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Event-triggered and self-triggered H_∞ output tracking control for discrete-time linear parameter-varying systems with network-induced delays

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ABSTRACT

In this paper, we address the event-triggered and self-triggered H_∞ output tracking problem of discrete LPV systems with network-induced delay. Considering the time delay and external disturbance, we formulate the closed-loop event-triggered output tracking problem into a time-delayed discrete polytopic linear parameter-varying system with relative event-triggering mechanism. Then, by constructing the parameter-dependent Lyapunov–Krasovskii functional, we derive a sufficient condition such that the closed-loop system is global asymptotic stable and satisfies the H_∞ output tracking performance. Further, we develop an approach to design the event-triggering mechanism and H_∞ output tracking controller. The parameters of event-triggering mechanism and the gains of output tracking controller are obtained by solving linear matrix inequalities. Moreover, we extend the results of the event-triggered control to the self-triggered H_∞ output tracking control. This not only mitigates the usage of network bandwidth but also avoids the use of additional hardware. Finally, the numerical simulation is included to illustrate the usefulness and effectiveness of the proposed approach.

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H_∞ output tracking control; linear parameter-varying system; networked control system; event-triggered control; self-triggered control

1. Introduction

Networked control systems (NCSs) refer to a class of systems that connect sensors, controllers and actuators through a shared communication channel to transmit data and exchange information, so as to execute real-time feedback control on controlled objects (Ge et al., 2017). NCSs have become one of the future development trends in the control field due to their potential advantages such as flexible structure, distributed control, resource sharing and intelligent nodes (Antsaklis & Baillieul, 2007; Jiang et al., 2011; F. Li et al., 2015; S. B. Li et al., 2015; Park & Park, 2011; Ulusoy et al., 2011). However, there are some intractable problems with NCSs. One stubborn issue, for instance, is the network-induced delay. Depending on the networked environment, multi-user shares the communication channel so as to make the information flux changes become irregular. Common data networks not only transmit the control information in the closed-loop control systems, but also the other data, which are irrespective with the control tasks. Resources competition and network congestion inevitably produce the network-induced delays. From the control point of view, network-induced delays may deteriorate the control system performance, and even make the systems become unstable. From the scheduling point of view, network-induced delays make the information unable to arrive the destinations on time, lose the deadline, and even produce the domino effect. The network-induced delay are sums of several small delays, for example the sensor-controller delay, the controller-actuator delay and the computation delay. Furthermore, the computation delay is usually time-varying, but its size and changes are small compared with the sensor-controller delay and the controller-actuator delay. During the design of NCSs, we may select appropriate hardware and high efficiency software coding and make the influence of the computation delay decrease the least extent. So, extensive researches concerning the network-induced delay have progressed in the past years. As to uncertain time-delay, Q. Zhang and Guo (2010) have modelled the NCS to an uncertain discrete-time linear system with delay, and designed the H_∞ dynamic output

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feedback controller. As to the delay and the packet loss exist simultaneously, a new continuous-time switch linear system model (W. A. Zhang & Li, 2008) and the discrete-time switch linear system model (J. Wang & Yang, 2011) were constructed to study the dynamic performance of the network system. However, the delay was assumed to be less than one sampling period in the aforementioned researches. Since the delays are random in most cases, the delay constraint increases the conservatism of the system. To reduce conservatism, on the basis of the discrete-time switched system model, Bai et al. (2012) relaxed the delay constraints and obtained the stabilisation criterion by using the average dwell time technique. Jia (2013) adopted another approach to deal with time-delay, i.e. modelling random delay to the homogeneous Markov chains, and proposed DR Model based NCS to analysis stability of the system. Furthermore, to tackle the case that the delay varies within one sampling period, Peng et al. (2008) have modelled NCSs using delayed differential equation.

With the development of NCSs, network-based output tracking control has gradually attracted a significant interest recently (Y. B. Gao et al., 2015; Hua et al., 2007; C. Li et al., 2017; Z. Li & Ma, 2017; D. Zhang et al., 2018). Different from traditional output tracking control, network-based output tracking control shares information through the network, so that the control system tracks the output of the given reference model as much as possible. Therefore, tracking performance depends not only on the designed controller, but also on network communication resources, which brings great challenges to the network tracking control. In order to reduce the influence of external disturbances and parameters-varying, the H_∞ output tracking method was proposed (H. J. Gao & Chen, 2008). Moreover, in the networked control systems, processing time and network bandwidth are scarce resources, while the traditional networked control system sampling relies on the time-triggering mechanism (called periodic trigger). In the time-triggering mechanism, the sampling period is selected based on the worst case, and control task is executed at the same frequency. The communication resources will be used even if it is not necessary from the stability/performance perspectives (Hajshirmohamadi et al., 2016). This is a waste. Therefore, a reduction of the data transmission rate is meaningful in wireless sensor systems so as to avoid the network traffic congestion and decrease the energy consumption of the sensor units (H. Liu & Yu, 2017). To tackle this issue, the event-triggering mechanism (ETM) is introduced into the H_∞ output tracking control as an alternative time-triggered control. ETM not only saves the system's effective bandwidth and computing resources, but also ensures the internal execution time strictly greater than zero (Donkers & Heemels, 2012). For example, in D. W. Zhang et al. (2015), a state-dependent ETM was introduced to reduce the transmission of data packets for the network-based output tracking control through a T-S fuzzy model, and the controller was jointly designed together with the event-triggering mechanism. In R. J. Liu et al. (2019), another event-triggering mechanism based on the form of *delta* sampling was proposed in synthesise a full-state feedback tracking controller. Furthermore, an observer-based event-triggered control mechanism was proposed for leader-follower systems with time delay to make all followers track leaders asymptotically in W. Liu et al. (2016). In addition, a novel hybrid-triggered reliable dissipative controller design is developed for the SNCCSs with randomly occurring cyber-attacks, actuator saturation and actuator faults in Sathishkumar and Liu (2020). Although event triggering can reduce the amount of data transmissions effectively, it requires some special hardwares or even a large amount of special hardware equipment in some extreme conditions to detect trigger conditions in real time, which will lead to more resources are used. To avoid this disadvantage of event-triggering control, a software realisation of the event-triggering technique called self-triggering scheme has come into use recently. Under the self-triggering mechanism, the next task release time is predicted by the processing computer based on the previous dynamic information of the system. In C. Li et al. (2017) and Z. Li Ma (2017), NCS with the network-induced short delay and packet loss is modelled as a switching system. Then, based on the switching system model, the switching rules and the conditions are obtained to complete the event-triggering mechanism and discrete-time switching controller jointly, in which self-triggering conditions are provided to guarantee the exponentially stability of the system. In Shao et al. (2013), for the event-triggered tracking control problem and self-triggered tracking control problem of linear systems, event-triggering conditions based on state errors are proposed to guarantee that the control system asymptotically tracks the output of the given reference system. Many significant results concerning event-triggered and self-triggered control design are reported in the literature (Almeida et al., 2014; Garcia & Antsaklis, 2013; Peng et al., 2016; Tallapragada & Chopra, 2014; X. Wang et al., 2019; Z. Wang & Chen, 2015), however, most of them are developed for linear time-invariant systems. In fact, the ETC in the frame of discrete-time system has its inherent benefits, for instance, the non-zero minimum inter-event time could always be guaranteed. As a consequence, a discrete-time plant model is important. On the other hand, linear parameter-varying (LPV) systems represent a very special class of linear systems whose dynamics depends on a priori unknown, but online measurable time-varying parameters. Due to their effectiveness on modelling and control of nonlinear systems, LPV systems have been extensively studied in the literature. For example, in F. Li et al. (2015) and S. B. Li et al. (2015), for the LPV system, a parameter-dependent sufficient condition is proposed such that the networked control system with mixed event-triggering mechanism is global uniform ultimate bounded. Based on the literature (F. Li et al., 2015; S. B. Li et al., 2015), the closed-loop event-triggered control system was modelled as a time-delayed LPV system with mixed event-triggering mechanism and parameter uncertainty, and then, scholars continue to study the optimisation problem between robustness and resource utilisation of networked control systems (Xie et al., 2018). Nevertheless, to the best of our knowledge, the event-triggering mechanism has not yet gained adequate attention in the H_∞ output tracking control problem for discrete-time linear parameter-varying systems with network-induced delays despite its clear practical insight, and there are not related research on self-triggered H_∞ output tracking control based on network-induced delays. In this paper, we use a time-delayed discrete polytopic LPV model to describe the networked control system, and address the event-triggered and self-triggered H_∞ output tracking control problem under the influence of network-induced delay and external disturbance. The contribution is that a design algorithm of output tracking controller based on event-triggered and self-triggered scheme is developed for discrete polytopic LPV system in a network environment, which ensures that the closed-loop system is global asymptotic stable and satisfies the H_∞ output tracking performance. Finally, several numerical examples show that the proposed event-triggered and self-triggered H_∞ output tracking control strategy is effective.

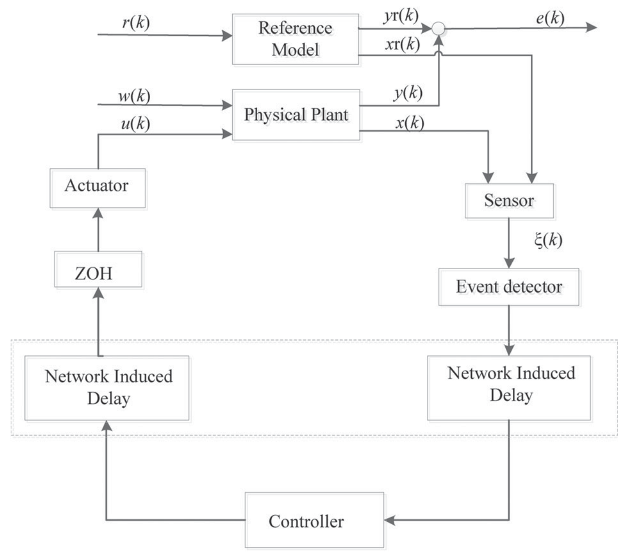

Figure 1. Considered networked tracking control system under ETM.

