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Robust H∞ Output-feedback Vehicle Yaw Control Using an Active Front Wheel Steering

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Abstract: In this paper, we investigate the vehicle stability control problem under the side wind disturbance. To improve the vehicle lateral stability, an active front wheel steering (AFS) is designed based on the robust H∞ output-feedback control. The robust controller takes the wind disturbance and uncertain parameters of the cornering stiffness into consideration while system modeling and controller designing. The simulation with obstacles suddenly encountering shows the vehicle can avoid the obstacles safely and effectively with side wind disturbances. By this way, the controller can improve the vehicle lateral stability and drivability significantly, and reduce the workload of the driver. In addition, good robustness is achieved while uncertain parameters exist.

Key words: Active Front Wheel Steering, Robust H∞ output-feedback control, Lateral Stabilization Control

1 Introduction

Currently, the vehicle stability control system achieves effective yaw moment by controlling the driving or braking torque. It can help drivers avoid the accidents in some extreme working conditions while it may have a great impact on the longitudinal performance of the vehicle[1]. Therefore, the active front wheel steering (AFS) is proposed to achieve a reasonable distribution of yaw moment and improve the stability of the vehicle which almost has no influence on the longitudinal performance.

The vehicle control system is a nonlinear time-varying system with parameter uncertainty and external disturbances. The general engineering control methods, like PID controller cannot achieve the vehicle stability control robustly. Therefore, there are a lot of studies investigating the robust control strategy of AFS. Fukao. T. et al. [2] considered the relationship between the slip ratio and lateral force on roads with different adhesion coefficients, and designed an active steering system based on model reference adaptive control. Do et al. [3] adopted a robust sliding mode learning control algorithm, which improves the vehicle Steer-by-wire steering performance even does not know the values of the uncertain parameter. Nam et al.[4] used a robust control method based on the disturbance model to achieve active steering control to improve vehicle stability.

The robust control method can suppress the parametric uncertainties of model caused by external disturbance. The robust H∞ output- feedback control is one of the hotspots in the research fields. In the early stage, the controllers were solved mainly by the Riccati method whose parameters need to be determined in advance, otherwise it may affect the control effects [5]. In this paper, we proposed a controller in transforming the nonlinear matrix inequality into linear matrix inequality (LMI), whose parameters can be solved by LMI toolbox. Based on H∞ loop shaping theory, Mammar et al. [6] designed a robust controller for the active front steering, which takes the changes of vehicle speed and tire cornering stiffness into consideration. It can also separate the certainties and uncertainties of the system by linear fractional transformation.

The state feedback controller requires sensors to measure the state of the vehicle, which requires a large amount of costs [7]. In addition, the accurate measurement of slip angle is very important for vehicle stability. Therefore, this paper presents output feedback control with no need of measuring the slip angle.

The rest of this paper is organized as follows. The vehicle dynamics and controller design are described in Section 2. The simulation based on the Carsim-Simulink platform is conducted in Section 3. The conclusion is presented in Section 4.

2 Vehicle Dynamics Model and Controller Design

In this section, we use a two-degree-of-freedom(2DOF) vehicle model combined with the side wind disturbance in

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the actual drive for the controller design.

2.1 Vehicle Dynamics Model

Assuming that the vehicle's front steering angle is not large, the vehicle dynamics model can be expressed as:

$$\begin{cases} m(\dot{v}+u\gamma)=F_{yf}+F_{yr}+F_i \\ I_z\dot{\gamma}=l_fF_{yf}-l_rF_{yr}-e_iF_i \end{cases} \tag{1}$$

Where m is the vehicle mass, u and v are the longitudinal and lateral velocity, γ is the yaw rate, F_{yf} and F_{yr} are the longitudinal tire force of front and rear wheels, F_i is the force of lateral wind disturbance, e_i is the distance of the center of gravity from the point of lateral wind, the longitudinal tire force of front and rear wheels can be expressed as

$$\begin{cases} F_{yf}=-C_f\alpha_f, F_{yr}=-C_r\alpha_r \\ \alpha_f=\beta+\dfrac{l_f\gamma}{u}-\delta_f, \alpha_r=\beta-\dfrac{l_r\gamma}{u} \end{cases} \tag{2}$$

Where C_f and C_r are the cornering stiffness of front and rear tire, α_f and α_r are the tire slip angles, δ_f is the steering angle of the front wheel, β is the slip angle, where $\beta=v/u$[8], tire cornering stiffness will be affected by the friction coefficient, tire pressure and other factors, with uncertainty.

