ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Delay-dependent guaranteed-cost control based on combination of Smith predictor and equivalent-input-disturbance approach

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ARTICLE INFO

Article history:
Received 16 January 2015
Received in revised form
29 January 2016
Accepted 8 February 2016
This paper was recommended for publication by Dr. Oscar Camacho

Keywords:
Cone complementary linearization
Equivalent input disturbance (EID)
Free-weighting matrix
Guaranteed-cost control
Smith predictor
Time delay

ABSTRACT

This paper presents a new system configuration and a design method to improve control performance for a system with an input time delay and disturbances. The equivalent-input-disturbance approach is extended to handle a time-delay system. It is combined with the Smith predictor to reject disturbances. A delay-dependent stability condition is devised in terms of a matrix inequality by using the free-weighting matrix approach. The gain of the observer is designed by applying the cone complementary linearization method to the matrix inequality. A numerical example demonstrates the validity of the method.

1. Introduction

A time delay is often encountered in many practical systems, such as chemical processes, biological mechanisms, and mechanical apparatus [1–3]. Since a time delay decreases the stability margin of a closed-loop system, the design of a robust time-delay control system is a challenging problem, and has been drawing considerable attentions [4–7].

Various methods have been proposed to improve the robust performance of a closed-loop time-delay system [8–10]. The Smith predictor (SP) [11–13] among them is the one that has been widely used. It equivalently removes a time delay out of the closed control loop, and stabilizes the time-delay system. However, disturbance-rejection performance is not satisfactory for the SP. While sliding mode control (SMC) [14,15] is an effective method to solve this problem, it may cause high-frequency oscillation. This makes it difficult to implement the control law for a mechatronic system.

Some methods that actively compensate disturbances have been widely noticed. The equivalent-input-disturbance (EID) approach is one among them. It was devised to reject both matched and unmatched disturbances effectively [16–18]. An EID is a signal on the control input channel of a system that produces the same effect on the output as actual disturbances do. This approach does not require a prior information about disturbances. And since it does not use the inverse dynamics of a plant, it avoids the cancelation of unstable poles and zeros, which happens in a disturbance observer.

This paper considers a guaranteed-cost control problem for a plant with both of an input time delay and a non-stationary disturbance. The control system combines the SP with the EID approach, which is called the SP-EID control system here after, to improve control performance. Since the guaranteed-cost control method provides a control law that not only stabilizes a time-delay system but also ensures an adequate level of control performance, such a control law is presented in this paper to guarantee the upper bound of a specified linear integral-quadratic cost function. A delay-dependent sufficient stability condition is derived in terms of a matrix inequality. And the gain of the observer is obtained from the condition using the cone complementary linearization method. The validity of the method is demonstrated through simulations.

In the rest of the paper, $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$ is indicated by $\begin{bmatrix} A & B \\ \star & C \end{bmatrix}$.

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http://dx.doi.org/10.1016/j.isatra.2016.02.008
0019-0578/

2. Configuration of SP-EID of control system

Consider the following linear time-invariant time-delay plant:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t-\tau) + B_d d(t), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state; $u(t) \in \mathbb{R}^m$ is the control input; $y(t) \in \mathbb{R}^q$ is the output; $d(t) \in \mathbb{R}^{n_d}$ is a disturbance; and A, B, B_d, and C are constant matrices with suitable dimensions. τ is a scalar representing the delay in the system. The initial condition is $x(t)|_{t=0} = x(0)$.

The following assumptions are made for (A, B, C). They are standard in control system design.

Assumption 1. (A, B, C) is controllable and observable.

Assumption 2. (A, B, C) has no zeros on the imaginary axis.

The configuration of the SP-EID control system is shown in Fig. 1. The system contains the plant, a controller $\overline{C}(s)$, the SP, a state observer, and an EID estimator. The EID estimator is extended from its original form in [18] to the one in the figure so as to handle the time delay in the plant. Note that

$$B^+ := (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}} \tag{2}$$

in the estimator.

According to the definition of the EID [18], we introduce an EID, $d_e(t)$, on the control input channel, and describe the plant as

$$\begin{cases} \dot{x}(t) = Ax(t) + B[u(t-\tau) + d_e(t)], \\ y(t) = Cx(t). \end{cases} \tag{3}$$

In the above equation, we abuse the notation a little bit, and use the same variable, $x(t)$, to indicate the state of both the original plant and that in (3). This should not cause confusion.

Let $G(s) = C(sI - A)^{-1}B$. Then, the transfer function of the plant from $u(t)$ to $y(t)$ is given by $P(s) = G(s)e^{-\tau s}$.

A full-order observer is used to estimate the EID. The state-space representation of the observer is

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu_f(t-\tau) + L[y(t) - \hat{y}(t)], \\ \hat{y}(t) = C\hat{x}(t), \end{cases} . \tag{4}$$

where $\hat{x}(t)$ is a reconstruction state of $x(t)$.

Following the same line as that in [18], it is easy to show that an estimate of the EID is given by

$$\hat{d}_e(t) = B^+ LC\tilde{x}(t) + u_f(t-\tau) - u(t-\tau), \tag{5}$$

where

$$\tilde{x}(t) = x(t) - \hat{x}(t). \tag{6}$$

A low-pass filter, $F(s)$,

$$\begin{cases} \dot{x}_F(t) = A_F x_F(t) + B_F \hat{d}_e(t), \\ \tilde{d}_e(t) = C_F x_F(t), \end{cases} \tag{7}$$

is used to select the angular-frequency band width for the EID estimation. It satisfies

$$|F(j\omega)| \approx 1, \quad \forall \omega \in [0, \omega_r], \tag{8}$$

where ω_r is the highest angular frequency of the disturbance. A suitable filter has its cutoff angular frequency being more than 5–10 times larger than ω_r. The filtered disturbance, $\tilde{d}_e(t)$, is given by

$$\tilde{D}_e(s) = F(s)\hat{D}_e(s), \tag{9}$$

where $\tilde{D}_e(s)$ and $\hat{D}_e(s)$ are the Laplace transformations of $\tilde{d}_e(t)$ and $\hat{d}_e(t)$ respectively.

