac{\gamma^2}{\varepsilon_i}\bar{P}_i(I - \bar{F}_i\bar{C})(I - \bar{F}_i\bar{C})^T\bar{P}_i + \varepsilon_i I_n < 0 \tag{25}$$

*where $\bar{F}_i = WF_i, \bar{L}_i = WL_i, \bar{P}_i = WP_iW^T$ and $\bar{Q}_i = WQ_iW^T$.*

**Proof.** Please see the Appendix B. □

Now, decompose state $\bar{x}$, matrices $\bar{A}_i, \bar{F}_i, \bar{D}_i, \bar{P}_i, \bar{Q}_i$, $i \in \Gamma$, into blocks as follows:

$$\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}, \bar{A}_i = \begin{bmatrix} \bar{A}_{i,11} & \bar{A}_{i,12} \\ \bar{A}_{i,21} & \bar{A}_{i,22} \end{bmatrix}, \bar{F}_i = \begin{bmatrix} \bar{F}_{i,1} \\ \bar{F}_{i,2} \end{bmatrix}, \bar{D}_i = \begin{bmatrix} \bar{D}_{i,1} \\ \bar{D}_{i,2} \end{bmatrix},$$
$$\bar{P}_i = \begin{bmatrix} \bar{P}_{i,1} & \bar{P}_{i,2} \\ \bar{P}_{i,2}^T & \bar{P}_{i,3} \end{bmatrix}, \bar{Q}_i = \begin{bmatrix} \bar{Q}_{i,1} & \bar{Q}_{i,2} \\ \bar{Q}_{i,2}^T & \bar{Q}_{i,3} \end{bmatrix},$$

where $\bar{x}_1 \in \mathbb{R}^p, \bar{A}_{i,11} \in \mathbb{R}^{p\times p}, \bar{F}_{i,1} \in \mathbb{R}^{p\times p}, \bar{D}_{i,1} \in \mathbb{R}^{p\times q}, \bar{P}_{i,1} \in \mathbb{R}^{p\times p}$ and $\bar{Q}_{i,1} \in \mathbb{R}^{p\times p}$.

**Lemma 3.** *For any $i \in \Gamma$, if (25) holds, then the following Riccati equation holds:*

$$-\bar{Q}_{i,3} := \bar{P}_{i,3}(\bar{A}_{i,22} - \Theta_i\bar{A}_{i,12}) + (\bar{A}_{i,22} - \Theta_i\bar{A}_{i,12})^T\bar{P}_{i,3} + \frac{\gamma^2}{\varepsilon_i}\bar{P}_{i,3}(\Theta_i\Theta_i^T + I)\bar{P}_{i,3} + \varepsilon_i I_{n-p} < 0, \tag{26}$$

*where $\Theta_i = -\bar{P}_{i,3}^{-1}\bar{P}_{i,2}^T(I - \bar{F}_{i,1}S) + \bar{F}_{i,2}S$.*

**Proof.** Please see the Appendix C. □

**Lemma 4.** *For any $i \in \Gamma$, we have*

$$\Theta_i\bar{D}_{i,1} = \bar{D}_{i,2}. \tag{27}$$

**Proof.** Please see the Appendix D. □

**Remark 6.** It should be emphasized that the relationship of matrices in Lemma 4 which are derived from Riccati Eq. (13) is a convenient condition for reduced-order observer construction. Although the dynamics of both $\bar{x}_1$ and $\bar{x}_2$ are influenced by the unknown inputs in (24) in form, it is possible for us to decouple the unknown inputs from part of the states based on Lemma 4, and thus design a reduced-order observer.

Take the following state transformation

$$\varsigma(t) = \begin{bmatrix} \varsigma_1(t) \\ \varsigma_2(t) \end{bmatrix} = \begin{bmatrix} I_p & 0 \\ -\Theta_{\sigma(t)} & I_{n-p} \end{bmatrix}\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} \tag{28}$$

where $\varsigma_1 \in \mathbb{R}^p$. Eq. (28) implies that $\varsigma_1 = \begin{bmatrix} I_p & 0 \end{bmatrix}\bar{x}$ and $\varsigma_2 = \begin{bmatrix} -\Theta_{\sigma(t)} & I_{n-p} \end{bmatrix}\bar{x}$, and this gives that when $t \neq \tau_k$,

$$\begin{aligned}
\dot{\varsigma}_1(t) &= \begin{bmatrix} I_p & 0 \end{bmatrix}\dot{\bar{x}} = \begin{bmatrix} I_p & 0 \end{bmatrix}\Big(\bar{A}_{\sigma(t)}\bar{x}(t) + \bar{B}_{\sigma(t)}u(t) + W\Phi(x(t)) + \bar{D}_{\sigma(t)}\omega(t)\Big) \\
&= (\bar{A}_{\sigma(t),11} + \bar{A}_{\sigma(t),12}\Theta_{\sigma(t)})\varsigma_1(t) + \bar{A}_{\sigma(t),12}\varsigma_2(t) + \begin{bmatrix} I_p & 0 \end{bmatrix}\big(\bar{B}_{\sigma(t)}u(t) + W\Phi(x)\big) \\
&\quad + \bar{D}_{\sigma(t),1}\omega(t)
\end{aligned}$$