Notation: Let $\mathbb{R}^n$ stands for n-dimensional Euclidean space, and the European norm of $x \in \mathbb{R}^n$ is denoted by $\|x\| = \sqrt{x^T x}$; $\mathbb{N}$ is the set of non-negative integer numbers. The superscript 'T' stands for matrix transposition; $P > 0 (\geqslant 0)$ means that P is real symmetric and positive definite (semidefinite); diag$\{\cdot\}$ is used to denoted a block-diagonal matrix and an asterisk $(*)$ to represent a term induced by symmetric matrix in symmetric matrices; sym$\{A\}$ is used to denote the expression $A + A^T$; the maximum and minimum eigenvalue of a symmetric real matrix A are denoted by $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$, respectively. A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class $\mathcal{K}$ if it is continuous, strictly increasing and $\alpha(0) = 0$. The linear space of square-integrable vector function over $[0, \infty)$ is denoted by $\mathfrak{L}_2[0, \infty)$.

2. System modelling

2.1. Problem statement

The configuration of considered networked tracking control system is shown in Figure 1 with signal transmission delay and event-triggering mechanism.

The plant considered here is a discrete polytopic LPV system whose dynamics is given as:

$$\begin{aligned} x(k+1) &= A(\theta_k)x(k) + B(\theta_k)u(k) + Ew(k), \\ y(k) &= C(\theta_k)x(k) + Dw(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^r$ and $y(k) \in \mathbb{R}^p$ are the state vector, the control input and the output vector, respectively. $w(k) \in \mathbb{R}^q$ is the disturbance input that satisfies $w(k) \in \mathfrak{L}_2[0, \infty)$, time-varying parameter $\theta_k \in \mathbb{R}^k, k \in \mathbb{N}$ lies in some set $\Theta \subset \mathbb{R}^k$. Assume that Θ is measurable set in real-time and the system matrix is a function of the time-varying parameter θ_k. The above LPV system (1) can be cast into the following polytopic LPV model as in Baranyi and Varkonyi-Koczy (2005) and Sathishkumar et al. (2017):

$$\begin{bmatrix} A(\theta_k) & B(\theta_k) & E \\ C(\theta_k) & 0 & D \end{bmatrix} \in \Re$$
$$:= \left\{ \sum_{i=1}^{N} \alpha_i(\theta_k) \begin{bmatrix} A_i & B_i & E \\ C_i & 0 & D \end{bmatrix}, \alpha_i(\theta_k) > 0, \sum_{i=1}^{N} \alpha_i(\theta_k) = 1 \right\}$$

For system (1), $\Re$ is a given convex bounded polyhedral domain described by N vertices. (A_i, B_i, E, C_i, D) is the representation of the system (1) at the ith $(i = 1, 2, \ldots, N)$ vertex. Assuming that the output $y(k)$ of system (1) tracks the output signal $y_r(k)$ of the given reference system (2) through the network.

$$\begin{aligned} x_r(k+1) &= G(\theta_k)x_r(k) + r(k), \\ y_r(k) &= H(\theta_k)x_r(k), \end{aligned} \tag{2}$$

where $x_r(k) \in \mathbb{R}^m$ is the reference state, $r(k) \in \mathbb{R}^m$ is the energy bounded reference input, $y_r(k) \in \mathbb{R}^p$ is the output signal of reference system, $G(\theta_k)$ and $H(\theta_k)$ are appropriate dimensional parameter-dependent matrices with $G(\theta_k)$ Hurwitz. It is assumed that both $x(k)$ and $x_r(k)$ are online measurable.

Defining augmented matrix vector $\xi(k) = \begin{bmatrix} x(k) \\ x_r(k) \end{bmatrix}$ track error $\bar{e}(k) = y(k) - y_r(k)$. Considering system (1) and system (2), we get the following augmented discrete LPV system (3):

$$\begin{aligned} \xi(k+1) &= \bar{A}(\theta_k)\xi(k) + \bar{B}(\theta_k)u(k) + \bar{E}\bar{w}(k), \\ \bar{e}(k) &= \bar{C}(\theta_k)\xi(k) + \bar{D}\bar{w}(k), \end{aligned} \tag{3}$$

where

$$\bar{A}(\theta_k) = \begin{bmatrix} A(\theta_k) & 0 \\ 0 & G(\theta_k) \end{bmatrix}, \quad \bar{B}(\theta_k) = \begin{bmatrix} B(\theta_k) \\ 0 \end{bmatrix},$$
$$\bar{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \quad \bar{w}(k) = \begin{bmatrix} w(k) \\ r(k) \end{bmatrix}$$
$$\bar{C}(\theta_k) = [C(\theta_k) \quad -H(\theta_k)], \quad \bar{D} = [D \quad 0]$$

For the sake of actual situations and calculation simplicity, the following assumptions are given for system (3):

Assumption 2.1: Sensors are time-triggered with a constant sampled period h, $h \in \mathbb{N}$.

Assumption 2.2: The sampled data is transmitted in a single packet, and neither packet losses and nor disorder occurs in transmission.

In order to reduce the frequency of data transmission, the task release time $k_i \in \mathbb{N}$ $(i = 0, 1, 2 \ldots)$ is determined by event-triggering equipment. When the event detected, the state sequence ξ_{k_i} is transmitted to the controller through the network channel, then the control law is updated, and a new setpoint is sent to the zero-order holder (ZOH) via network again. ZOH keeps the current value as the control input at the actuator until the next transmitting datum come.

For ease of presentation, the sensor sampling sequence is described by the set $S_1 = \{0, h, 2h, \ldots, kh, \ldots\}$, $(k \in \mathbb{N})$, the transmission sequence at the sensors is described by the set $S_2 = \{0, k_1 h, k_2 h, \ldots, k_i h, \ldots\}$, $(k_i \in \mathbb{N})$.

The ETM is implemented as the violation of the following inequality condition:

$$e^T(k)M_1(\theta_k)e(k) < \delta\xi^T(k)M_2(\theta_k)\xi(k), \quad k \in [k_i, k_{i+1}), \tag{4}$$

where $e(k) = \xi(k_i) - \xi(k)$, $M_1(\theta_k) = \sum_{i=1}^N \alpha_i(\theta_k)M_{1i}$, $M_2(\theta_k) = \sum_{i=1}^N \alpha_i(\theta_k)M_{2i}$. $M_1(\theta_k)$ and $M_2(\theta_k)$ are quadratic positive weighting matrices on $e(k)$ and $\xi(k)$ to be determined, respectively. M_{1i}, M_{2i} are the value matrices at the ith vertex, and $\delta > 0$ is scalar. Hence the next event-triggering time instant k_{i+1} is defined:

$$k_{i+1} = \inf\left\{ k \geqslant k_i + h \,\middle|\, e^T(k)M_1(\theta_k)e(k) \geqslant \delta\xi^T(k)M_2(\theta_k)\xi(k), k \in \mathbb{N} \right\} \tag{5}$$

Remark 2.1: According to (4), the sampling data is not sent to the controller unless satisfying the trigger condition. This may cost a bit of computing time of (4), but can considerably relieve the transmission pressure and save the bandwidth use of the network. In networked control systems, the bandwidth resources are usually limited, and communication consumes more energy than information processing does. Therefore, it is significant to improve the bandwidth utilisation by the event-triggering mechanism.

Remark 2.2: As defined in (5), the lower bound of the triggering interval is the sampling period h, $(h > 0)$, which basically avoids Zeno phenomenon (Dashkovskiy & Feketa, 2017).

Remark 2.3: Up to now, various event-triggering mechanisms have been proposed in the literature, for example, hybrid-triggered scheme is executed by making use of random switch connecting time- and event-trigger scheme. And it is more suitable when the system state fluctuates wildly at the sampled instant; Another triggering mechanisms called memory event-triggered scheme is proposed when historic released signals are needed. And it is more suitable when the difference between two adjacent samplings is very small near the vertex of the response curve. However, the proposed event-triggering mechanisms $e^T(k)M_1(\theta_k)e(k) < \delta\xi^T(k)M_2(\theta_k)\xi(k), k \in [k_i, k_{i+1})$ is more suitable for discrete polytopic LPV system in this paper. It adopts parameter-dependent quadratic positive weighting matrices $M_1(\theta_k)$ and $M_2(\theta_k)$. The conservativeness is reduced and the resource utilisation is improved compared with the other two cases including $M_1(\theta_k){=}M_2(\theta_k){=}M(\theta_k)$ and $M_1(\theta_k) = M_1, M_2(\theta_k) = M_2$. Furthermore, based on general event-triggered scheme, an additional internal dynamic variable is introduced to formulate the dynamic event-triggered strategy. Due to the influence of time-varying parameter θ_k, the event-triggering mechanism is also dynamic in this paper. In addition, event triggering requires some special hardwares or even a large amount of special hardware equipment in some extreme conditions to detect trigger conditions in real time, which will lead to more resources are used. Further, self-triggering scheme proposed can avoid this disadvantage of event-triggering control in this paper.