A new control law of the control system is

$$u(t) = u_f(t) - \tilde{d}_e(t). \tag{10}$$

3. Stability analysis and system design of SP-EID control system

This section first analyzes the stability of the SP-EID control system, then presents a design method based on the analysis result.

3.1. Stability analysis

Let the reference input and the disturbance be zero, that is,

$$r(t) = 0, \quad d(t) = 0. \tag{11}$$

The plant is described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t-\tau), \\ y(t) = Cx(t). \end{cases} \tag{12}$$

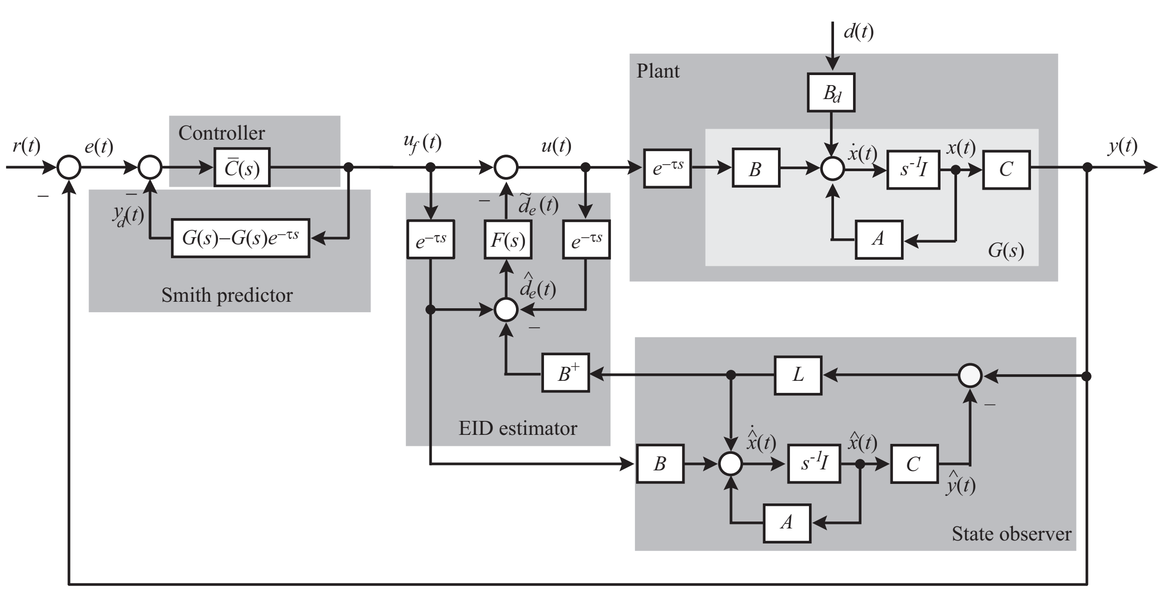

Fig. 1. Configuration of SP-EID control system.

A state-space form of $\overline{C}(s)$ is

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c[e(t) - y_d(t)], \\ u_f(t) = C_c x_c(t). \end{cases} \tag{13}$$

And a state-space form of the SP is

$$\begin{cases} \dot{x}_d(t) = A_d x_d(t) + B_d[u_f(t) - u_f(t-\tau)], \\ y_d(t) = C_d x_d(t). \end{cases} \tag{14}$$

First, recall the following definition and lemmas.

An infinite horizon quadratic cost function

$$J = \int_0^{\infty} [x^{\mathrm{T}}(t) Q x(t) + u^{\mathrm{T}}(t) R u(t)]\, \mathrm{d}t \tag{15}$$

was considered in this study. In (15), Q and R are given positive-definite symmetric matrices. A guaranteed-cost controller is defined as follows.

Definition 1. If there exist a control law, $u(t)$, and a positive scalar, γ, such that the closed-loop system is stable and the cost function, (15), of the system satisfies $J \le \gamma$; then γ is said to be a guaranteed cost and $u(t)$ is said to be a guaranteed cost controller for the system.

Lemma 1 (*Schur complement, Khargonek et al. [19]*). *For a given symmetric matrix*

$$\chi = \begin{bmatrix} \chi_{11} & \chi_{12} \\ \chi_{12}^{\mathrm{T}} & \chi_{22} \end{bmatrix}, \tag{16}$$

the following statements are equivalent:

(i) $\chi < 0$;
(ii) $\chi_{11} < 0$ *and* $\chi_{22} - \chi_{12}^{\mathrm{T}} \chi_{11}^{-1} \chi_{12} < 0$; *and*
(iii) $\chi_{22} < 0$ *and* $\chi_{11} - \chi_{12} \chi_{22}^{-1} \chi_{12}^{\mathrm{T}} < 0$.

Assume that the singular-value decomposition of a matrix Π is

$$\Pi = \overline{U} \begin{bmatrix} \overline{S} & 0 \end{bmatrix} \overline{T}^{\mathrm{T}}, \tag{17}$$

where $\overline{S}$ is a positive-definite matrix, and $\overline{U}$ and $\overline{T}$ are unitary matrices.

The following lemma presents a condition for the matrix equation $\Pi X = \overline{X} \Pi$.