and

$$\begin{aligned}
\dot{\varsigma}_2(t) &= \begin{bmatrix} -\Theta_{\sigma(t)} & I_{n-p} \end{bmatrix}\dot{\bar{x}} = \begin{bmatrix} -\Theta_{\sigma(t)} & I_{n-p} \end{bmatrix}\Big(\bar{A}_{\sigma(t)}\bar{x}(t) + \bar{B}_{\sigma(t)}u(t) + W\Phi(x(t)) + \bar{D}_{\sigma(t)}\omega(t)\Big) \\
&= (\bar{A}_{\sigma(t),22} - \Theta_{\sigma(t)}\bar{A}_{\sigma(t),12})\varsigma_2(t) \\
&\quad + \Big(\bar{A}_{\sigma(t),21} - \Theta_{\sigma(t)}\bar{A}_{\sigma(t),11} + (\bar{A}_{\sigma(t),22} - \Theta_{\sigma(t)}\bar{A}_{\sigma(t),12})\Theta_{\sigma(t)}\Big)\varsigma_1(t) \qquad (29) \\
&\quad + \begin{bmatrix} -\Theta_{\sigma(t)} & I_{n-p} \end{bmatrix}\big(\bar{B}_{\sigma(t)}u(t) + W\Phi(x)\big),
\end{aligned}$$

where $\varsigma_1(t) = S^{-1}y(t)$. Next, a reduced-order UIO can be constructed in the following Theorem.

**Theorem 2.** *Under Assumptions 1–3, the following system*

$$\begin{aligned}
\dot{\hat{\varsigma}}_2(t) &= (\bar{A}_{\sigma(t),22} - \Theta_{\sigma(t)}\bar{A}_{\sigma(t),12})\hat{\varsigma}_2(t) \\
&\quad + \Big(\bar{A}_{\sigma(t),21} - \Theta_{\sigma(t)}\bar{A}_{\sigma(t),11} + (\bar{A}_{\sigma(t),22} - \Theta_{\sigma(t)}\bar{A}_{\sigma(t),12})\Theta_{\sigma(t)}\Big)S^{-1}y(t) \\
&\quad + \begin{bmatrix} -\Theta_{\sigma(t)} & I_{n-p} \end{bmatrix}\Big(\bar{B}_{\sigma(t)}u(t) + W\Phi\big(\hat{x}\big)\Big),\ t \neq \tau_k \qquad (30)
\end{aligned}$$

$$\hat{\varsigma}_2(\tau_k^+) = \hat{\varsigma}_2(\tau_k^-) - (\Theta_{\sigma(\tau_k^+)} - \Theta_{\sigma(\tau_k^-)})S^{-1}y(\tau_k), \quad t = \tau_k \qquad (31)$$

$$\hat{x}(t) = W^T\begin{bmatrix} S^{-1}y(t) \\ \hat{\varsigma}_2(t) + \Theta_{\sigma(t)}S^{-1}y(t) \end{bmatrix} \qquad (32)$$

*is a reduced-order impulsive reset switching observer of system (1), such that* $\lim_{t\to\infty}(x(t) - \hat{x}(t)) = 0$.

**Proof.** From the output equation in (24) and the Eq. (28), we get $\varsigma_1 = \bar{x}_1 = S^{-1}y$. Since $\bar{x} = Wx$, now based on (28) we can reach

$$x = W^T\bar{x} = W^T\begin{bmatrix} I & 0 \\ \Theta_{\sigma(t)} & I \end{bmatrix}\begin{bmatrix} \varsigma_1 \\ \varsigma_2 \end{bmatrix} = W^T\begin{bmatrix} \varsigma_1 \\ \varsigma_2 + \Theta_{\sigma(t)}\varsigma_1 \end{bmatrix} = W^T\begin{bmatrix} S^{-1}y(t) \\ \varsigma_2 + \Theta_{\sigma(t)}S^{-1}y(t) \end{bmatrix},$$

and this leads to

$$\begin{aligned}
\tilde{x}(t) = x(t) - \hat{x}(t) &= W^T\begin{bmatrix} S^{-1}y(t) \\ \varsigma_2(t) + \Theta_{\sigma(t)}S^{-1}y(t) \end{bmatrix} - W^T\begin{bmatrix} S^{-1}y(t) \\ \hat{\varsigma}_2(t) + \Theta_{\sigma(t)}S^{-1}y(t) \end{bmatrix} \\
&= M^T(\varsigma_2(t) - \hat{\varsigma}_2(t)).
\end{aligned}$$

Therefore, in order to prove $\lim_{t\to\infty}(x(t) - \hat{x}(t)) = 0$, what we need to do is to show $\lim_{t\to\infty}(\varsigma_2(t) - \hat{\varsigma}_2(t)) = 0$. Let $\tilde{\varsigma}_2(t) = \varsigma_2(t) - \hat{\varsigma}_2(t)$ be the estimation error. By (28) we have $\varsigma_2(t) = -\Theta_{\sigma(t)}S^{-1}y(t) + \bar{x}_2(t)$ and note that $y(t)$ and $\bar{x}_2(t) = Mx(t)$ are continuous at switching time instant $\tau_k$, we have

$$\varsigma_2(\tau_k^+) = -\Theta_{\sigma(\tau_k^+)}S^{-1}y(\tau_k) + \bar{x}_2(\tau_k),$$

and

$$\varsigma_2(\tau_k^-) = -\Theta_{\sigma(\tau_k^-)} S^{-1} y(\tau_k) + \bar{x}_2(\tau_k),$$

which imply

$$\varsigma_2(\tau_k^+) = \varsigma_2(\tau_k^-) - (\Theta_{\sigma(\tau_k^+)} - \Theta_{\sigma(\tau_k^-)}) S^{-1} y(\tau_k). \tag{33}$$