2.2. Closed-loop system modelling

Due to the influence of the network-induced delay, the state sequence ξ_{k_i} is sent at time k_i by the event-triggering mechanism and arrives at the actuator at the moment $k_i h + \tau_{k_i}$, afterwards, the control input will remain a constant through ZOH until the next states sequence $\xi_{k_{i+1}}$ arrives. Denote $\tau_{k_i} = \tau_{k_i}^{sc} + \tau_{k_i}^{ca} \in [\tau_m, \tau_M]$, $\tau_m, \tau_M \in \mathbb{N}$, where $\tau_{k_i}^{sc}$ is the transport delay from sensors to the controllers, and $\tau_{k_i}^{ca}$ is the transport delay from controllers to actuators. The state feedback control law is chosen as follows:

$$\begin{aligned} u(k) &= \bar{K}(\theta_k)\xi(k_i) \\ &= K_1(\theta_k)x(k_i) + K_2(\theta_k)x_r(k_i) \\ &\qquad k \in [k_i + \tau_{k_i}, k_{i+1} + \tau_{k_{i+1}}), \end{aligned} \tag{6}$$

where $\bar{K}(\theta_k) = [K_1(\theta_k)\ \ K_2(\theta_k)]$, $K_1(\theta_k)$ and $K_2(\theta_k)$ are parameter-dependent state feedback gains.

The time intervals I_a partitioned as $\mathrm{I}_a = \bigcup_{m=k_i}^{k_{i+1}-1} \mathrm{I}_m$, with $\mathrm{I}_m = [m + \tau_m, m + 1 + \tau_{m+1})$, $k_i \in \mathbb{N}$ and $\tau_m \leqslant 1 + \tau_{m+1}$, which ensures that the sequence $m + \tau_m$ is strictly increasing.

For $\forall k \in \mathrm{I}_m$, we denote its overall time delay η including both the network-induced delays and packet-losses caused by the ETM. Given that η is bounded by $\underline{\eta}$ and $\bar{\eta}$, respectively. Then, according to (6), the feedback control law is given by $u(k) = \bar{K}(\theta_k)(e(k - \eta_k) + \xi(k - \eta_k))$.

Applying to system (3), we get the following augmented closed-loop time-delayed LPV system (7):

$$\begin{aligned} \xi(k+1) &= \bar{A}(\theta_k)\xi(k) + \bar{B}(\theta_k)\bar{K}(\theta_k)\xi(k - \eta_k) \\ &\quad + \bar{B}(\theta_k)\bar{K}(\theta_k)e(k - \eta_k) + \bar{E}\bar{w}(k), \\ \bar{e}(k) &= \bar{C}(\theta_k)\xi(k) + \bar{D}\bar{w}(k) k \in I_m, \end{aligned} \tag{7}$$

where $\bar{K}(\theta_k) = [K_1(\theta_k)\ \ K_2(\theta_k)]$.

Our study is the H_∞ output tracking control of the closed-loop system (7) with the event-triggering mechanism (4). This is, design a state feedback controller (6) such that when the parameter θ varying within all its range, the augmented closed-loop system (7) satisfies:

(1) When the disturbance signal $\bar{w}(k) = 0$, the closed-loop system (7) is global asymptotically stable.
(2) Otherwise, given a scalar $\gamma > 0$, the output tracking error $\bar{e}(k)$ can be attenuated below the level γ in H_∞ sense, which implies $\|\bar{e}(k)\|_2 \leqslant \gamma \|\bar{w}(k)\|_2$.

We say that the output $y(k)$ of control system asymptotically tracks output $y_r(k)$ of the given reference system if closed-loop system in (7) is global asymptotically stable (Fridman et al., 2008). Hence, the problem of asymptotically tracking control is turned into the problem of the global asymptotically stability of the augmented system.

3. Event-triggered H_∞ output tracking control

3.1. Stability analysis

Lemma 3.1 (Moon et al., 2001): *Assume that vectors $a(\cdot)$, $b(\cdot)$ and $L(\cdot)$, for any matrices V, Y, and S the following holds*:

$$-2a^TLb \leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} V & Y-L \\ Y^T-L^T & S \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

where $\begin{bmatrix} V & Y \\ Y^T & S \end{bmatrix} \geq 0$.

Lemma 3.2 (Moon et al., 2001): *A special choice of Y and Z such that $Y = L = I$ and $V = S^{-1}$ in Lemma* 3.1, *the following holds*:

$$2a^Tb \leq a^TVa + b^TV^{-1}b$$

Theorem 3.3: *Consider closed-loop system* (7), *given positive scalars $\bar{\eta} \geqslant \underline{\eta}, \gamma, \delta$, if there exist a positive scalar δ_1, parameter-dependent symmetric $P(\theta_k)$, positive definite matrices function $M_1(\theta_k)$, $M_2(\theta_k)$ and symmetric positive definite matrices Q, N, U_i, $(i = 1, 2, 3)$, symmetric matrices R_{ii} $(i = 1, 2, 3)$ and matrices R_{12}, R_{13}, R_{23}, S_i $(i = 1, 2, 3)$, such that*

$$\begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} & U_1\bar{E} & \bar{C}^T & U_1\bar{B}(\theta_k)\bar{K}(\theta_k) & F_1 \\ * & \varepsilon_{22} & \varepsilon_{23} & U_2\bar{E} & 0 & U_2\bar{B}(\theta_k)\bar{K}(\theta_k) & F_2 \\ * & * & \varepsilon_{33} & U_3\bar{E} & 0 & U_3\bar{B}(\theta_k)\bar{K}(\theta_k) & F_3 \\ * & * & * & -\gamma I & \bar{D}^T & 0 & F_4 \\ * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & -M_1(\theta_k) & 0 \\ * & * & * & * & * & * & -\delta_1 I \end{bmatrix} < 0 \quad (8)$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & S_1 \\ * & R_{22} & R_{23} & S_2 \\ * & * & R_{33} & S_3 \\ * & * & * & N \end{bmatrix} \geq 0 \quad (9)$$

where

$$\varepsilon_{11} = P(\theta_{k+1}) - P(\theta_k) + (1 + \bar{\eta} - \underline{\eta})Q + \mathrm{sym}\{U_1(\bar{A}(\theta_k) - I)\} + \mathrm{sym}\{S_1\} + \bar{\eta}R_{11}$$

$$\varepsilon_{12} = U_1\bar{B}(\theta_k)\bar{K}(\theta_k) + (\bar{A}(\theta_k) - I)^TU_2 - S_1 + S_2^T + \bar{\eta}R_{12}$$

$$\varepsilon_{13} = P(\theta_{k+1}) - U_1 + (\bar{A}(\theta_k) - I)^TU_3 + S_3^T + \bar{\eta}R_{13}$$

$$\varepsilon_{22} = \delta M_2(\theta_k) - Q + \mathrm{sym}\{U_2\bar{B}(\theta_k)\bar{K}(\theta_k)\} + \mathrm{sym}\{-S_2\} + \bar{\eta}R_{22}$$

$$\varepsilon_{23} = -U_2 + \bar{K}^T(\theta_k)\bar{B}^T(\theta_k)U_3 - S_3^T + \bar{\eta}R_{23}$$

$$\varepsilon_{33} = \bar{\eta}N + P(\theta_{k+1}) + \mathrm{sym}\{-U_3\} + \bar{\eta}R_{33}$$

$$F_i = \begin{bmatrix} 0 & \cdots & 0 & \underbrace{I}_{i-th\ column} & 0 & \cdots & 0 \end{bmatrix}, \quad i = 1, 2, 3, 4$$

holds with parameter θ_k varying within its range, then the closed-loop system (7) *is global asymptotically stable with γ-disturbance attenuation property.*

Proof: The proof is given in Appendix. ■

Remark 3.1: The choice of an appropriate Lyapunov–Krasovskii functional is the key-point for deriving of stability criteria. It is known that the general form of this functional leads to a complicated system of partial differential equations. That is why many authors considered special forms of Lyapunov–Krasovskii functional and thus derived simpler (but more conservative) sufficient conditions. There are nonquadratic Lyapunov functions (J. Li & Yang, 2019; J. Li & Zhang, 2019) and quadratic Lyapunov functions. Furthermore, quadratic Lyapunov functions are divide into delay-independent and delay-dependent conditions. In this paper, the parameter-dependent Lyapunov function is selected. But only V_1 is parameter-dependent, V_2 and V_3 are not parameter-dependent. The purpose is to simplify the calculation, but at the same time, it makes the solution conditions more stringent and increase conservatism.

3.2. Design of state feedback controller and event-triggering mechanism

Based on Theorem 3.3, we give the following sufficient conditions for design of controller gain matrix and event-triggering mechanism.