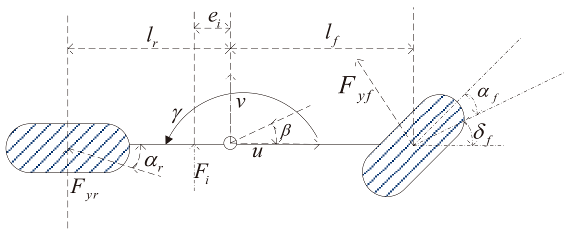

Fig 1:2DOF vehicle model

By defining state x as $[\beta\ \gamma]^T$, control input as δ_f, and disturbance input as F_i, the dynamics model (1) can be expressed as:

$$\dot{x}=Ax+Bu+B_2d \tag{3}$$

$$A=\begin{bmatrix} -\dfrac{C_r+C_f}{mu} & \dfrac{(l_rC_r-l_fC_f)}{mu^2}-1 \\ \dfrac{l_rC_r-l_fC_f}{I_z} & -\dfrac{l_r^2C_r-l_f^2C_f}{I_zu} \end{bmatrix}\ B=\begin{bmatrix} \dfrac{C_f}{mu} \\ \dfrac{C_fl_f}{I_z} \end{bmatrix}\ B_1=\begin{bmatrix} \dfrac{1}{mu} \\ \dfrac{e_i}{I_z} \end{bmatrix} \tag{4}$$

2.2 Controller Design

The controller proposed in this paper is to track the ideal yaw rate to realize lateral stabilization , so we need to get the vehicle's output yaw rate and the state-space model of the vehicle can be written as

$$\begin{cases} \dot{x}=Ax+Bu+B_1d \\ y=C_1x \\ z=C_2x \end{cases} \tag{5}$$

The controller tracking the ideal yaw rate can be described as H∞ performance of the system:

$$\|Z\|_\infty < \gamma\|W\|_2 \tag{6}$$

Where $C_1=C_2=[0\ \ 1]$, γ is the performance of the controller.

Due to the uncertainty of the cornering stiffness, we need to ensure the performance of the system, by defining

$$A=A_o+\Delta A \quad B=B_0+\Delta B \tag{7}$$

And
$$A_o=\begin{bmatrix} -\dfrac{C_{fm}+C_{rm}}{mu} & \dfrac{(l_rC_{rm}-l_fC_{fm})}{mu^2}-1 \\ \dfrac{(l_rC_{rm}-l_fC_{fm})}{I_z} & -\dfrac{l_r^2C_{rm}-l_fC_{fm}}{I_zu} \end{bmatrix}$$

$$\Delta A=\begin{bmatrix} -\dfrac{\lambda_1\tilde{C}_f+\lambda_2\tilde{C}_r}{mu} & \dfrac{(l_r\lambda_2\tilde{C}_r-l_f\tilde{C}_f)}{mu^2}-1 \\ \dfrac{(l_r\lambda_2\tilde{C}_r-l_f\tilde{C}_f)}{I_z} & -\dfrac{l_r^2\lambda_2\tilde{C}_r-l_f^2\tilde{C}_f}{I_zu} \end{bmatrix}$$

$$B_0=\begin{bmatrix} \dfrac{C_{fm}}{mu} \\ \dfrac{l_fC_{fm}}{I_z} \end{bmatrix},\quad \Delta B=\begin{bmatrix} \dfrac{\lambda_1\tilde{C}_f}{mu} \\ \dfrac{l_f\lambda_1\tilde{C}_f}{I_z} \end{bmatrix} \tag{8}$$