Lemma 2 (*Ho and Lu [20]*). *For the given matrix* $\Pi \in \mathbb{R}^{p \times n}$ *with* rank$(\Pi) = p$, *there exists a matrix* $\overline{X} \in \mathbb{R}^{p \times p}$ *such that* $\Pi X = \overline{X} \Pi$ *holds for any* $X \in \mathbb{R}^{n \times n}$ *if and only if* X *can be decomposed as*

$$X = \overline{T} \begin{bmatrix} \overline{X}_{11} & 0 \\ 0 & \overline{X}_{22} \end{bmatrix} \overline{T}^{\mathrm{T}}, \tag{18}$$

where $\overline{T} \in \mathbb{R}^{n \times n}$ *is a unitary matrix,* $\overline{X}_{11} \in \mathbb{R}^{p \times p}$, *and* $\overline{X}_{22} \in \mathbb{R}^{(n-p) \times (n-p)}$.

Considering that the states in the SP-EID control system in Fig. 1 are $\hat{x}(t)$, $\tilde{x}(t)$, $x_F(t)$, $x_d(t)$, and $x_c(t)$, we define

$$\varphi(t) = \begin{bmatrix} \hat{x}^{\mathrm{T}}(t) & \tilde{x}^{\mathrm{T}}(t) & x_F^{\mathrm{T}}(t) & x_d^{\mathrm{T}}(t) & x_c^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}} \tag{19}$$

and use it to describe the system. Since

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu_f(t-\tau) + LC\tilde{x}(t),$$
$$\dot{\tilde{x}}(t) = (A - LC)\tilde{x}(t) - BC_F x_F(t-\tau),$$
$$\dot{x}_F(t) = A_F x_F(t) + B_F B^+ LC\tilde{x}(t) + B_F C_F x_F(t-\tau),$$
$$\dot{x}_d(t) = A_d x_d(t) + B_d C_c x_c(t) - B_d C_c x_c(t-\tau),$$
$$\dot{x}_c(t) = A_c x_c(t) - B_c C\tilde{x}(t) - B_c C\hat{x}(t) - B_c C_d x_d(t),$$

the state-space representation of the closed-loop system is

$$\dot{\varphi}(t) = A_1 \varphi(t) + A_2 \varphi(t-\tau) + B_1 u_f(t-\tau), \tag{20}$$

where

$$A_1 = \begin{bmatrix} A & LC & 0 & 0 & 0 \\ 0 & A-LC & 0 & 0 & 0 \\ 0 & B_F B^+ LC & A_F & 0 & 0 \\ 0 & 0 & 0 & A_d & B_d C_c \\ -B_c C & -B_c C & 0 & -B_c C_d & A_c \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -BC_F & 0 & 0 \\ 0 & 0 & B_F C_F & 0 & 0 \\ 0 & 0 & 0 & 0 & -B_d C_c \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} B \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The initial condition is $\varphi(t)|_{t=0} = \varphi(0)$. The state-feedback control law is

$$u_f(t) = K\varphi(t), \tag{21}$$

where

$$K = [0 \;\; 0 \;\; 0 \;\; 0 \;\; C_c].$$

Assume that the singular-value decomposition of the output matrix is

$$C = U \begin{bmatrix} \hat{S} & 0 \end{bmatrix} \hat{T}^{\mathrm{T}}, \tag{22}$$

where $\hat{S}$ is a semi-positive definite matrix, and U and $\hat{T}$ are unitary matrices. Letting $\hat{T}$ be

$$\hat{T} = \begin{bmatrix} \hat{T}_1 & \hat{T}_2 \end{bmatrix}, \tag{23}$$

we have the following theorem.

Theorem 1. *For given* $Q > 0$, $R > 0$, *and* $\alpha > 0$, *if there exist symmetric positive-definite matrices* Y_i $(i = 1, \ldots, 5)$, Δ_i $(i = 1, \ldots, 5)$, X_1, X_{11}, X_{22}, X_3, X_4, X_5, *and appropriate matrices* W, Λ_i $(i = 1, \ldots, 5)$, M_i $(i = 1, \ldots, 5)$, *such that the following inequality holds:*

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \tau\Lambda & \tau\Psi_{14} & \Psi_{15} & \Psi_{16} \\ \star & -\Psi_{22} & \tau M & \tau\Psi_{24} & 0 & 0 \\ \star & \star & -\tau\Psi_{33} & 0 & 0 & 0 \\ \star & \star & \star & -\tau\Delta & 0 & 0 \\ \star & \star & \star & \star & -Q^{-1} & 0 \\ \star & \star & \star & \star & \star & -R^{-1} \end{bmatrix} < 0, \tag{24}$$

where

$$\Psi_{11} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & 0 & \Phi_{15} \\ \star & \Phi_{22} & \Phi_{23} & 0 & \Phi_{25} \\ \star & \star & \Phi_{33} & 0 & 0 \\ \star & \star & \star & \Phi_{44} & \Phi_{45} \\ \star & \star & \star & \star & \Phi_{55} \end{bmatrix},$$

$$\Phi_{11} = \alpha A X_1 + \alpha X_1 A^{\mathrm{T}} + Y_1 + \Lambda_1 + \Lambda_1^{\mathrm{T}},$$
$$\Phi_{12} = WC,$$
$$\Phi_{15} = -\alpha X_1 C^{\mathrm{T}} B_c^{\mathrm{T}},$$
$$\Phi_{22} = AX_2 + X_2 A^{\mathrm{T}} - WC - C^{\mathrm{T}} W^{\mathrm{T}} + \Lambda_2 + \Lambda_2^{\mathrm{T}} + Y_2,$$
$$\Phi_{23} = C^{\mathrm{T}} W^{\mathrm{T}} B^{+\mathrm{T}} B_F^{\mathrm{T}},$$
$$\Phi_{25} = -X_2 C^{\mathrm{T}} B_c^{\mathrm{T}},$$
$$\Phi_{33} = X_3 A_F^{\mathrm{T}} + A_F X_3 + \Lambda_3 + \Lambda_3^{\mathrm{T}} + Y_3,$$
$$\Phi_{44} = X_4 A_d^{\mathrm{T}} + A_d X_4 + \Lambda_4 + \Lambda_4^{\mathrm{T}} + Y_4,$$
$$\Phi_{45} = B_d C_c X_5 - X_4 C_d^{\mathrm{T}} B_c^{\mathrm{T}},$$