It follows from (31) and (33) that at each switching instant $\tau_k$, we have $\tilde{\varsigma}_2(\tau_k^+) = \tilde{\varsigma}_2(\tau_k^-)$. According to (29) and (30), when $t \neq \tau_k$, the error dynamic of $\tilde{\varsigma}_2(t)$ is obtained as

$$\begin{aligned}\dot{\tilde{\varsigma}}_2(t) &= (\bar{A}_{\sigma(t),22} - \Theta_{\sigma(t)} \bar{A}_{\sigma(t),12}) \tilde{\varsigma}_2(t) + \begin{bmatrix} -\Theta_{\sigma(t)} & I_{n-p} \end{bmatrix} W \\ &\quad \times \left( \Phi\left( W^T \begin{bmatrix} S^{-1} y(t) \\ \varsigma_2(t) + \Theta_{\sigma(t)} S^{-1} y(t) \end{bmatrix} \right) - \Phi\left( W^T \begin{bmatrix} S^{-1} y(t) \\ \hat{\varsigma}_2(t) + \Theta_{\sigma(t)} S^{-1} y(t) \end{bmatrix} \right) \right).\end{aligned} \tag{34}$$

Choose Lyapunov function candidate $V_2(t) = \tilde{\varsigma}_2^T(t) \bar{P}_{\sigma(t),3} \tilde{\varsigma}_2(t)$. We assume $\sigma(t) = i$ for $t \in [\tau_k, \tau_{k+1})$ and $\sigma(t) = j$ for $t \in [\tau_{k-1}, \tau_k)$. Then, for $t \in [\tau_k, \tau_{k+1})$, the derivative of $V_2(t)$ alone the error dynamic Eq. (34) is

$$\begin{aligned}\dot{V}_2(t) &= \tilde{\varsigma}_2^T(t) \big( \bar{P}_{i,3} (\bar{A}_{i,22} - \Theta_i \bar{A}_{i,12}) + (\bar{A}_{i,22} - \Theta_i \bar{A}_{i,12})^T \bar{P}_{i,3} \big) \tilde{\varsigma}_2(t) + 2 \tilde{\varsigma}_2^T(t) \bar{P}_{i,3} [-\Theta_i \quad I] W \\ &\quad \times \left( \Phi\left( W^T \begin{bmatrix} S^{-1} y(t) \\ \varsigma_2(t) + \Theta_i S^{-1} y(t) \end{bmatrix} \right) - \Phi\left( W^T \begin{bmatrix} S^{-1} y(t) \\ \hat{\varsigma}_2(t) + \Theta_i S^{-1} y(t) \end{bmatrix} \right) \right).\end{aligned} \tag{35}$$

In view of the Lipschitz condition (2), we obtain

$$\begin{aligned}&2 \tilde{\varsigma}_2^T(t) \bar{P}_{i,3} \begin{bmatrix} -\Theta_i & I \end{bmatrix} W \left( \Phi\left( W^T \begin{bmatrix} S^{-1} y(t) \\ \varsigma_2(t) + \Theta_i S^{-1} y(t) \end{bmatrix} \right) - \Phi\left( W^T \begin{bmatrix} S^{-1} y(t) \\ \hat{\varsigma}_2(t) + \Theta_i S^{-1} y(t) \end{bmatrix} \right) \right) \\ &\quad \leq 2\gamma \cdot \| \tilde{\varsigma}_2^T(t) \bar{P}_{i,3} \begin{bmatrix} -\Theta_i & I \end{bmatrix} W \| \cdot \| M^T \tilde{\varsigma}_2(t) \|.\end{aligned} \tag{36}$$

For any scalar $\varepsilon_i > 0$, it follows from (36) that

$$\begin{aligned}&2\gamma \cdot \| \tilde{\varsigma}_2^T(t) \bar{P}_{i,3} \begin{bmatrix} -\Theta_i & I \end{bmatrix} W \| \cdot \| M^T \tilde{\varsigma}_2(t) \| \\ &\quad \leq \frac{\gamma^2}{\varepsilon_i} \| \tilde{\varsigma}_2^T(t) \bar{P}_{i,3} \begin{bmatrix} -\Theta_i & I \end{bmatrix} W \|^2 + \varepsilon_i \| M^T \tilde{\varsigma}_2(t) \|^2 \\ &\quad = \tilde{\varsigma}_2^T(t) \left( \frac{\gamma^2}{\varepsilon_i} \bar{P}_{i,3} (\Theta_i \Theta_i^T + I) \bar{P}_{i,3} + \varepsilon_i M M^T \right) \tilde{\varsigma}_2(t) \\ &\quad = \tilde{\varsigma}_2^T(t) \left( \frac{\gamma^2}{\varepsilon_i} \bar{P}_{i,3} (\Theta_i \Theta_i^T + I) \bar{P}_{i,3} + \varepsilon_i I \right) \tilde{\varsigma}_2(t).\end{aligned} \tag{37}$$

Then, combining (35) with (36) and (37), one can obtain

$$\begin{aligned}\dot{V}_2(t) &\leq -\tilde{\varsigma}_2^T(t) \bigg( \bar{P}_{i,3} (\bar{A}_{i,22} - \Theta_i \bar{A}_{i,12}) + (\bar{A}_{i,22} - \Theta_i \bar{A}_{i,12})^T \bar{P}_{i,3} \\ &\qquad + \frac{\gamma^2}{\varepsilon_i} \bar{P}_{i,3} \begin{bmatrix} -\Theta_i & I \end{bmatrix} \begin{bmatrix} -\Theta_i^T \\ I \end{bmatrix} \bar{P}_{i,3} + \varepsilon_i I \bigg) \tilde{\varsigma}_2(t) \\ &\leq -\tilde{\varsigma}_2^T(t) \bar{Q}_{i,3} \tilde{\varsigma}_2(t).\end{aligned}$$