Theorem 3.4: *Consider the closed-loop system* (7), *given positive scalars $\bar{\eta} \geqslant \underline{\eta}, \gamma, \delta, c_1, c_2$, if there exist a positive scalar δ_1, parameter-dependent symmetric positive definite matrices function $\hat{P}(\theta_k), \hat{M}_1(\theta_k), \hat{M}_2(\theta_k)$, parameter-dependent matrix $Y(\theta_k)$ and symmetric positive definite matrices $\hat{Q}, \hat{N}, W$, symmetric matrices*

$\hat{R}_{ii}$ $(i=1,2,3)$ *and matrices* $\hat{R}_{12}, \hat{R}_{13}, \hat{R}_{23}, \hat{S}_i$ $(i=1,2,3)$ *satisfying* (10) *and* (11), *the closed-loop system* (7) *is global asymptotically stable with* γ*-disturbance attenuation property.*

$$\Phi(\theta_k) \triangleq \begin{bmatrix} \hat{\varepsilon}_{11} & \hat{\varepsilon}_{12} & \hat{\varepsilon}_{13} & \bar{E} & W\bar{C}^T & \bar{B}(\theta_k)Y(\theta_k) & WF_1 \\ * & \hat{\varepsilon}_{22} & \hat{\varepsilon}_{23} & c_1\bar{E} & 0 & c_1\bar{B}(\theta_k)Y(\theta_k) & WF_2 \\ * & * & \hat{\varepsilon}_{33} & c_2\bar{E} & 0 & c_2\bar{B}(\theta_k)Y(\theta_k) & WF_3 \\ * & * & * & -\gamma I & \bar{D}^T & 0 & F_4 \\ * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & -\hat{M}_1(\theta_k) & 0 \\ * & * & * & * & * & * & -\delta_1 I \end{bmatrix} < 0 \quad (10)$$

$$\Upsilon \triangleq \begin{bmatrix} \hat{R}_{11} & \hat{R}_{12} & \hat{R}_{13} & \hat{S}_1 \\ * & \hat{R}_{22} & \hat{R}_{23} & \hat{S}_2 \\ * & * & \hat{R}_{33} & \hat{S}_3 \\ * & * & * & \hat{N} \end{bmatrix} \geq 0 \quad (11)$$

where

$$\hat{\varepsilon}_{11} = \hat{P}(\theta_{k+1}) - \hat{P}(\theta_k) + (1+\bar{\eta}-\underline{\eta})\hat{Q} + \mathrm{sym}\{(\bar{A}(\theta_k)-I)W\} + \mathrm{sym}\{\hat{S}_1\} + \bar{\eta}\hat{R}_{11}$$

$$\hat{\varepsilon}_{12} = \bar{B}(\theta_k)Y(\theta_k) + c_1 W(\bar{A}(\theta_k)-I)^T - \hat{S}_1 + \hat{S}_2^T + \bar{\eta}\hat{R}_{12}$$

$$\hat{\varepsilon}_{13} = \hat{P}(\theta_{k+1}) - W + c_2 W(\bar{A}(\theta_k)-I)^T + \hat{S}_3^T + \bar{\eta}\hat{R}_{13}$$

$$\hat{\varepsilon}_{22} = \delta\hat{M}_2(\theta_k) - \hat{Q} + \mathrm{sym}\{c_1\bar{B}(\theta_k)Y(\theta_k)\} + \mathrm{sym}\{-\hat{S}_2\} + \bar{\eta}\hat{R}_{22}$$

$$\hat{\varepsilon}_{23} = -c_1 W + c_2 Y^T(\theta_k)\bar{B}^T(\theta_k) - \hat{S}_3^T + \bar{\eta}\hat{R}_{23}$$

$$\hat{\varepsilon}_{33} = \bar{\eta}\hat{N} + \hat{P}(\theta_{k+1}) + \mathrm{sym}\{-c_2 W\} + \bar{\eta}\hat{R}_{33}$$

$$F_i = \begin{bmatrix} 0 & \cdots 0 & \underbrace{I}_{i-th\ column} & 0 & \cdots & 0 \end{bmatrix}, \quad i = 1,2,3,4$$

Moreover, the state feedback controller (6) *can be obtained by* $\bar{K}(\theta_k) = Y(\theta_k)W^{-1}$ *and ETM* (4) *can be obtained by* $M_1(\theta_k) = W^{-1}\hat{M}_1(\theta_k)W^{-1}$ *and* $M_2(\theta_k) = W^{-1}\hat{M}_2(\theta_k)W^{-1}$.

Proof: Let $WU_1 = I, WU_2 = c_1 I, WU_3 = c_2 I$, $\hat{W} = \mathrm{diag}\{W\ W\ W\ W\}$ and $\bar{W} = \mathrm{diag}\{W\ W\ W\ I\ I\ W\ I\}$, then, pro-multiplying and post-multiplying $\bar{W}$ and $\bar{W}$ to (8) in Theorem 3.3, respectively, as well as $\hat{W}$ and $\hat{W}$ to (9). Denoting $WQW = \hat{Q}$, $WNW = \hat{N}$, $WR_{ij}W = \hat{R}_{ij}$ $(i,j=1,2,3)$, $WS_jW = \hat{S}_j$ $(j=1,2,3)$, $WP(\theta_k)W = \hat{P}(\theta_k)$, $\bar{K}(\theta_k)W = Y(\theta_k)$, it then yields $\Phi(\theta_k) < 0$, $\Upsilon \geqslant 0$. This completes the proof. ■

Theorem 3.4 explains the LMI method to solve the design of H_∞ output tracking controller (6) and ETM (4), but the LMI condition (10) is of infinite dimensions and non-convex with respect to the scheduling parameter θ; so, it causes difficulty for implementing the controller. In order to solve this problem and reduce conservatism simultaneously, we introduce a convex optimisation approach. At the vertex of the given bounded time-delayed polytopic LPV system, a finite dimensional parameterised LMI is presented, and then by means of convex relaxation, the desired LMI that satisfies the solution conditions is obtained.

Theorem 3.5: *Consider the closed-loop system* (7), *given positive scalars* $\bar{\eta} \geqslant \underline{\eta} > 0$, $\gamma > 0$, $\delta > 0$, $c_1 > 0$, $c_2 > 0$, *if there exist a positive scalar* δ_1, *symmetric positive definite matrices* $\hat{P}_j, Y_j, \hat{M}_{1j}, \hat{M}_{2j}, \hat{Q}, \hat{N}, W$ *symmetric matrices* $\hat{R}_{ii}$ $(i=1,2,3)$ *and matrices* $\hat{R}_{12}, \hat{R}_{13}, \hat{R}_{23}$, $\hat{S}_i$ $(i=1,2,3)$ *satisfying* (12), (13) *and* (14), *then the closed-loop system* (7) *is global asymptotically stable with* γ*-disturbance attenuation property.*

$$\Phi_{ii}^l < 0, \quad i,l \in \{1,2,\dots L\} \quad (12)$$

$$\frac{2}{L-1}\Phi_{ii}^l + \Phi_{ij}^l + \Phi_{ji}^l < 0, \quad i,j,l \in \{1,2,\dots L\},\ i \neq j \quad (13)$$

$$\Upsilon \triangleq \begin{bmatrix} \hat{R}_{11} & \hat{R}_{12} & \hat{R}_{13} & \hat{S}_1 \\ * & \hat{R}_{22} & \hat{R}_{23} & \hat{S}_2 \\ * & * & \hat{R}_{33} & \hat{S}_3 \\ * & * & * & \hat{N} \end{bmatrix} \geq 0 \quad (14)$$

where

$$\Phi_{ij}^l \triangleq \begin{bmatrix} \tilde{\varepsilon}_{11} & \tilde{\varepsilon}_{12} & \tilde{\varepsilon}_{13} & \bar{E} & W\bar{C}^T & \bar{B}_jY_i & WF_1 \\ * & \tilde{\varepsilon}_{22} & \tilde{\varepsilon}_{23} & c_1\bar{E} & 0 & c_1\bar{B}_jY_i & WF_2 \\ * & * & \tilde{\varepsilon}_{33} & c_2\bar{E} & 0 & c_2\bar{B}_jY_i & WF_3 \\ * & * & * & -\gamma I & \bar{D}^T & 0 & F_4 \\ * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & -\hat{M}_{1i} & 0 \\ * & * & * & * & * & * & -\delta_1 I \end{bmatrix}$$

Table 1. The computational complexity of LPV system of Theorem 3.5.

Decision variables	No. of LMIs	The maximum order of LMI
$12(n+m)^2 + 3L(n+m)^2 + 7$	$L^3 + 1$	$5(n+m+1)$

$$\begin{aligned}
\tilde{\varepsilon}_{11} &= \hat{P}_l - \hat{P}_i + \left(1 + \bar{\eta} - \underline{\eta}\right)\hat{Q} + \text{sym}\left\{\left(\bar{A}_j - I\right)W\right\} \\
&\quad + \text{sym}\{\hat{S}_1\} + \bar{\eta}\hat{R}_{11} \\
\tilde{\varepsilon}_{12} &= \bar{B}_j Y_i + c_1 W\left(\bar{A}_j - I\right)^T - \hat{S}_1 + \hat{S}_2^T + \bar{\eta}\hat{R}_{12} \\
\tilde{\varepsilon}_{13} &= \hat{P}_l - W + c_2 W\left(\bar{A}_j - I\right)^T + \hat{S}_3^T + \bar{\eta}\hat{R}_{13} \\
\tilde{\varepsilon}_{22} &= \delta\hat{M}_{2i} - \hat{Q} + \text{sym}\left\{c_1\bar{B}_j Y_i\right\} \\
&\quad + \text{sym}\left\{-\hat{S}_2\right\} + \bar{\eta}\hat{R}_{22} \\
\tilde{\varepsilon}_{23} &= -c_1 W + c_2 Y_i^T \bar{B}_j^T - \hat{S}_3^T + \bar{\eta}\hat{R}_{23} \\
\tilde{\varepsilon}_{33} &= \bar{\eta}\hat{N} + \hat{P}_l + \text{sym}\left\{-c_2 W\right\} + \bar{\eta}\hat{R}_{33} \\
F_i &= \begin{bmatrix} 0 & \cdots & 0 & \underbrace{I}_{i-\text{th column}} & 0 & \cdots & 0 \end{bmatrix}, \\
&\quad i = 1, 2, 3, 4
\end{aligned}$$

Then the state feedback controller (6) *can be obtained by* $K_i = Y_i W^{-1}$ *and ETM* (4) *can be obtained by* $M_{1i} = W^{-1}\hat{M}_{1i}W^{-1}$ *and* $M_{2i} = W^{-1}\hat{M}_{2i}W^{-1}$.