$$\Phi_{55}=X_5A_c^{\mathrm{T}}+A_cX_5+\Lambda_5+\Lambda_5^{\mathrm{T}}+Y_5,$$

$$\Psi_{12}=\begin{bmatrix}\Omega_{11} & 0 & 0 & 0 & BC_cX_5\\ 0 & -\Lambda_2+M_2^{\mathrm{T}} & \Omega_{23} & 0 & 0\\ 0 & 0 & \Omega_{33} & 0 & 0\\ 0 & 0 & 0 & -\Lambda_4+M_4^{\mathrm{T}} & 0\\ 0 & 0 & 0 & 0 & -\Lambda_5+M_5^{\mathrm{T}}\end{bmatrix},$$

$$\Omega_{11}=-\Lambda_1+M_1^{\mathrm{T}},$$

$$\Omega_{23}=-BC_FX_3,$$

$$\Omega_{33}=-\Lambda_3+M_3^{\mathrm{T}}+B_FC_FX_3,$$

$$\Psi_{15}=X=\mathrm{diag}\{\alpha X_1,X_2,X_3,X_4,X_5\},$$

$$\Psi_{16}=\begin{bmatrix}0\\0\\0\\0\\X_5C_c^{\mathrm{T}}\end{bmatrix},$$

$$\Psi_{14}=\begin{bmatrix}\alpha X_1A^{\mathrm{T}} & 0 & 0 & 0 & -\alpha X_1C^{\mathrm{T}}B_c^{\mathrm{T}}\\ C^{\mathrm{T}}W^{\mathrm{T}} & -C^{\mathrm{T}}W^{\mathrm{T}}+X_2A^{\mathrm{T}} & C^{\mathrm{T}}W^{\mathrm{T}}B^{+\mathrm{T}}B_F^{\mathrm{T}} & 0 & -X_2C^{\mathrm{T}}B_c^{\mathrm{T}}\\ 0 & 0 & X_3A_F^{\mathrm{T}} & 0 & 0\\ 0 & 0 & 0 & X_4A_d^{\mathrm{T}} & -X_4C_d^{\mathrm{T}}B_c^{\mathrm{T}}\\ 0 & 0 & 0 & X_5C_c^{\mathrm{T}}B_d^{\mathrm{T}} & X_5A_c^{\mathrm{T}}\end{bmatrix},$$

$$\Psi_{22}=Y+M+M^{\mathrm{T}},\quad Y=\mathrm{diag}\{Y_1,Y_2,Y_3,Y_4,Y_5\},$$

$$\Psi_{24}=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & & \\ 0 & -X_3C_F^{\mathrm{T}}B^{\mathrm{T}} & X_3C_F^{\mathrm{T}}B_F^{\mathrm{T}} & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ X_5C_c^{\mathrm{T}}B^{\mathrm{T}} & 0 & 0 & -X_5C_c^{\mathrm{T}}B_d^{\mathrm{T}} & 0\end{bmatrix},$$

$$\Psi_{33}=X\Delta^{-1}X,$$

$$\Delta=\mathrm{diag}\{\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5\},$$

$$\Lambda=\mathrm{diag}\{\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_4,\Lambda_5\},$$

$$M=\mathrm{diag}\{M_1,M_2,M_3,M_4,M_5\},$$

then the system (20) is asymptotically stable for any constant time-delay $\tau>0$. The cost function satisfies

$$J\le\varphi^{\mathrm{T}}(0)P\varphi(0)+\int_{-\tau}^{0}\varphi^{\mathrm{T}}(s)S\varphi(s)\,\mathrm{d}s+\int_{-\tau}^{0}\int_{\theta}^{0}\dot{\varphi}^{\mathrm{T}}(s)Z\dot{\varphi}(s)\,\mathrm{d}s\,\mathrm{d}\theta, \tag{25}$$

and the singular-value decomposition of X_2 is

$$X_2=\begin{bmatrix}\hat{T}_1 & \hat{T}_2\end{bmatrix}\begin{bmatrix}X_{11} & 0\\ 0 & X_{22}\end{bmatrix}\begin{bmatrix}\hat{T}_1^{\mathrm{T}}\\ \hat{T}_2^{\mathrm{T}}\end{bmatrix}. \tag{26}$$

Moreover, the gain of the observer is

$$L=WU\hat{S}X_{11}^{-1}\hat{S}^{-1}U^{\mathrm{T}}. \tag{27}$$

Proof. The closed-loop system with the control law (20) is

$$\dot{\varphi}(t)=A_1\varphi(t)+(A_2+B_1K)\varphi(t-\tau). \tag{28}$$

Choose a Lyapunov functional candidate to be

$$V(\varphi(t))=\varphi^{\mathrm{T}}(t)P\varphi(t)+\int_{t-\tau}^{t}\varphi^{\mathrm{T}}(s)S\varphi(s)\,\mathrm{d}s+\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{\varphi}^{\mathrm{T}}(s)Z\dot{\varphi}(s)\,\mathrm{d}s\,\mathrm{d}\theta, \tag{29}$$

where $P_i\ (i=1,\ldots,5)$, $S_i\ (i=1,\ldots,5)$, $Z_i\ (i=1,\ldots,5)$, $P=\mathrm{diag}\{\frac{1}{\alpha}P_1,P_2,P_3,P_4,P_5\}$, $S=\mathrm{diag}\{S_1,S_2,S_3,S_4,S_5\}$, and $Z=\mathrm{diag}\{Z_1,Z_2,Z_3,Z_4,Z_5\}$ are symmetric positive-definite matrices to be determined, and $\alpha>0$.