Where

$$C_{fm}=\frac{C_{f\max}+C_{f\min}}{2},\ C_{rm}=\frac{C_{r\max}+C_{r\min}}{2}$$

$$\tilde{C}_f=C_{f\max}-C_{fm},\ \tilde{C}_r=C_{r\max}-C_{rm},$$

$$C_f=C_{fm}+\lambda_1\tilde{C}_f,\ C_r=C_{rm}+\lambda_1\tilde{C}_r$$

The controller is designed as the following equation.

$$\begin{cases} \dot{x}=A_k\hat{x}+B_ky \\ u=C_k\hat{x}+D_ky \end{cases} \tag{9}$$

Where $\hat{x}$ is the state of the controller, A_k, B_k, C_k and D_k are the undetermined controller parameter matrices, ΔA and ΔB are matrices of uncertain parameters. Suppose $\Delta A=HFE_1$, $\Delta B=HFE_2$, H, E_1 and E_2 are known constant matrices that reflect the structural information of uncertain parameters, and F satisfies the condition of

$FF^T \leq I$. By Combining (4) and (5), we can get the following equation.

$$\begin{cases} \dot{\varepsilon} = A_{c1}\varepsilon + B_{c1}d \\ z = C_{c1}\varepsilon \end{cases} \tag{10}$$

Where

$$\varepsilon = [x, \hat{x}]^T \quad A_{c1} = \overline{A}_{c1} + \overline{H}F\overline{E}$$

$$B_{c1} = [B_1 \ \ 0]^T \quad C_{c1} = [C_2, 0]$$

$$\overline{A_{c1}} = \begin{bmatrix} A_0 + B_0 D_k C_1 & B_0 C_k \\ B_k C_1 & A_k \end{bmatrix} \quad \overline{H} = \begin{bmatrix} H \\ 0 \end{bmatrix}$$

$$\overline{E} = [E_1 + E_2 D_k C_1, E_2 C_k]$$

Through *Lemma 1* [9]and *Lemma 2* [10], the controller parameters exist such that the bounded real lemma condition holds for γ if and only if there exists a symmetric positive define matrix X_{c1} satisfying the following conditions.

$$\begin{bmatrix} dua\{A_{c1}{}^T X_{c1}\} & X_{c1}\overline{B} & \overline{C}^T & X_{c1}\overline{H} & \overline{E}^T \\ * & -\gamma I & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 \\ * & * & 0 & -\varepsilon^{-1}I & 0 \\ * & * & * & * & \varepsilon I \end{bmatrix} < 0 \tag{11}$$

Where $dua\{\cdot\} = \cdot + \cdot^T$ and $*$ represents symmetric matrix. The matrix X_{c1} and its inverse matrix is subjected to the following blocks

$$X_{c1} = \begin{bmatrix} Y & N \\ N^T & W \end{bmatrix} \quad X_{c1}{}^{-1} = \begin{bmatrix} X & M \\ M^T & Z \end{bmatrix}$$

$$X_{c1}\begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix} = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix}$$

By defining

$$F_1 = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}, \ F_2 = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix}$$

$$\begin{bmatrix} dua\{A_0 X + B_0 \hat{C}\} & \hat{A}^T + (A_0 + B_0 \hat{D} C_1) & B_1 & X C_2^T & H & X E_1^T + \hat{C} E_2^T \\ * & dua\{Y A_0 \hat{B} C_2\} & Y B_1 & C_2^T & YH & E_1^T + C_1 \hat{D}^T E_2^T \\ * & * & -\gamma I & 0 & 0 & 0 \\ * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & -\varepsilon^{-1} I & 0 \\ * & * & * & * & * & \varepsilon I \end{bmatrix} < 0 \tag{12}$$

The equivalent matrix inequality can be obtained by multiplying the upper left matrix diag$\{F_1^T, I, I, I, I\}$ and the right multiplying matrix diag$\{F_1, I, I, I, I\}$. We can get the following equivalent matrix equation (12).