Proof: The parameter-dependent LMI (12) can be written as (15)

$$\begin{aligned}
\Phi(\theta_k) &= \sum_{l=1}^{N}\alpha_l(\theta_{k+1})\left(\sum_{j=1}^{N}\sum_{i=1}^{N}\alpha_i(\theta_k)\alpha_j(\theta_k)\Phi_{ij}^l\right) \\
&= \sum_{l=1}^{N}\alpha_l(\theta_{k+1})\left(\sum_{i=1}^{N}\alpha_i^{\,2}(\theta_k)\Phi_{ii}^l\right. \\
&\quad \left. + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i(\theta_k)\alpha_j(\theta_k)(\Phi_{ij}^l + \Phi_{ij}^l)\right) < 0
\end{aligned} \tag{15}$$

In terms of Lemma 3.1 in Guerra et al. (2012), $\Phi(\theta_k) < 0$ is satisfied if (12) and (13) hold. ■

The numerical complexity of stability criteria depends on number of scalar decision variables, number of LMIs and maximal order of LMI. The numerical complexity of the Theorem 3.5 is summarised in Table 1.

Remark 3.2: From Table 1, we know that as the vertical number L of convex polytopes increase, the computational complexity of Theorem 3.5 increases considerably. The conditions (12) and (13) provide a convex condition with L^3 LMIs in terms of the polytope vertices, which can be solved numerically. It should be noted that the computation complexity of the proposed theorems is mainly caused by the coupling term $\bar{B}(\theta_k)Y(\theta_k)$. If the control gain $\bar{K}(\theta_k)$ in Equation (6) is independent of the time-varying parameter θ_k, the conditions (12) and (13) will provide a convex condition with L^2 LMIs, which efficiently reduces the computation complexity, however, it increases the conservatism of the proposed results.

4. Self-triggered output tracking control strategy

Self-triggered control strategy does not require additional hardware compared to event-triggered control. We can extend the above ETM to obtain a corresponding self-triggered control scheme. We can derive the sufficient conditions for the system with network-induced delay to asymptotically track the output of given reference system under the self-triggering mechanism.

Corollary 4.1: *Define event-triggering conditions as*:

$$|\|e(k)\|^2 \le \rho\|\xi(k_i)\|^2, \quad k \in [k_i, k_{i+1}) \tag{16}$$

where, $e(k) = \xi(k_i) - \xi(k)$, $\rho = \mu/2(1+\mu)$, $\mu = \delta(\lambda_{\min}(M_{1i}))^{-1}\lambda_{\max}(M_{2i})$, $\delta > 0$, $\gamma > 0$, *then the closed-loop system* (7) *is global asymptotically stable with* γ*-disturbance attenuation property if condition* (16) *is satisfied.*

Proof: Referring to (4), (16) can written as

$$\begin{aligned}
e^T(k)e(k) &\le -\mu e^T(k)e(k) + \frac{\mu}{2}\xi^T(k_i)\xi(k_i) \\
&= -\mu e^T(k)e(k) \\
&\quad + \frac{\mu}{2}(e(k)+\xi(k))^T(e(k)+\xi(k)) \\
&\le -\frac{\mu}{2}e^T(k)e(k) + \frac{\mu}{2}\xi^T(k)\xi(k) \\
&\quad + \frac{\mu}{2}e^T(k)e(k) + \frac{\mu}{2}\xi^T(k)\xi(k) \\
&= \mu\xi^T(k)\xi(k)
\end{aligned}$$

$e^T(k)\lambda_{\min}(M_{1i})e(k) \le \delta\xi^T(k)\lambda_{\max}(M_{2i})\xi(k)$ is obtained through proper transformation, namely $\|e(k)\|^2 \le$

$\mu\|\xi(k)\|^2$. Therefore, (16) can guarantee the same performance as the even-triggering conditions (4) in Theorem 3.3, where the closed-loop system (7) is global asymptotically stable with γ-disturbance attenuation property. ■

Theorem 4.2: *Consider the closed-loop system* (7), *given positive scalars* $\bar{\eta} \geqslant \underline{\eta} > 0, \gamma > 0, \delta > 0, c_1 > 0, c_2 > 0$, *if there exist a positive scalar* δ_1, *symmetric positive definite matrices* $\hat{P}_j, Y_j, \hat{M}_{1j}, \hat{M}_{2j}, \hat{Q}, \hat{N}, W$, *symmetric matrices* $\hat{R}_{ii}$ $(i = 1, 2, 3)$, *and matrices* $\hat{R}_{12}, \hat{R}_{13}, \hat{R}_{23}, \hat{S}_i$ $(i = 1, 2, 3)$ *satisfying* (12), (13) *and* (14), *the self-triggering mechanism is designed as*:

$$k_{i+1} = k_i + \max\left\{h, \log_{\|\bar{A}(\theta_k)\|} \times \left(1 - \frac{\sqrt{\rho}\left(1 - \|\bar{A}(\theta_k)\|\right)\|\xi(k_i)\|}{\vartheta\left(\xi(k_i)\right)}\right)\right\}, \quad k \in [k_i, k_{i+1}) \tag{17}$$

then the closed-loop system (7) *is global asymptotically stable with* γ*-disturbance attenuation property, where,*

$$\vartheta(\xi(k_i)) = \|I - \bar{A}(\theta_k) - \bar{B}(\theta_k)\bar{K}(\theta_k)\|\|\xi(k_i)\|.$$

Proof: In time interval $[k_i, k_{i+1})$, we can get:

$$\begin{aligned}
e(k+1) &= \xi(k_i) - \xi(k+1)\\
&= \xi(k_i) - [\bar{A}(\theta_k)\xi(k) + \bar{B}(\theta_k)\bar{K}(\theta_k)\xi(k_i)]\\
&= -\bar{A}(\theta_k)\xi(k) + (I - \bar{B}(\theta_k)\bar{K}(\theta_k))\xi(k_i)\\
&= (I - \bar{A}(\theta_k) - \bar{B}(\theta_k)\bar{K}(\theta_k))\xi(k_i)\\
&\quad + \bar{A}(\theta_k)e(k)\\
\|e(k+1)\| &\leq \|\bar{A}(\theta_k)\|\|e(k)\| + \|I - \bar{A}(\theta_k)\\
&\quad - \bar{B}(\theta_k)\bar{K}(\theta_k)\|\|\xi(k_i)\|\\
&= \|\bar{A}(\theta_k)\|\|e(k)\| + \vartheta(\xi(k_i))
\end{aligned}$$

Hence, in time interval $[k_i, k_{i+1})$,

$$\begin{aligned}
\|e(k)\| &\leq \|\bar{A}(\theta_k)\|\|e(k-1)\| + \vartheta(\xi(k_i))\\
&\leq \|\bar{A}(\theta_k)\|\{\|\bar{A}(\theta_k)\|\|e(k-2)\| + \vartheta(\xi(k_i))\}\\
&\quad + \vartheta(\xi(k_i))\\
&\quad \cdots\\
&\leq \|\bar{A}(\theta_k)\|^{k-k_i}\|e(k_i)\| + (1 - \|\bar{A}(\theta_k)\|)^{-1}\\
&\quad \times (1 - \|\bar{A}(\theta_k)\|^{k-k_i})\vartheta(\xi(k_i))\\
&\leq (1 - \|\bar{A}(\theta_k)\|)^{-1}(1 - \|\bar{A}(\theta_k)\|^{k-k_i})\vartheta(\xi(k_i))
\end{aligned}$$

Supposing $(1 - \|\bar{A}(\theta_k)\|)^{-1}(1 - \|\bar{A}(\theta_k)\|^{k-k_i})\vartheta(\xi(k_i)) \leq \sqrt{\rho}\|\xi(k_i)\|,\ k \in [k_i, k_{i+1})$, we have $\|e(k)\|^2 \leq \rho\|\xi(k_i)\|^2$. So,

$$\begin{aligned}
T_k &\overset{=}{\Delta} k_{i+1} - k_i\\
&= \log_{\|\bar{A}(\theta_k)\|}\left(1 - \frac{\sqrt{\rho}(1 - \|\bar{A}(\theta_k)\|)\|\xi(k_i)\|}{\vartheta(\xi(k_i))}\right).
\end{aligned}$$

Based on the above analysis, self-triggering condition (17) can guarantee that the event-triggering condition (4) will be satisfied. Therefore, Theorem 4.2 can be obtained by Theorem 3.5. ■

Remark 4.1: From the definition of $\vartheta(\xi(k_i)) = \|I - \bar{A}(\theta_k) - \bar{B}(\theta_k)\bar{K}(\theta_k)\|\|\xi(k_i)\|$, we can obtain

$$\begin{aligned}
T_k &= \log_{\|\bar{A}(\theta_k)\|}\left(1 - \frac{\sqrt{\rho}(1 - \|\bar{A}(\theta_k)\|)\|\xi(k_i)\|}{\vartheta(\xi(k_i))}\right)\\
&= \log_{\|\bar{A}(\theta_k)\|}\left(1 - \frac{\sqrt{\rho}(1 - \|\bar{A}(\theta_k)\|)}{\|I - \bar{A}(\theta_k) - \bar{B}(\theta_k)\bar{K}(\theta_k)\|}\right)
\end{aligned}$$

This implies that T_k is only relevant to the scheduling parameter θ_k.