Calculating the derivative of $V(\varphi(t))$ along the solution of the system in (28) yields

$$\dot{V}(\varphi(t))=2\varphi^{\mathrm{T}}(t)P\dot{\varphi}(t)+\varphi^{\mathrm{T}}(t)S\varphi(t)-\varphi^{\mathrm{T}}(t-\tau)S\varphi(t-\tau)+\tau\dot{\varphi}^{\mathrm{T}}(t)Z\dot{\varphi}(t)-\int_{t-\tau}^{t}\dot{\varphi}^{\mathrm{T}}(s)Z\dot{\varphi}(s)\,\mathrm{d}s. \tag{30}$$

Applying the free-weighting matrix approach [21], the following equation holds for any matrices N_1 and N_2:

$$2\left[\varphi^{\mathrm{T}}(t)N_1+\varphi^{\mathrm{T}}(t-\tau)N_2\right]\left[\varphi(t)-\int_{t-\tau}^{t}\dot{\varphi}(s)\,\mathrm{d}s-\varphi(t-\tau)\right]=0. \tag{31}$$

And for

$$\eta(t)=\begin{bmatrix}\varphi(t)\\ \varphi(t-\tau)\end{bmatrix},\quad \overline{X}=\begin{bmatrix}\overline{X}_{11} & \overline{X}_{12}\\ \star & \overline{X}_{22}\end{bmatrix}\ge 0, \tag{32}$$

the following also holds

$$\tau\eta^{\mathrm{T}}(t)\overline{X}\eta(t)-\int_{t-\tau}^{t}\eta^{\mathrm{T}}(t)\overline{X}\eta(t)\,\mathrm{d}s\ge 0. \tag{33}$$

Incorporating (31) and (33) into (30) yields

$$\dot{V}(\varphi(t))\le 2\varphi^{\mathrm{T}}(t)P\dot{\varphi}(t)+\varphi^{\mathrm{T}}(t)S\varphi(t)-\varphi^{\mathrm{T}}(t-\tau)S\varphi(t-\tau)+\tau\dot{\varphi}^{\mathrm{T}}(t)Z\dot{\varphi}(t)-\int_{t-\tau}^{t}\dot{\varphi}^{\mathrm{T}}(s)Z\dot{\varphi}(s)\,\mathrm{d}s+2\left[\varphi^{\mathrm{T}}(t)N_1+\varphi^{\mathrm{T}}(t-\tau)N_2\right]\times\left[\varphi(t)-\int_{t-\tau}^{t}\dot{\varphi}(s)\,\mathrm{d}s-\varphi(t-\tau)\right]+\tau\eta^{\mathrm{T}}(t)\overline{X}\eta(t)-\int_{t-\tau}^{t}\eta^{\mathrm{T}}(t)\overline{X}\eta(t)\,\mathrm{d}s. \tag{34}$$

So,

$$\dot{V}(\varphi(t))+\varphi^{\mathrm{T}}(t)[Q+K^{\mathrm{T}}\mathrm{R}K]\varphi(t)\le\eta^{\mathrm{T}}(t)\Xi\eta(t)-\int_{t-\tau}^{t}\eta_1^{\mathrm{T}}(t,s)\Gamma\eta_1(t,s)\,\mathrm{d}s, \tag{35}$$

where

$$\Xi=\begin{bmatrix}\Theta_{11} & \Theta_{12}\\ \star & \Theta_{22}\end{bmatrix},$$

$$\Gamma=\begin{bmatrix}\overline{X}_{11} & \overline{X}_{12} & N_1\\ \star & \overline{X}_{22} & N_2\\ \star & \star & Z\end{bmatrix},$$

$$\Theta_{11}=PA_1+A_1^{\mathrm{T}}P+S+\tau A_1^{\mathrm{T}}ZA_1+N_1+N_1^{\mathrm{T}}+\tau\overline{X}_{11}+Q+K^{\mathrm{T}}RK,$$

$$\Theta_{12}=P(A_2+B_1K)-N_1+N_2^{\mathrm{T}}+\tau A_1^{\mathrm{T}}Z(A_2+B_1K)+\tau\overline{X}_{12},$$

$$\Theta_{22}=-S-N_2-N_2^{\mathrm{T}}+\tau(A_2+B_1K)^{\mathrm{T}}Z(A_2+B_1K)+\tau\overline{X}_{22},$$

$$\eta_1(t)=\begin{bmatrix}\varphi(t)\\ \varphi(t-\tau)\\ \dot{\varphi}(s)\end{bmatrix}.$$

It is clear from (35) that the asymptotic stability of the closed-loop system is guaranteed by $\Xi<0$ and $\Gamma\ge 0$ because they provide

$$\dot{V}(\varphi(t))<-\varphi^{\mathrm{T}}(t)[Q+K^{\mathrm{T}}RK]\varphi(t). \tag{36}$$