By solving the linear matrix inequality, we can get a feasible solution of and Y. From the above we can see that $MN^T = I - XY$, and matrices M and N can be obtained through the singular value decomposition. Then we can get the equation.

$$\begin{cases} D_k = \hat{D} \\ C_k = (\hat{C} - D_k C_1 S)(M^T)^{-1} \\ B_k = N^{-1}(\hat{B} - Y B_0 D_k) \\ A_k = N^{-1}\left[\hat{A} - Y(A_0 + B_0 D_k C_1)X\right](M^T)^{-1} - B_k C_1 X (M^T)^{-1} - N^{-1} Y B_0 C_k \end{cases} \tag{13}$$

2.3 Controller Structure Design

Block diagram of the controller is as follows in fig 2, δ_f is the front wheel steering angle, δ_{fd} is driver's steering input angle, δ_{fc} is the front wheel steering angle generated by the controller and γ_d is the ideal yaw rate.

And $\delta_f = \delta_{fd} + \delta_{fc}, \gamma_d = \dfrac{u}{(l_f + l_r)(1 + k_{us} u^2)} \delta_{fd}$ (14)

Where k_{us} [8]is the vehicle stability factor which can be determined by the ideal parameters of the linear 2 degree-of-freedom vehicle model. In the simulations, it is assumed that the tire cornering stiffness varies from 30% of normal and the vehicle drives straight at a speed of 90km/h.

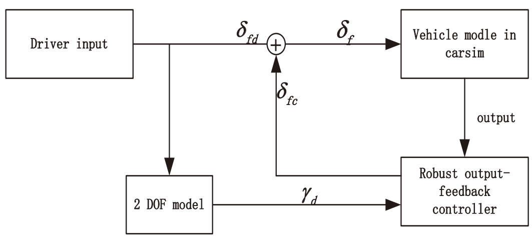

Fig 2: Block diagram of the controller

3 Simulations Analyses

Carrying out simulation in Simulink and Carsim to verify performance of the controller, and regard zero slip angle as a reference value. Vehicle parameters used in the simulation shows in table 1.

3.1 Steering Input with a Step Signal

Assuming that the driver inputs a steering angle of 4 degree at 1 second and returns to 0 at 3 second to simulate a vehicle suddenly encounter obstacles. Figures 3 and 4 show the response of the vehicle's yaw rate and slip angle respectively. It is obvious that the yaw rate follow the ideal curve well over a small overshoot and it also can be seen the system stabilizes quickly and the slip angle is controlled to a very small extent with the controller. The results indicate that AFS robust control can significantly improve the stability of vehicles in emergency conditions.

Table 1: Vehicle parameters used in the simulation

Parameters	Values
Vehicle mess, m	1111kg
Inertia moment of the vehicle	2031kg·m^2
Distance of CG from front axle, l_f	1.04m
Distance of CG from rear axle, l_r	1.56m
Tire cornering stiffness of front	40000N/rad
Tire cornering stiffness of rear	30000N/rad