5. Simulation results

Example 5.1: A numerical example is used to illustrate the effectiveness of the proposed strategy relevant to the event-triggered and self-triggered H_∞ output tracking control. Firstly, we evaluate the event-triggered H_∞ output tracking control strategy by two relatively simple cases: the non-zero initial conditions without external disturbances ($w(t) = 0$) and zero initial conditions with external disturbances ($w(t) \neq 0$). We mainly analyse the stability and disturbance attenuation performance. Then, we continue our evaluation in the case of non-zero initial conditions with external disturbances ($w(t) \neq 0$). Finally, we examine the tracking performance of the control system under the self-triggering mechanism. Consider the following polytopic discrete LPV system:

$$x(k+1) = \begin{bmatrix} -2 - \theta_k & 1 & -1\\ 2 & -2 + \theta_k & -1.5\\ -2 & 2(1 - \theta_k) & -2 \end{bmatrix} x(k)$$

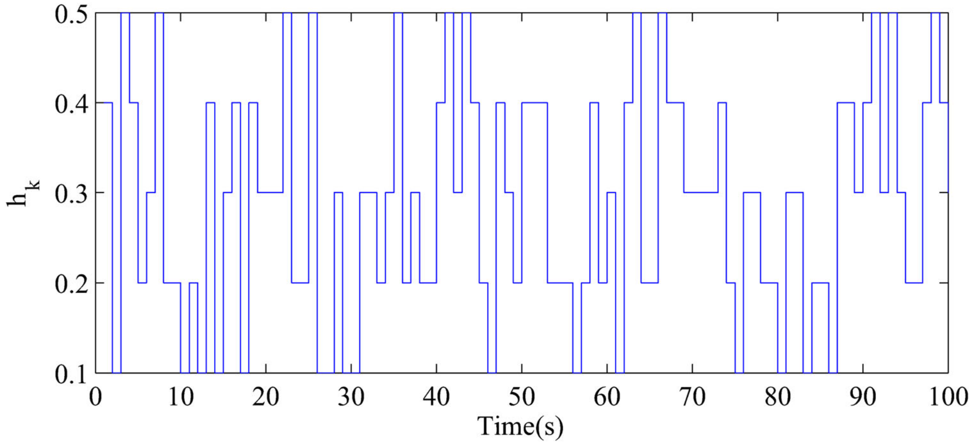

Figure 2. Network-induced η_k.

$$+\begin{bmatrix} -2+\theta_k \\ -1 \\ -\theta_k \end{bmatrix} u(k) + \begin{bmatrix} -1 \\ 1 \\ -0.5 \end{bmatrix} w(k)$$
$$y(k) = [-0.5 \quad 0 \quad -2]x(k) + 0.1w(k)$$

Suppose the reference model is given by

$$x_r(k+1) = -x_r(k) + r(k),$$
$$y_r(k) = 0.5x_r(k).$$

It is assumed that the sampling period $h = 0.1s$, time-varying parameter $\theta_k = |\sin(10k\pi)|$, $0 \leq \theta_k \leq 1$. Network-induced delays are bounded by $\bar{\eta} = 0.5s$ and $\underline{\eta} = 0.1s$. Other parameters are assumed to be $c_1 = 0.1, c_2 = 0.01, \gamma = 0.2$. We may choose $\alpha^1(\theta_k) = 1 - \theta_k$, $\alpha^2(\theta_k) = \theta_k$ and event-triggering parameter $\delta = 0.1$ in (4). The state of the discrete LPV system is initialised to $x_0 = \left[\begin{smallmatrix} 1 \\ -1 \\ 0.5 \end{smallmatrix}\right]$, and the initial state of the reference model is $x_{r0} = -1$. The time-parameter network-induced delays are uniformly distributed within $[\underline{\eta}, \bar{\eta}]$ as shown in Figure 2. In addition, the update rate of the control signal under the ETM is denoted as $f_k = (n_s/n_k) \times 100\%$, where n_s and n_k represent the amount of packets sent and sampled, respectively.

According to the solution conditions in Theorem 3.5, we give the event-triggering conditions and design the state feedback controller. Using MATLAB LMI toolbox, the free weight matrices of the ETM (4) and the gain matrices of the controller (6) are obtained as following:

$$M_{11} = \begin{bmatrix} 0.0424 & -0.0001 & 0.1116 & 0.0278 \\ -0.0001 & 0.0156 & -0.0003 & 0.0000 \\ 0.1116 & -0.0003 & 0.4591 & 0.1111 \\ 0.0278 & 0.0000 & 0.1111 & 0.0416 \end{bmatrix}$$

$$M_{12} = \begin{bmatrix} 0.0424 & -0.0001 & 0.1116 & 0.0280 \\ -0.0001 & 0.0160 & -0.0003 & 0.0000 \\ 0.1116 & -0.0003 & 0.4591 & 0.1111 \\ 0.0280 & 0.0000 & 0.1111 & 0.0416 \end{bmatrix}$$

$$M_{21} = \begin{bmatrix} 3.3998 & -0.0084 & 13.2945 & 3.3190 \\ -0.0084 & 0.0727 & -0.0329 & -0.0077 \\ 13.2945 & -0.0329 & 53.0546 & 13.2301 \\ 3.3190 & -0.0077 & 13.2301 & 3.3630 \end{bmatrix}$$

$$M_{22} = \begin{bmatrix} 3.3400 & -0.0084 & 13.2950 & 3.3190 \\ -0.0084 & 0.0727 & -0.0330 & -0.0077 \\ 13.2950 & -0.0330 & 53.0543 & 13.2300 \\ 3.3190 & -0.0077 & 13.2300 & 3.3630 \end{bmatrix}$$

$$K_1 = [-0.0173 \quad 0.0064 \quad -0.0124 \quad -0.0013]$$
$$K_2 = [0.0112 \quad -0.0030 \quad 0.0062 \quad 0.0009]$$

When there is no external disturbance ($w(t) = 0$), the state responses of the event-triggered tracking control system are shown in Figure 3, indicating that all states of the augmented closed-loop system converge to zero under actions of the proposed ETM. The system is global asymptotically stable. The outputs of the control system and reference system under ETM are shown in Figure 4. It is clear to see from Figure 4 that $y(k)$ tracks $y_r(k)$ well under the non-zero initial conditions without external disturbances ($w(t) = 0$). Figure 5 shows the evolution of the proposed ETM, where $\kappa = e^T(k-\tau_k)M_1(\theta_k)e(k-\tau_k)$

Table 2. The update rates of the control signal with different values of δ under ETM.

δ	0.01	0.05	0.1	0.15	0.2
$M_1(\theta_k) \neq M_2(\theta_k)$ (ETM (4) in this paper)	14.1%	6.4%	4.5%	3.3%	3.2%
$M_1(\theta_k) = M_2(\theta_k)$ (ETM in B. G. Li & Xu, 2013)	48.3%	24.3%	17.3%	14.5%	12.6%

and

$\varsigma = \delta\xi^T(k-\tau_k)M_2(\theta_k)\xi(k-\tau_k)$. Figure 6 shows inter-event interval of ETM. The value of each stem represents the length of the time period between the current event and the previous one, which illustrates a reduction in data transmission.

The update rates of the control signal for different δ under ETM are listed in Table 2. From Table 2, it is clear that the ETM proposed in this paper has a lower percentage of packets transmission than that in B. G. Li and Xu (2013). Meanwhile, as displayed in Table 2, with the increasing of δ, the amount of packets transmission becomes smaller. It is reasonable since a larger δ make it more difficult to be triggered.

Remark 5.1: In order to obtain the update rates of the control signal, two cases for the quadratic positive weighting matrices in ETM are given as (i) the constraints with $M_1(\theta_k) \neq M_2(\theta_k)$, (ii) the constraints with $M_1(\theta_k) = M_2(\theta_k)$. At the same time, the scalar δ is given according to the Table 2 each time. Assuming that the amount of packets sampled is n_k, we know that n_s of them is successfully transmitted by solving Theorem 3.5, and then the update rates of the control signal are got according to $f_k = (n_s/n_k) \times 100\%$.

Assume that the initial state of the augmented system is

$$\xi_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

exogenous disturbance is assumed to be

$$w(k) = \begin{cases} 1.5\sin(5k) & 0 \leqslant k \leqslant 100 \\ 0 & \text{otherwise} \end{cases},$$

the input of the reference model

$$r(k) = \begin{cases} \sin(0.5k) & 0 \leqslant k \leqslant 100 \\ 0 & \text{otherwise} \end{cases}.$$

The state responses of the augmented system are shown in Figure 7, indicating that the system is asymptotically stable again when the disturbance disappears. The outputs of the control system and reference system under ETM are shown in Figure 8, from which we can see that the tracking performance is good. In addition, by calculation, $\sum_{k=0}^{100}\|\bar{e}(k)\|_2 = 12.1924$, $\sum_{k=0}^{100}\|\bar{w}(k)\|_2 = \sum_{k=0}^{100}\|w(k)\|_2 + \sum_{k=0}^{100}\|v(k)\|_2 = 130.1358$, which yields

$$\sum_{k=0}^{100}\|\bar{e}(k)\|_2 \Big/ \sum_{k=0}^{100}\|\overline{w}(k)\|_2 = 0.0937 < \gamma = 0.2$$

showing the effectiveness of the H_∞ tracking controller design. Figure 9 shows inter-event interval of ETM. That indicates ETM effectively reduces the amount of data transmission significantly.

Then, in order to further verify the output tracking performance in the case of non-zero initial conditions with external disturbance ($w(t) \neq 0$), considering the $w(k) = 1.5\sin(5k)$ and reference input $r(k) = \sin(0.5k)$, the output $y(k)$ of the control system and $y_r(k)$ of the given reference system under the actions of ETM are shown in Figure 10. From Figure 10, we can see that though the initial condition is nonzero, the tracking performance is good.