This implies that $V(\varphi(t))$ is a Lyapunov–Krasovskii functional. Furthermore, integrating both sides of (36) from 0 to ∞ with the initial condition yields

$$J\le\varphi^{\mathrm{T}}(0)P\varphi(0)+\int_{-\tau}^{0}\varphi^{\mathrm{T}}(s)S\varphi(s)\,\mathrm{d}s+\int_{-\tau}^{0}\int_{\theta}^{0}\dot{\varphi}^{\mathrm{T}}(s)Z\dot{\varphi}(s)\,\mathrm{d}s\,\mathrm{d}\theta. \tag{37}$$

Now, we show that (24) guarantees $\Xi<0$ and $\Gamma\ge 0$. If we select $Z>0$ and $\overline{X}$ such that

$$\overline{X}=\begin{bmatrix}N_1\\N_2\end{bmatrix}Z^{-1}\begin{bmatrix}N_1\\N_2\end{bmatrix}^{\mathrm{T}}, \tag{38}$$

then $\overline{X}\ge 0$ and $\Gamma\ge 0$. By Lemma 1, $\Xi<0$ is equivalent to

$$\begin{bmatrix}\overline{\Theta}_{11} & \overline{\Theta}_{12} & \tau N_1 & \tau A_1^{\mathrm{T}}Z & I & K^{\mathrm{T}}\\ \star & \overline{\Theta}_{22} & \tau N_2 & \tau(A_2+B_1K)^{\mathrm{T}}Z & 0 & 0\\ \star & \star & -\tau Z & 0 & 0 & 0\\ \star & \star & \star & -\tau Z & 0 & 0\\ \star & \star & \star & \star & -Q^{-1} & 0\\ \star & \star & \star & \star & \star & -R^{-1}\end{bmatrix}<0, \tag{39}$$

where

$\overline{\Theta}_{11}=PA_1+A_1^{\mathrm{T}}P+S+N_1+N_1^{\mathrm{T}}$,
$\overline{\Theta}_{12}=P(A_2+B_1K)-N_1+N_2^{\mathrm{T}}$,
$\overline{\Theta}_{22}=-S-N_2-N_2^{\mathrm{T}}$.

Let $P_i^{-1}=X_i\ (i=1,\ldots,5)$, $Z_i^{-1}=\Delta_i\ (i=1,\ldots,5)$, $X=\mathrm{diag}\{\alpha X_1,X_2,X_3,X_4,X_5\}$, and $\Delta=\mathrm{diag}\{\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5\}$.

Pre- and post-multiplying (39) by $\mathrm{diag}\{P^{-1},P^{-1},P^{-1},Z^{-1},I,I\}=\mathrm{diag}\{X,X,X,\Delta,I,I\}$, and letting $\Lambda=XN_1X=\mathrm{diag}\ \{\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_4,\Lambda_5\}$, $M=XN_2X=\mathrm{diag}\{M_1,M_2,M_3,M_4,M_5\}$, $Y=XSX=\mathrm{diag}\{Y_1,Y_2,Y_3,Y_4,Y_5\}$, and $\overline{W}=KX$ yield

$$\begin{bmatrix}\Upsilon_{11} & \Upsilon_{12} & \tau\Lambda & \tau XA_1^{\mathrm{T}} & X & \overline{W}^{\mathrm{T}}\\ \star & \Upsilon_{22} & \tau M & \Upsilon_{24} & 0 & 0\\ \star & \star & -\tau X\Delta^{-1}X & 0 & 0 & 0\\ \star & \star & \star & -\tau\Delta & 0 & 0\\ \star & \star & \star & \star & -Q^{-1} & 0\\ \star & \star & \star & \star & \star & -R^{-1}\end{bmatrix}<0, \tag{40}$$

where

$\Upsilon_{11}=A_1X+XA_1^{\mathrm{T}}+Y+\Lambda+\Lambda^{\mathrm{T}}$,
$\Upsilon_{12}=A_2X+B_1\overline{W}-\Lambda+M^{\mathrm{T}}$,
$\Upsilon_{22}=-Y-M-M^{\mathrm{T}}$,
$\Upsilon_{24}=\tau(\overline{W}^{\mathrm{T}}B_1^{\mathrm{T}}+XA_2^{\mathrm{T}})$.

Applying Lemma 2 to (22) gives

$$\overline{X}_2=U\hat{S}X_{11}\hat{S}^{-1}U^{\mathrm{T}}, \tag{41}$$

with

$$CX_2=\overline{X}_2C. \tag{42}$$

Letting

$$L\overline{X}_2=W, \tag{43}$$

and substituting (20), (42), and (43) into (40) yield (24).

This completes the proof. □

Note that the inequality (24) is not a linear matrix inequality (LMI) due to the terms such as $\Psi_{33}=X\Delta^{-1}X$, and the problem is a non-convex problem.

To solve this problem, we introduce a new variable, H, such that $X\Delta^{-1}X>H$. So, we have $H^{-1}>X^{-1}\Delta X^{-1}$. According to the Schur complement, it is equal to

$$\begin{bmatrix}H^{-1} & X^{-1}\\ X^{-1} & \Delta^{-1}\end{bmatrix}>0. \tag{44}$$

The original design problem is converted to the following optimization problem involving LMI conditions:

$\min \mathrm{Tr}(HT+XJ+\Delta D)$

subject to

$$\begin{bmatrix}\Psi_{11} & \Psi_{12} & \tau\Lambda & \tau\Psi_{14} & \Psi_{15} & \Psi_{16}\\ \star & -\Psi_{22} & \tau M & \tau\Psi_{24} & 0 & 0\\ \star & \star & -\tau H & 0 & 0 & 0\\ \star & \star & \star & -\tau\Delta & 0 & 0\\ \star & \star & \star & \star & -Q^{-1} & 0\\ \star & \star & \star & \star & \star & -R^{-1}\end{bmatrix}<0, \tag{45}$$

$$\begin{bmatrix}T & J\\ J & D\end{bmatrix}>0,\quad \begin{bmatrix}H & I\\ I & T\end{bmatrix}>0, \tag{46}$$

$$\begin{bmatrix}X & I\\ I & J\end{bmatrix}>0,\quad \begin{bmatrix}\Delta & I\\ I & D\end{bmatrix}>0, \tag{47}$$

where $\mathrm{Tr}(\cdot)$ means the trace of a square matrix, which is the sum of the elements on the main diagonal of the matrix.