Therefore, we have

$$\dot{V}_2(t) \leq -\tilde{\varsigma}_2^T(t) \bar{Q}_{i,3} \tilde{\varsigma}_2(t) \leq -\frac{\lambda_{\min}(\bar{Q}_{i,3})}{\lambda_{\max}(\bar{P}_{i,3})} V_2(t) \leq -\min_{i \in \Gamma} \left( \frac{\lambda_{\min}(\bar{Q}_{i,3})}{\lambda_{\max}(\bar{P}_{i,3})} \right) V_2(t) = -\bar{\theta}_1 \cdot V_2(t), \tag{38}$$

where $\bar{\theta}_1 = \min_{i\in\Gamma}\left(\frac{\lambda_{\min}(\bar{Q}_{i,3})}{\lambda_{\max}(\bar{P}_{i,3})}\right)$. Meanwhile, at switching time instant $\tau_k$, it is derived that

$$V_2(\tau_k^+) = \tilde{\varsigma}_2^T(\tau_k^+)\bar{P}_{i,3}\tilde{\varsigma}_2(\tau_k^+) = \tilde{\varsigma}_2^T(\tau_k^-)\bar{P}_{i,3}\tilde{\varsigma}_2(\tau_k^-) \le \frac{\lambda_{\max}(\bar{P}_{i,3})}{\lambda_{\min}(\bar{P}_{j,3})}V_2(\tau_k^-)$$
$$\le \max_{(i,j)\in(\Gamma\times\Gamma)}\left(\frac{\lambda_{\max}(\bar{P}_{i,3})}{\lambda_{\min}(\bar{P}_{j,3})}\right)V_2(\tau_k^-) = \bar{\theta}_2\cdot V_2(\tau_k^-) \tag{39}$$

where $\bar{\theta}_2 = \max_{(i,j)\in(\Gamma\times\Gamma)}\left(\frac{\lambda_{\max}(\bar{P}_{i,3})}{\lambda_{\min}(\bar{P}_{j,3})}\right)$.

On the one hand, we have

$$\frac{\lambda_{\min}(\bar{Q}_{i,3})}{\lambda_{\max}(\bar{P}_{i,3})} \ge \frac{\lambda_{\min}(\bar{Q}_i)}{\lambda_{\max}(\bar{P}_i)} = \frac{\lambda_{\min}(W^T\bar{Q}_iW)}{\lambda_{\max}(W^T\bar{P}_iW)} = \frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)}.$$

Therefore,

$$\bar{\theta}_1 = \min_{i\in\Gamma}\left(\frac{\lambda_{\min}(\bar{Q}_{i,3})}{\lambda_{\max}(\bar{P}_{i,3})}\right) \ge \min_{i\in\Gamma}\left(\frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)}\right) = \theta_1.$$

Furthermore, for any $(i, j) \in \Gamma \times \Gamma$,

$$\frac{\lambda_{\max}(\bar{P}_{i,3})}{\lambda_{\min}(\bar{P}_{j,3})} \le \frac{\lambda_{\max}(\bar{P}_i)}{\lambda_{\min}(\bar{P}_j)} = \frac{\lambda_{\max}(W^T\bar{P}_iW)}{\lambda_{\min}(W^T\bar{P}_jW)} = \frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)}.$$

Thus,

$$\bar{\theta}_2 = \max_{(i,j)\in\Gamma\times\Gamma}\left(\frac{\lambda_{\max}(\bar{P}_{i,3})}{\lambda_{\min}(\bar{P}_{j,3})}\right) \le \max_{(i,j)\in\Gamma\times\Gamma}\left(\frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)}\right) = \theta_2.$$

This means that provided $\tau_a > \tau^*$, we have $\tau_a > \frac{\ln\bar{\theta}_2}{\bar{\theta}_1}$. And by using the conclusion of Lemma 1 we have $\lim_{t\to\infty}\tilde{\varsigma}_2(t) = 0$. The proof is completed. □

**Remark 7.** The full-order observer ((10)–(12)) and reduced-order observer (30)–(32) are called impulsive reset switching observers, because at each switching time instant $\tau_k$, impulsive reset equations (Eq. (11) of full-order observer and Eq. (31) of reduced-order observer) are designed for observer state vector $v(t)$ and $\hat{\varsigma}_2(t)$. It should be emphasized that the impulsive reset Eqs. (11) and (31) play important roles in observer designs. If there were no (11) and (31), the state estimate $\hat{x}$ in both full-order and reduced-order observers would always jump at each switching instant, which may deteriorate the estimation performances. It is the reset Eqs. (11) and (31) that makes the state estimations continuous at each switched point. Indeed, for full-order observer (10)–(12), we have

$$\hat{x}(\tau_k^+) = v(\tau_k^+) + F_{\sigma(\tau_k^+)}y(\tau_k) = v(\tau_k^-) + (F_{\sigma(\tau_k^-)} - F_{\sigma(\tau_k^+)})y(\tau_k) + F_{\sigma(\tau_k^+)}y(\tau_k)$$
$$= v(\tau_k^-) + F_{\sigma(\tau_k^-)}y(\tau_k) = \hat{x}(\tau_k^-),$$

and for reduced-order observer (30)–(32), we have

$$\hat{x}(\tau_k^+) = W^T\begin{bmatrix} S^{-1}y(\tau_k) \\ \hat{\varsigma}_2(\tau_k^+) + \Theta_{\sigma(\tau_k^+)}S^{-1}y(\tau_k)\end{bmatrix}$$
$$= W^T\begin{bmatrix} S^{-1}y(\tau_k) \\ \hat{\varsigma}_2(\tau_k^-) - (\Theta_{\sigma(\tau_k^+)} - \Theta_{\sigma(\tau_k^-)})S^{-1}y(\tau_k) + \Theta_{\sigma(\tau_k^+)}S^{-1}y(\tau_k)\end{bmatrix}$$

$$= W^T \begin{bmatrix} S^{-1}y(\tau_k) \\ \hat{\varsigma}_2(\tau_k^-) + \Theta_{\sigma(\tau_k^-)} S^{-1}y(\tau_k) \end{bmatrix} = \hat{x}(\tau_k^-).$$