The above simulation results focus on the analysis of the H_∞ output tracking control performance under the actions of ETM, next we will verify the output tracking performance under the self-triggering mechanism. Apply self-triggering mechanism:

$$k_{i+1} = k_i + \max\left\{h, \log_{\|\bar{A}(\theta_k)\|} \times \left(1 - \frac{\sqrt{\rho}(1-\|\bar{A}(\theta_k)\|)\|\xi(k_i)\|}{\vartheta(\xi(k_i))}\right)\right\},$$
$$k \in [k_i, k_{i+1}),$$

where $\delta, M_{11}, M_{12}, M_{21}, M_{22}, K_1, K_2$ are the same as those in the event-triggering mechanism. Assume the sampling period $h = 0.01$ s. Figure 11 shows the inter-event interval under the self-triggered control strategy. From Figure 11, we can see that the average task period is 0.0312 s, the minimum task period is 0.0310 s and the update rate of control signal under the self-triggering mechanism is about 25%. Figure 12 shows the output $y(k)$ of the control system and $y_r(k)$ of the given reference system under self-triggered control strategy.

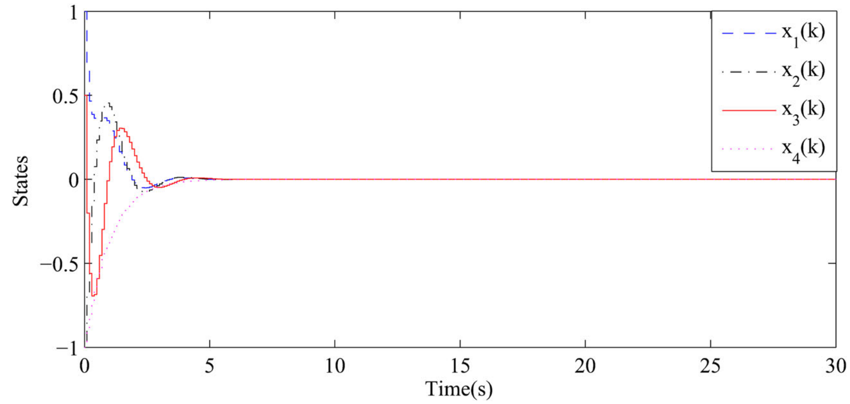

Figure 3. The state response curves of event-triggered tracking control.

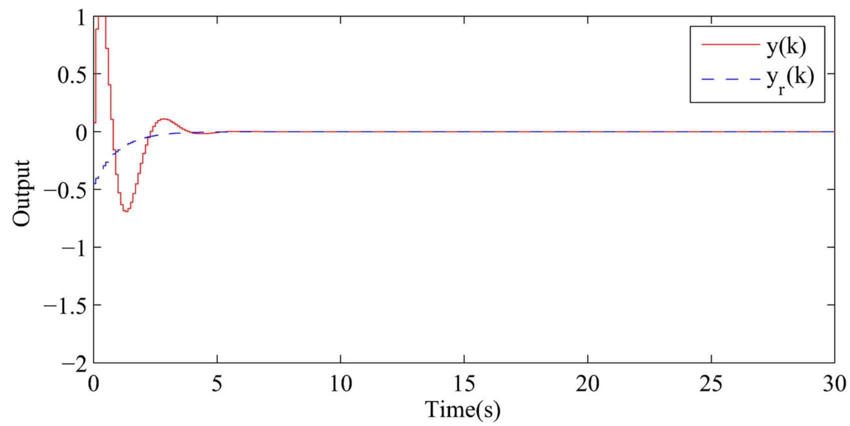

Figure 4. The outputs of the control system and reference system under ETM.

Example 5.2: As shown in Figure 13, we study the application of proposed track strategy to control an angular positioning system via a network remotely, namely, a networked angular positioning system. The system consists of a rotating antenna at the origin of the plane, driven by an electric motor. The control problem is to use the input voltage to the motor to rotate the antenna so that it always points in the direction of a moving object in the plane. The motion of the antenna can be described by the following discrete-time equations obtained from their continuous-time counterparts by discretisation, using a sampling time of 0.1 s and Euler's first-order approximation for the derivative (Han & Feng, 2019):

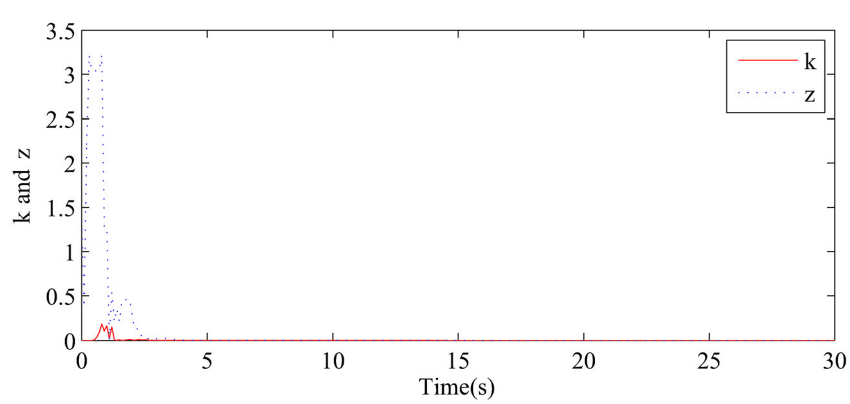

Figure 5. The evolution of the proposed ETM.

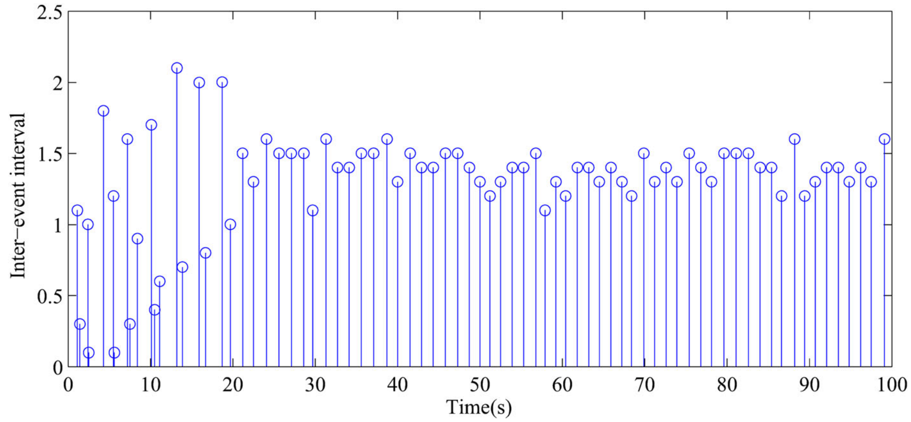

Figure 6. Inter-event interval of ETM.

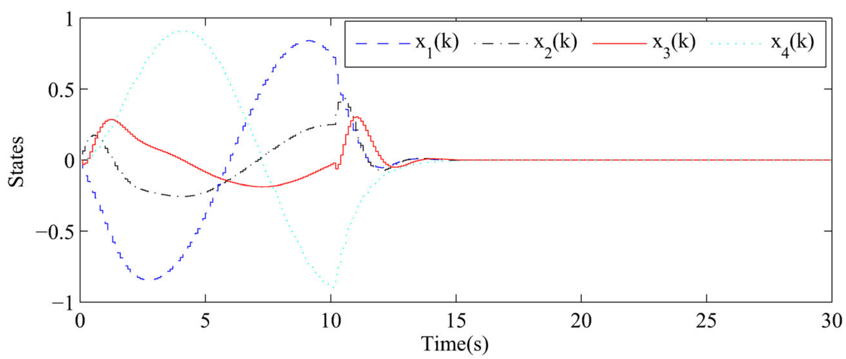

Figure 7. The state response curve of the augmented system.

$$\begin{bmatrix}\theta(k+1)\\ \dot{\theta}(k+1)\end{bmatrix}=\begin{bmatrix}1 & 0.1\\ 0 & 1-0.1\alpha(k)\end{bmatrix}\begin{bmatrix}\theta(k)\\ \dot{\theta}(k)\end{bmatrix}+\begin{bmatrix}0\\ 0.0787\end{bmatrix}u(k)+\begin{bmatrix}0.05\\ -0.1\end{bmatrix}w(k)$$
$$y(k)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\theta(k)\\ \dot{\theta}(k)\end{bmatrix}+0.01w(k),$$

where $\theta(k)$ is the angular position of the antenna, $\dot{\theta}(k)$ is the angular velocity of the antenna and $u(k)$ is the input voltage to the motor. The uncertain parameter $\alpha(k)$ is proportional to the coefficient of viscous friction in the rotating parts of the antenna. It is assumed to be arbitrarily time-varying in the range of $0.1 \leqslant \alpha(k) \leqslant 10$. We conclude that $A(k) \in \Omega$, where Ω is

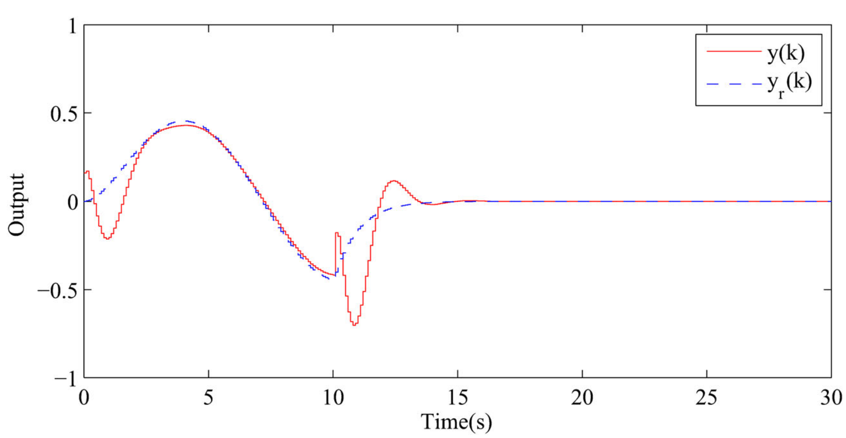

Figure 8. The outputs of the control system and reference system under ETM.