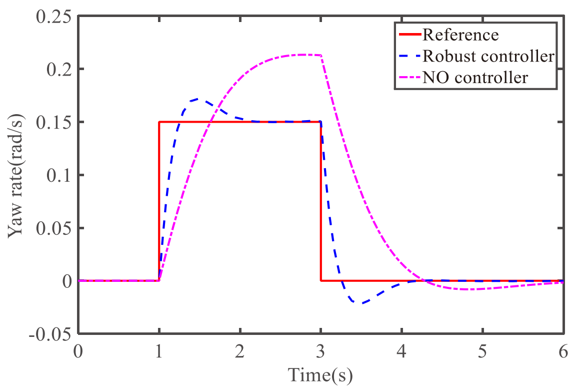

Fig 3. Response curves of yaw rate-time

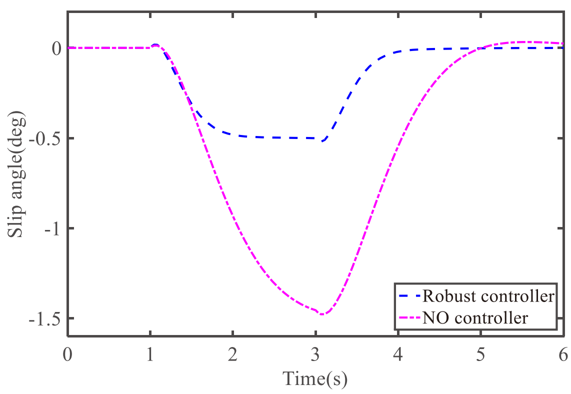

Fig 4. Response curves of slip angle-time

3.2 Side Wind Disturbance

Simulating a vehicle encountered wind disturbance at low attachment road. The side wind disturbance model uses common model as in fig.5[11]. Assuming the driver does not respond to any disturbances. Figures 6 and 7 show the response of the vehicle's yaw rate and slip angle respectively. It can be seen that the yaw rate and slip angle significantly reduced without human intervention. The vehicle maintains the normal running track basically. The results show that controller can resist the side wind disturbance effectively.

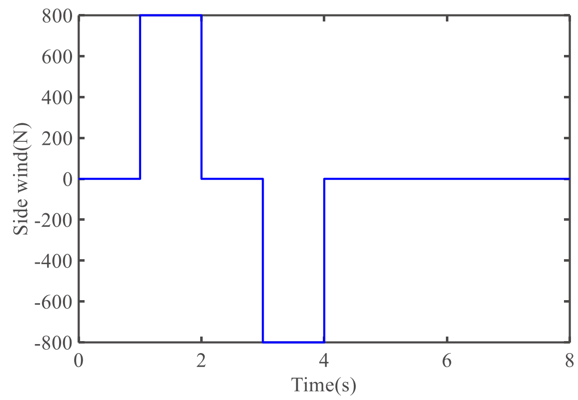

Fig 5. Curve of side wind-time

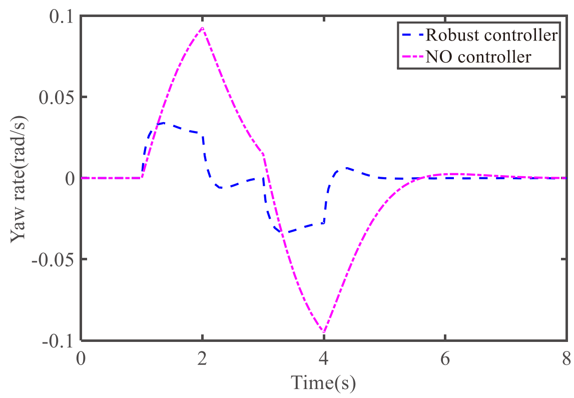

Fig 6. Response curves of yaw rate-time

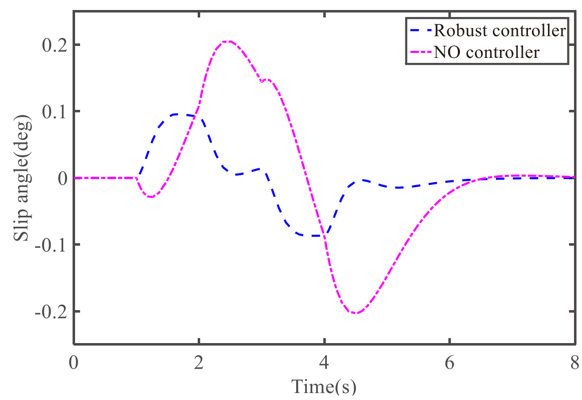

Fig 7. Response curves of slip angle-time

4 Conclusion

Active front wheel steering (AFS) based on robust H∞

output-feedback control is proposed to improve the vehicle lateral stability under the lateral wind disturbance. Uncertain parameter of cornering stiffness is considered in the controller design. The simulation conducted in MATLAB/Simulink and Carsim show that the proposed controller can improve the vehicle lateral stability and drivability significantly with lateral wind disturbance. The results also show that the proposed controller has good robustness to the parameter uncertainty. In addition, the cost of the proposed controller is reduced as it is unnecessary to measure slip angle compared to current vehicle stability control strategies.

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