**Remark 8.** Not only the full-order but also the reduced-order observers in the present paper, three issues had to be addressed. They are UI decoupling, observer error dynamics stability analysis and average dwell time determination. First, in full-order observer construction, Eq. (8) ensures the elimination of UIs in the error dynamics (16), and Eq. (27) is a convenient UI-decoupling condition for the reduced-order UIO. Here, (27) is just derived from (8). Secondly, Riccati Eq. (13) guarantees the stability of each subsystem of the full-order observer error dynamics. It can be seen from Lemma 2 that once (13) holds, there is also a Riccati Eq. (26) which ensures the reduced-order observer error dynamic subsystem stability. Moreover, each subsystem stability, which is guaranteed by the Riccati equation, together with ADT restriction condition ensures the asymptotical stability of the whole switched full-order observer error system. Finally, as shown in the proof of the Theorem 2, if the average dwell time ensures the existence of the full-order observer, it can also guarantee the existence of the corresponding reduced-order observer. For these reasons, it is sufficient to conclude that for switched nonlinear system (1), the conditions (the Riccati Eq. (13) together with the ADT restriction condition (15)) under which a full-order observer exists also guarantee the existence of a reduced-order observer.

**Remark 9.** In literature [47–49], the observer designs for several classes of uncertain nonlinear systems with unknown inputs (unknown non-smooth nonlinearities) were studied, where the uncertain nonlinearities were usually be approximated by fuzzy logic systems or neural networks, and the observation errors were bounded. By comparison, the state estimations obtained by both the full-order and the reduced-order observers designed in the present paper can asymptotically converge to their actual values.

**Remark 10.** Although the observer designs for switched linear or nonlinear systems were considered in [33–35,40–42,50], both the system model and the results of our paper are different from those in [33–35,40–42,50]. First, in [50], the system considered is a switched nonlinear system which does not contain unknown input. In [34,35,40–42], only switched linear unknown input systems were considered but nonlinear terms were not included, and in [33], the discrete-time descriptor system was studied with arbitrary switching signal. While, in the present paper, both the unknown input and the nonlinearity are considered. Second, the relationship of the existence conditions between the full-order and the reduced-order unknown input observers is discussed in detail. Specifically, we point out that the ADT of the reduced-order observer is not larger than that of the full-order observer, and based on this, we conclude that the conditions under which the full-order observer exists can also guarantee the existence of a reduced-order observer.

**Remark 11.** The relationships of the existence condition between full-order and reduced-order observers are first discussed in [43] for a general Lipschitz nonlinear system, and later for one-sided Lipschitz nonlinear systems [51] and [52]. Compared with the work in [43,51,52], the work given in the present paper is more challenging because we have to deal with not only the unknown inputs but also the switching influences. In order to deal with the unknown inputs, we have to find a coordinate transformation (28) to not only ensure the stability of the obtained subsystem (29) (i.e., ensure matrix $\bar{A}_{\sigma(t),22} - \Theta_{\sigma(t)}\bar{A}_{\sigma(t),12}$ being Hurwitz) but also guarantee the unknown inputs being decoupled in (29). In order to deal with the switching influences, impulsive reset switching designs are considered such that the 'state

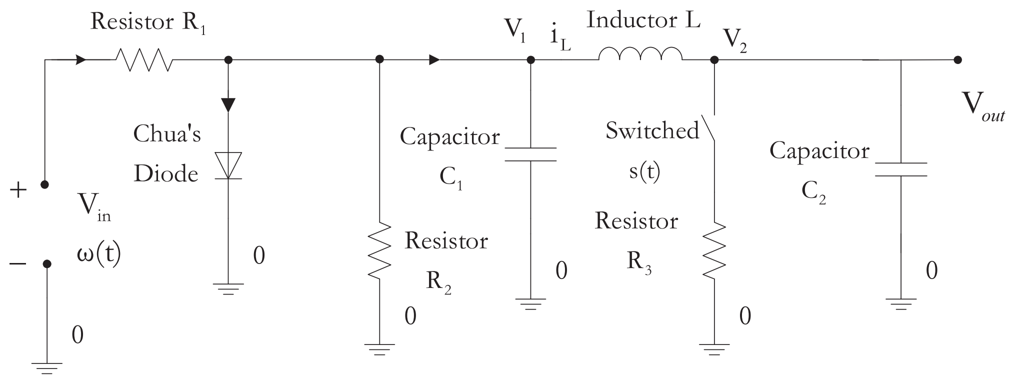


Fig. 1. An electronic circuit.

jump' phenomena at each switching point can be avoided in the state estimation. Moreover, detailed discussions on ADT and the comparisons of ADT between the full-order observer and the reduced-order observer are made, and thus some important conclusions about ADT are obtained.

## 4. A simulation example

In this section, we apply the proposed methods to a switched electronic circuit system.