The cone complementary linearization method is used to solve the above optimization problem. If the solution of the above optimization problem is $3n$, that is, $\mathrm{Tr}(HT+XJ+\Delta D)=3n$, then we know from Theorem 1 that the guaranteed-cost control problem is solvable [22].

3.2. Design algorithm

An algorithm of designing the control system that combines the SP and the EID approach is developed based on Theorem 1 as follows.

Step 1: Choose A_F, B_F, and C_F for the low-pass filter, (7), that satisfies (8).
Step 2: Choose a satisfactory controller $\overline{C}(s)$ based on the design of the SP.
Step 3: Find a feasible solution to (24), and calculate the observer gain L from (27).

4. Simulation verification

A numerical example is used to illustrate the design procedure and the validity of the method. The parameters of the plant (1) are

$$\begin{cases}A=\begin{bmatrix}-2 & 0\\ 4 & -5\end{bmatrix},\quad B=\begin{bmatrix}4\\0\end{bmatrix},\quad B_d=\begin{bmatrix}1\\1.2\end{bmatrix},\\ C=[5\ \ 1.2],\quad \tau=0.16\ \mathrm{s}.\end{cases} \tag{48}$$

The transfer function from the input to the output is

$$P(s)=G(s)e^{-0.16s},\quad G(s)=\frac{20s+119.2}{s^2+7s+10}. \tag{49}$$

Let the reference input be

$$r(t)=1200\times 1(t), \tag{50}$$

where $1(t)$ is the unit step function and the disturbance be

$$d(t)=\begin{cases}15\times(\sin 4\pi t+\cos 2\pi t+\sin \pi t+\sin 0.5\pi t), & 10\le t\le 20,\\ 0, & \text{otherwise}.\end{cases} \tag{51}$$

A first-order low-pass filter

$$F(s)=\frac{b}{s+a} \tag{52}$$

was selected for the EID estimator. Since $\omega_r=4\pi$ rad and $5\omega_r=62.8$ rad,

$a=101,\quad b=100$

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were chosen to meet condition (8). The state-space form of the filter is

$$A_F = -101,\quad B_F = 100,\quad C_F = 1. \tag{53}$$

Trial and error found the following parameters of $\overline{C}(s)$ that produced satisfactory results:

$$A_c = -0.05,\quad B_c = 0.12,\quad C_c = 1. \tag{54}$$

Note that, to guarantee the feasibility of the LMI (24), a small number but not 0 was chosen for A_c. Simulations showed that this choice provided satisfactory control precision.

$Q = \mathrm{diag}\{100I, I, I, I, I\}$ and $R = I$ were chosen for (15), and $\alpha = 1$ was selected for (29).

Using the cone complementary linearization method [22] yielded a feasible solution to the optimization problem, the gain of the observer was given by

$$L = [2.0850 \quad -4.3058]^{\mathrm{T}}. \tag{55}$$

The simulation results for the SP-EID control are shown in Fig. 2. The peak-to-peak value (PPV) of the steady-state tracking error during [10 s, 20 s] was 105. The spectrums of the disturbance, d (Fig. 2 (c)), and the EID estimate, $\tilde{d}_e$ (Fig. 2 (d)), are shown in Fig. 3. It is clear from the figure that, while the magnitude of these two signals are different due to the difference in their input channels, the frequency components of d are generated in $\tilde{d}_e$. So, the control input that incorporates $\tilde{d}_e$ suppresses the effect of the disturbance.

To demonstrate the validity of the SP-EID method, we compared it with the widely used SP and the SMC methods. First, the output of the SP control system is shown in Fig. 4. The PPV of the steady-state tracking error for the SP control during [10 s, 20 s] was 163, which is about 1.5 times larger than that of the SP-EID method.

Instead of using an EID estimator in Fig. 1, the SMC system inserted a sliding-mode controller before the plant in the forward path, and the controller was

$$u_S(t) = -K_S\,\mathrm{sgn}[S(t)], \tag{56}$$

where K_S is a positive real number, and $S(t)$ is a switching function and it was chosen to be

$$S(t) = -u_f(t) \tag{57}$$

in this study. $K_S = 100$ was used for comparison.

The simulation results for the SMC method are shown in Fig. 5. The PPV of the steady-state tracking error for the SMC method during [10 s, 20 s] was 143, which is less than that of the SP method, but is about 1.3 times larger than that of the SP-EID method. This demonstrates that the SP-EID method is effective in rejecting disturbances and is superior to the SP and the SMC methods.

One of the big differences between simulations and experiments is measurement noise. In order to demonstrate that the SP-EID method is useful for a real-time system, a relatively large noise with the amplitude being about 300, which was 25% of the steadstate output, was added in the output. The simulation results (Fig. 6) show that the system was stable and the steady-state tracking error during [10 s, 20 s] was 129. This demonstrates the validity of the SP-EID method for control engineering practice.