Consider the electronic circuit as is shown in Fig. 1, where $C_1$ and $C_2$ stand for capacitors, $R_1$, $R_2$ and $R_3$ are resistors, $L$ is an inductor and $i_L$ denotes the current through $L$. $\omega(t)$ is unknown value of the voltage resource. $V_1(t)$ and $V_2(t)$, the voltages of $C_1$ and $C_2$, are viewed as the measurement outputs. The Chua's diode is a nonlinear resistor [53], whose Voltage-Amperage curve is depicted by

$$I = g(U) = \begin{cases} -1.143U - 0.429, & U \le -1 \\ -0.714U, & -1 < U \le 1 \\ -1.143U + 0.429, & U > 1 \end{cases}$$

where $U$ and $I$ represent the voltage and the current of the Chua's diode, respectively. Then, the circuit system can be characterized by

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L}V_1(t) - \dfrac{1}{L}V_2(t) \\ \dfrac{dV_1}{dt} = -\dfrac{1}{C_1}i_L(t) - \dfrac{1}{C_1}g(V_1) - \dfrac{1}{C_1R_2}V_1(t) + \dfrac{1}{C_1R_1}\omega(t) \\ \dfrac{dV_2}{dt} = \dfrac{1}{C_2}i_L(t) - \dfrac{s(t)}{C_2R_3}V_2(t) \end{cases}$$

where $s(t) = \begin{cases} 1, & \text{switch on} \\ 0, & \text{switch off} \end{cases}$ is designed to produce the switching between the two modes of the electronic circuit. Let $x_1(t) = i_L(t), x_2(t) = V_1(t), x_3(t) = V_2(t)$, then the above circuit system can be described by

$$\begin{cases} \dot{x}(t) = A_i x(t) + \Phi(x) + D\omega(t) \\ y(t) = Cx(t), i = 1, 2 \end{cases}$$

where

$$A_1 = \begin{bmatrix} 0 & \frac{1}{L} & -\frac{1}{L} \\ -\frac{1}{C_1} & -\frac{1}{C_1R_2} & 0 \\ \frac{1}{C_2} & 0 & -\frac{1}{C_2R_3} \end{bmatrix}, A_2 = \begin{bmatrix} 0 & \frac{1}{L} & -\frac{1}{L} \\ -\frac{1}{C_1} & -\frac{1}{C_1R_2} & 0 \\ \frac{1}{C_2} & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ \frac{1}{C_1R_1} \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \Phi(x) = \begin{bmatrix} 0 \\ -\frac{1}{C_1}g(x_2) \\ 0 \end{bmatrix}.$$

Choose $C_1 = 50mF, C_2 = 20mF, R_1 = R_2 = 0.2\Omega, R_3 = 10\Omega, L = 800mH$, and $\omega(t) = 2sin(5t)$. Based on Algorithm 1, the observer gain matrices are calculated as:

$$P_1 = \begin{bmatrix} 0.1999 & 0 & 0 \\ 0 & 0.5825 & 0 \\ 0 & 0 & 0.5825 \end{bmatrix}, \Pi_1 = \begin{bmatrix} 0.2499 & -0.2499 \\ 5.1384 & -3.7363 \\ 3.7363 & 5.1384 \end{bmatrix},$$

$$\bar{Z}_1 = \begin{bmatrix} 0 & 0.1251 \\ 0 & 0 \\ 0 & 0.5825 \end{bmatrix}, \varepsilon_1 = 4.3408$$

$$P_2 = \begin{bmatrix} 0.2010 & 0 & 0 \\ 0 & 0.5780 & 0 \\ 0 & 0 & 0.5780 \end{bmatrix}, \Pi_2 = \begin{bmatrix} 0.2513 & 0.3930 \\ 5.2748 & -0.2929 \\ 0.2929 & 5.2748 \end{bmatrix},$$

$$\bar{Z}_2 = \begin{bmatrix} 0 & 0.1289 \\ 0 & 0 \\ 0 & 0.5780 \end{bmatrix}, \varepsilon_2 = 4.5313$$

$$F_1 = \begin{bmatrix} 0 & 0.6256 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, L_1 = \begin{bmatrix} 1.2500 & -1.2500 \\ 8.8212 & -6.4142 \\ 6.4142 & 8.8212 \end{bmatrix}, T_1 = \begin{bmatrix} 1 & 0 & -0.6256 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.7875 & 0 & 0 \\ 0 & 5.9359 & 0 \\ 0 & 0 & 5.9359 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 0 & 0.6410 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, L_2 = \begin{bmatrix} 1.2500 & 1.9552 \\ 9.1259 & -0.5068 \\ 0.5068 & 9.1259 \end{bmatrix}, T_2 = \begin{bmatrix} 1 & 0 & -0.6410 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 1.1075 & 0 & 0 \\ 0 & 6.0183 & 0 \\ 0 & 0 & 6.0183 \end{bmatrix}.$$

According to matrix $C$, one can obtain by using Smith orthogonal procedure that

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, W = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \Theta_1 = \begin{bmatrix} 0 & 0.6256 \end{bmatrix}, \Theta_2 = \begin{bmatrix} 0 & 0.6410 \end{bmatrix}$$

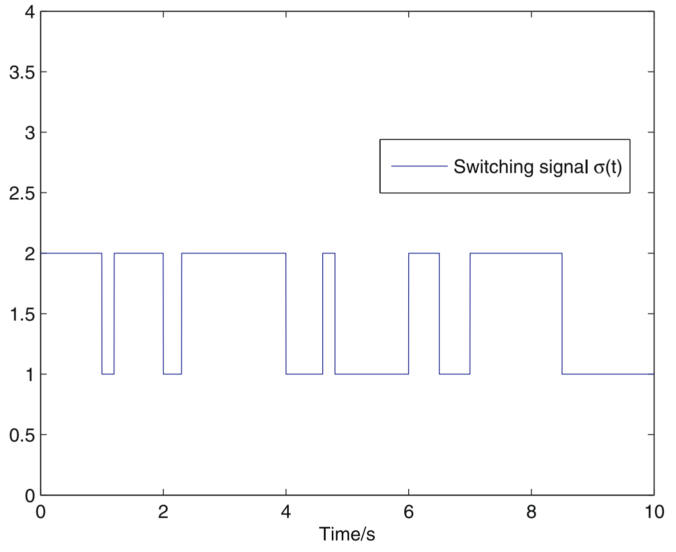


Fig. 2. Switching signal $\sigma(t)$.

and

$$\bar{A}_{1,11} = \begin{bmatrix} -100 & 0 \\ 0 & 0 \end{bmatrix}, \bar{A}_{1,12} = \begin{bmatrix} -20 \\ 50 \end{bmatrix}, \bar{A}_{1,21} = \begin{bmatrix} 1.25 & -1.25 \end{bmatrix}, \bar{A}_{1,22} = 0$$

$$\bar{A}_{2,11} = \begin{bmatrix} -100 & 0 \\ 0 & -5 \end{bmatrix}, \bar{A}_{2,12} = \begin{bmatrix} -20 \\ 50 \end{bmatrix}, \bar{A}_{2,21} = \begin{bmatrix} 1.25 & -1.25 \end{bmatrix}, \bar{A}_{2,22} = 0.$$

The full-order impulsive switching observer (10)–(12) and reduced-order impulsive switching observer (30)–(32) are designed under the ADT switching signals generated by $\sigma(t)$ with

$$\tau_a \geq \tau^* = 0.7869(\theta_1 = 1.3519 \text{ and } \theta_2 = 2.8977).$$

Set the initial values $x(0) = \begin{bmatrix} 2.5 & -2 & -5 \end{bmatrix}^T$ in system (1), $v(0) = \begin{bmatrix} -10 & 12 & 20 \end{bmatrix}^T$ in full-order observer (10)–(12), and $\varsigma_2(0) = -10$ in reduced-order observer (30)–(32), respectively. Fig. 2 shows the switching signal with respect to time $t$. Figs. 3 and 4 show the state estimating performances given by full-order and reduced-order impulsive reset observers. It can be seen that both the full-order and the reduced-order observers possess good estimation performances.

## 5. Conclusions

In this paper, the observer designs for a class of switched nonlinear systems are considered. A full-order and a reduced-order impulsive reset switching observers are constructed. The sufficient conditions of the full-order observer are formulated in terms of a Riccati equation. Moreover, we have shown that the existence conditions under which the full-order observer (10)–(12) exist also guarantees the existence of a corresponding reduced-order observer

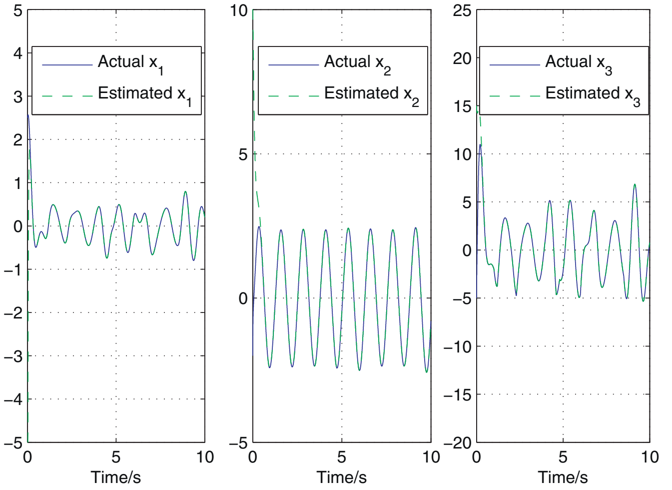


Fig. 3. State estimation via full-order observer.

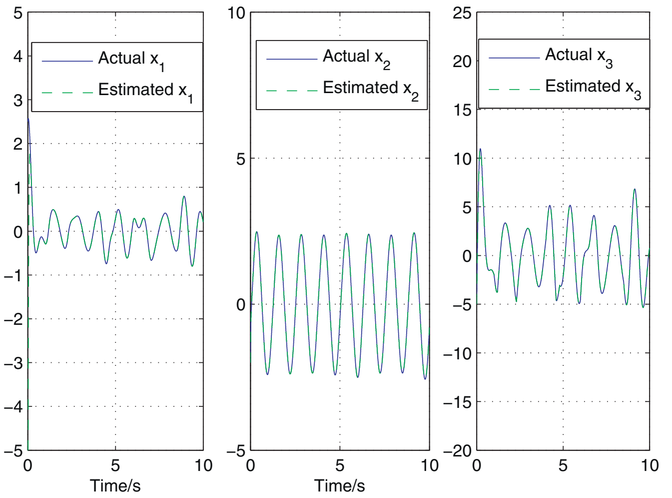


Fig. 4. State estimation via reduced-order observer.

(30)–(32). How to extend these methods to unknown input systems with switching nonlinear term will be next consideration.