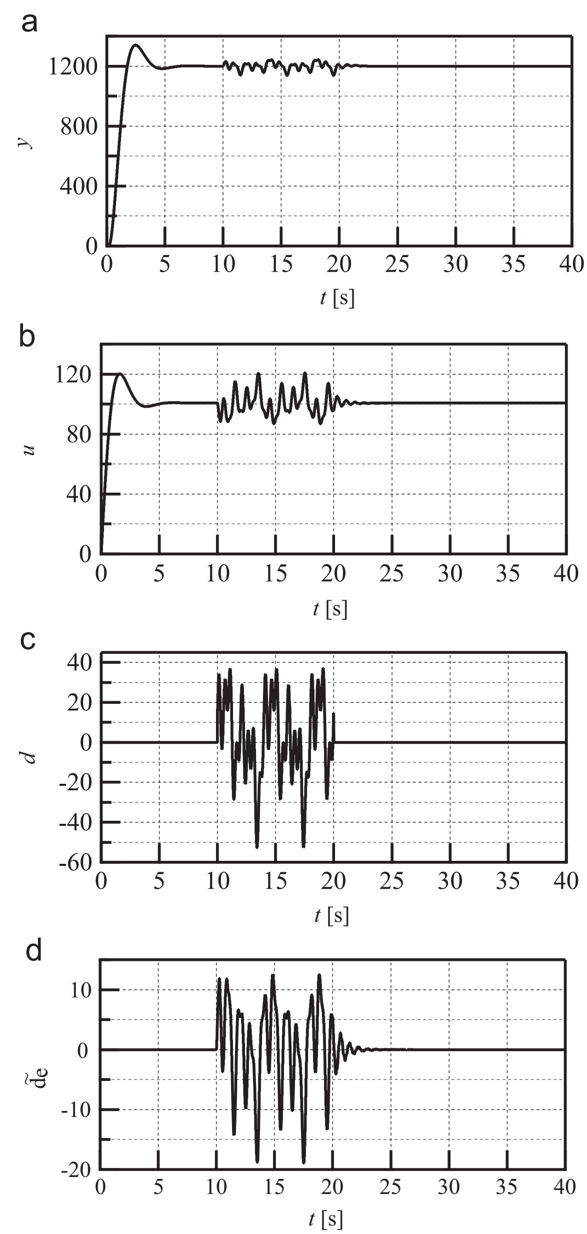

Fig. 2. Simulations results for (50) and (51) for SP-EID. (a) System output, y; (b) control input, u; (c) disturbance, d; and (d) EID estimate, $\tilde{d}_e$.

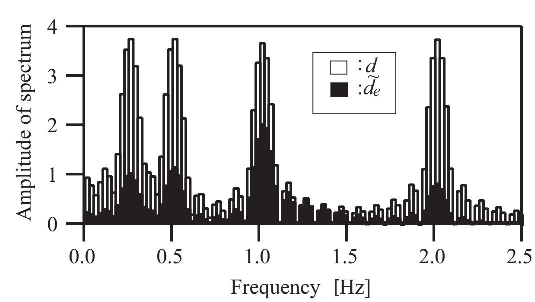

Fig. 3. Spectrums of d and $\tilde{d}_e$.

5. Conclusion

This paper presented a new active disturbance rejection method that combines the SP method with the EID approach to improve the disturbance rejection performance for a time-delay system. A delay-dependent stability condition was devised in terms of a nonlinear matrix inequality by using the free-weighting matrix approach. The problem of solving the nonlinear matrix inequality was converted to an optimization problem by introducing a new variable. The gain of the state observer was designed by applying the cone complementary linearization method to solve the optimization problem. A numerical example showed that the

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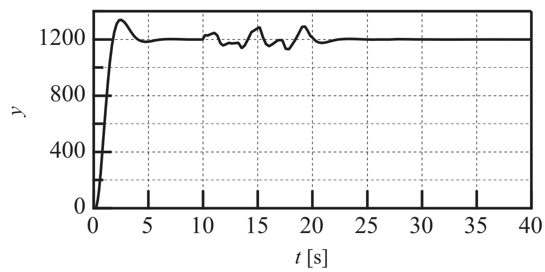

Fig. 4. System output for (50) and (51) for SP.

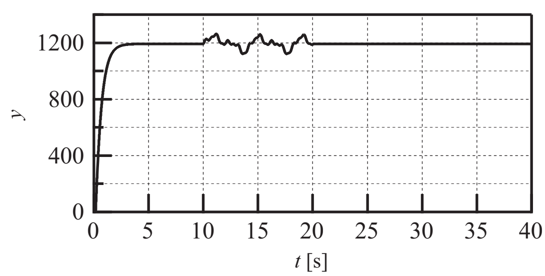

Fig. 5. System output for (50) and (51) for SMC.

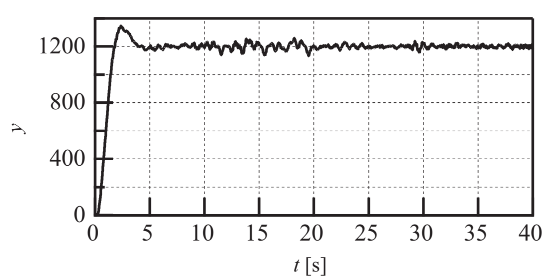

Fig. 6. System output for (50) and (51) with measurement noise for SP-EID.

SP-EID method provided satisfactory disturbance rejection performance. By comparison with the SP and the SMC methods, the SP-EID method showed the superiority over them. The advantages of the SP-EID method are as follows:

(a) The configuration of the control system is simple.
(b) It rejects a disturbance without requiring any prior information about the disturbance.
(c) It effectively compensates a disturbance in a time-delay system.

Acknowledgment

This work was supported by National Natural Science Foundation of China under Grants 61210011 and 61473313, and the Hubei Provincial Natural Science Foundation of China under Grant 2015CFA010, and by the Grant-in-Aid for Scientific Research (C), Japan Society for the Promotion of Science (JSPS) under Grant 26350673.

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