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## Appendix A. The computation of $\hat{C}$

Decompose matrix $C = \begin{bmatrix} c_1^T & c_2^T & \dots & c_p^T \end{bmatrix}^T$, where $c_i \in \mathbb{R}^{1\times n}$. By Smith orthogonal procedure, we obtain

$$\beta_1 = c_1^T,\ \beta_2 = c_2^T - \frac{< c_2^T, \beta_1 >}{< \beta_1, \beta_1 >}\beta_1,\ \dots,\ \beta_p = c_p^T - \sum_{i=1}^{p-1} \frac{< c_p^T, \beta_i >}{< \beta_i, \beta_i >}\beta_i \tag{40}$$

where $\beta_1, \beta_2, \dots, \beta_p \in \mathbb{R}^n$ are orthogonal vectors. Rewrite (40) as $C^T = \Upsilon\Omega$, where $\Upsilon = \begin{bmatrix} \beta_1 & \beta_2 & \dots & \beta_p \end{bmatrix}$ and

$$\Omega = \begin{bmatrix} 1 & \frac{< c_2^T, \beta_1 >}{< \beta_1, \beta_1 >} & \dots & \frac{< c_p^T, \beta_1 >}{< \beta_1, \beta_1 >} \\ 0 & 1 & \dots & \frac{< c_p^T, \beta_2 >}{< \beta_2, \beta_2 >} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

Then we have $\Upsilon^T\Upsilon = I_p$ and $C = \Omega^T\Upsilon^T$. Let $S = \Omega^T$ and $\hat{C} = \Upsilon^T$, then we have $C = S\hat{C}$ and $\hat{C}\hat{C} = I_p$.

## Appendix B. The proof of Lemma 2

**Proof.** Substituting (7), (8) into (13), and then pre-multiplying matrix $W$ and post-multiplying $W^T$ in (13), respectively, one can obtain Eq. (25) directly. □

## Appendix C. The proof of Lemma 3

**Proof.** Note that $\bar{C}\begin{bmatrix} 0 & I \end{bmatrix}^T = \begin{bmatrix} S & 0 \end{bmatrix}\begin{bmatrix} 0 & I \end{bmatrix}^T = 0$, then pre-multiplying matrix $\begin{bmatrix} 0_{(n-p)\times p} & I_{n-p} \end{bmatrix}$ and post-multiplying matrix $\begin{bmatrix} 0_{(n-p)\times p} & I_{n-p} \end{bmatrix}^T$ in Eq. (25), respectively gives

$$\begin{aligned} -\bar{Q}_{i,3} &= \begin{bmatrix} 0 & I \end{bmatrix}\bar{P}_i(\bar{A}_i - \bar{F}_i\bar{C}\bar{A}_i)\begin{bmatrix} 0 & I \end{bmatrix}^T + \begin{bmatrix} 0 & I \end{bmatrix}(\bar{A}_i - \bar{F}_i\bar{C}\bar{A}_i)^T\bar{P}_i\begin{bmatrix} 0 & I \end{bmatrix}^T \\ &\quad + \gamma^2/\varepsilon_i\begin{bmatrix} 0 & I \end{bmatrix}\bar{P}_i(I_n - \bar{F}_i\bar{C})(I_n - \bar{F}_i\bar{C})^T\bar{P}_i\begin{bmatrix} 0 & I \end{bmatrix}^T + \varepsilon_i I_{n-p} \\ &= \bar{P}_{i,3}(\bar{A}_{i,22} - \Theta_i\bar{A}_{i,12}) + (\bar{A}_{i,22} - \Theta_i\bar{A}_{i,12})^T\bar{P}_{i,3} + \gamma^2/\varepsilon_i\bar{P}_{i,3}(\Theta_i\Theta_i^T + I)\bar{P}_{i,3} + \varepsilon_i I_{n-p} < 0. \end{aligned}$$

The proof is completed. □

## Appendix D. The proof of Lemma 4

**Proof.** Since for any $i \in \Gamma$, we have $F_iCD_i = D_i$. Therefore, $WF_iCW^TWD_i = WD_i$, i.e., $\bar{F}_i\bar{C}\bar{D}_i = \bar{D}_i$. Then we get

$$(I - \bar{F}_{i,1}S)\bar{D}_{i,1} = \bar{D}_{i,1} - \bar{F}_{i,1}S\bar{D}_{i,1} = \bar{D}_{i,1} - \begin{bmatrix} I_p & 0_{p\times(n-p)} \end{bmatrix}\bar{F}_i\begin{bmatrix} S & 0 \end{bmatrix}\begin{bmatrix} \bar{D}_{i,1} \\ \bar{D}_{i,2} \end{bmatrix}$$

$$= \bar{D}_{i,1} - \begin{bmatrix} I_p & 0_{p\times(n-p)} \end{bmatrix} \bar{F}_i \bar{C} \bar{D}_i = \bar{D}_{i,1} - \begin{bmatrix} I_p & 0_{p\times(n-p)} \end{bmatrix} \bar{D}_i = 0$$

and

$$\bar{F}_{i,2} S \bar{D}_{i,1} = \begin{bmatrix} 0_{(n-p)\times p} & I_{n-p} \end{bmatrix} \bar{F}_i S \bar{D}_{i,1} = \begin{bmatrix} 0_{(n-p)\times p} & I_{n-p} \end{bmatrix} \bar{F}_i \begin{bmatrix} S & 0 \end{bmatrix} \begin{bmatrix} \bar{D}_{i,1} \\ \bar{D}_{i,2} \end{bmatrix}$$
$$= \begin{bmatrix} 0_{(n-p)\times p} & I_{n-p} \end{bmatrix} \bar{F}_i \bar{C} \bar{D}_i = \begin{bmatrix} 0_{(n-p)\times p} & I_{n-p} \end{bmatrix} \bar{D}_i = \bar{D}_{i,2}.$$

Thus, $\Theta_i \bar{D}_{i,1} = -\bar{P}_{i,3}^{-1} \bar{P}_{i,2}^T (I - \bar{F}_{i,1} S) \bar{D}_{i,1} + \bar{F}_{i,2} S \bar{D}_{i,1} = \bar{D}_{i,2}$, and this ends the proof. □

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