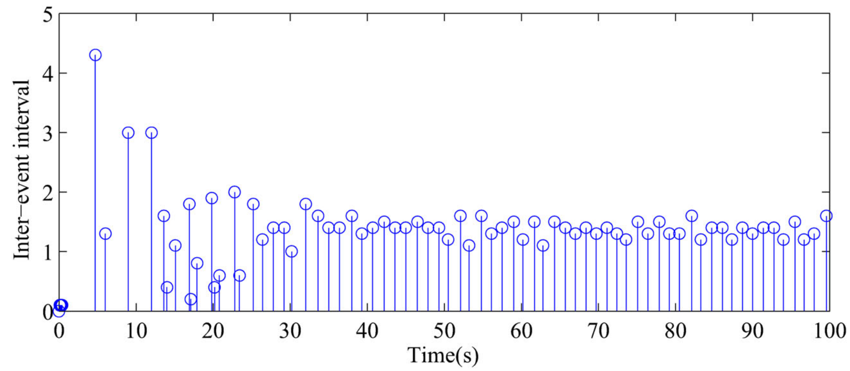

Figure 9. Inter-event interval of ETM.

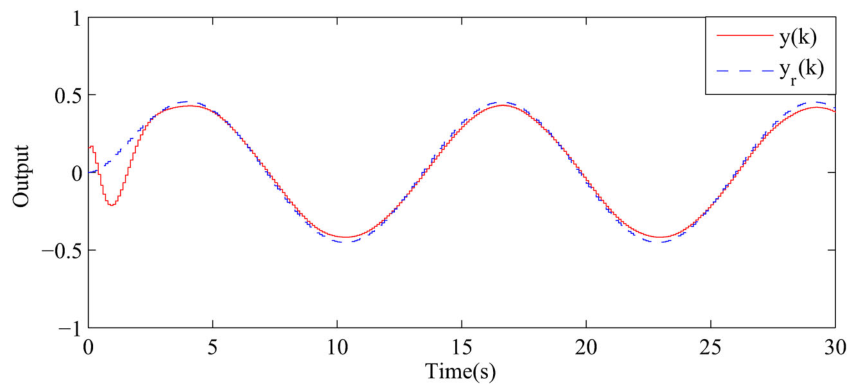

Figure 10. The outputs of the control system and reference system under ETM.

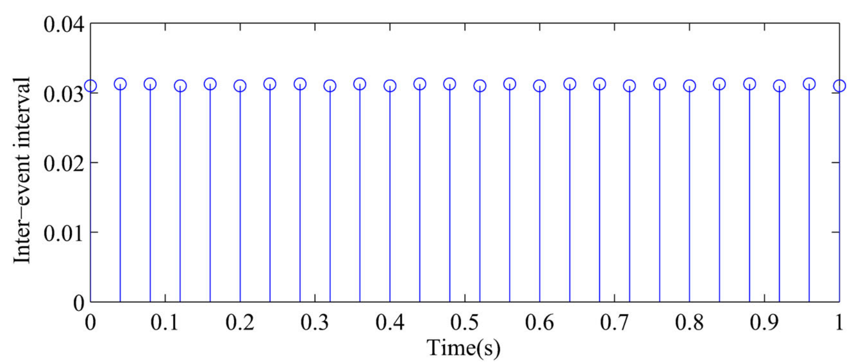

Figure 11. The inter-event interval under the self-triggered control strategy.

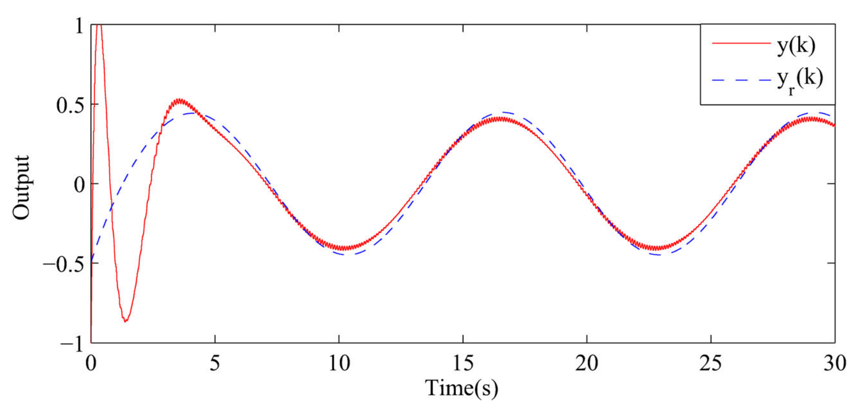

Figure 12. The outputs of the control system and reference system under self-triggered control strategy.

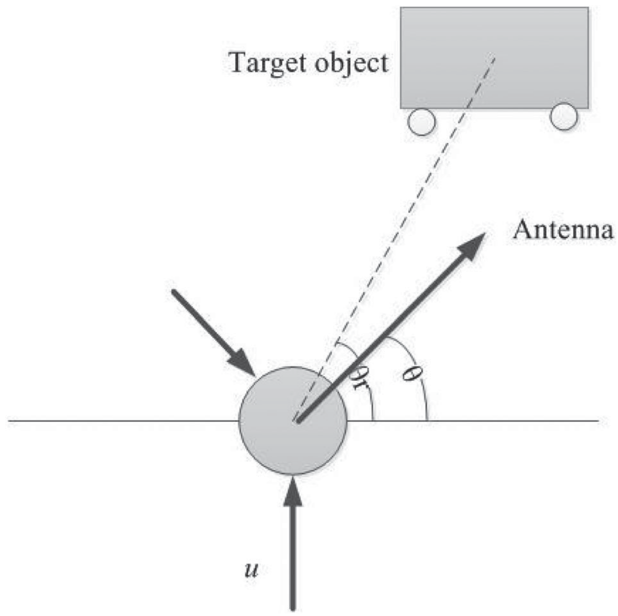

Figure 13. Networked angular positioning system.

given as follows

$$\Omega = Co\left\{\begin{bmatrix} 1 & 0.1 \\ 0 & 0.99 \end{bmatrix}, \begin{bmatrix} 1 & 0.1 \\ 0 & 0 \end{bmatrix}\right\}.$$

The trajectory of moving object to be tracked is given by

$$\theta_r(k+1) = -\theta_r(k) + r(k),$$
$$y_r(k) = \theta_r(k),$$

where $\theta_r(k)$ is the angular position of the moving object.

In the networked angular positioning system, we assume that network-induced delays are bounded by $\bar{\eta} = 3$ s and $\underline{\eta} = 1$ s. Other parameters are assumed to be $\gamma = 90$, $c_1 = 10^{-3}$, $c_2 = 1$ and event-triggering parameter $\delta = 0.1$. The state of the discrete LPV system is initialised to $\theta_0 = \left[\begin{smallmatrix} 0.05 \\ 0 \end{smallmatrix}\right]$ and the initial state of the reference model is $\theta_{r0} = -1$. The exogenous disturbance is assumed to be

$$w(k) = \begin{cases} 0.05\sin(0.01k) & 0 \leqslant k \leqslant 100 \\ 0 & \text{otherwise} \end{cases},$$

the input of the reference model

$$r(k) = \begin{cases} 0.02\sin(0.5k) & 0 \leqslant k \leqslant 100 \\ 0 & \text{otherwise} \end{cases}.$$

Using MATLAB LMI toolbox, the free weight matrices of the ETM (4) and the gain matrices of the controller (6) are obtained as following:

$$M_{11} = \begin{bmatrix} 0.7550 & -0.0001 & -0.1705 \\ -0.0001 & 0.4351 & 0.0000 \\ -0.1705 & 0.0000 & 0.5048 \end{bmatrix},$$

$$M_{12} = \begin{bmatrix} 0.7550 & -0.0001 & -0.1705 \\ -0.0001 & 0.4351 & 0.0000 \\ -0.1705 & 0.0000 & 0.5048 \end{bmatrix},$$

$$M_{21} = \begin{bmatrix} 4.9964 & -0.0007 & -1.6181 \\ -0.0007 & 1.9232 & 0.0002 \\ -1.6181 & 0.0002 & 2.6208 \end{bmatrix},$$

$$M_{22} = \begin{bmatrix} 4.9964 & -0.0007 & -1.6181 \\ -0.0007 & 1.9232 & 0.0002 \\ -1.6181 & 0.0002 & 2.6208 \end{bmatrix},$$

$$K_1 = [0.1348 \quad -0.1816 \quad -0.0106],$$
$$K_2 = [-0.2033 \quad 0.2906 \quad 0.0151]$$

Figure 14 shows inter-event interval of ETM. That indicates ETM effectively reduces the amount of data transmission significantly. Figure 15 depicts the closed-loop responses for networked angular positioning system, one can see that when there is no disturbance, the system will operate close to the origin. The output $y(k)$ of the control system and $y_r(k)$ of the given reference system are shown in Figure 16, indicating that the angular positions θ of the antenna tracks θ_r of the moving object well under the proposed ETM and output-feedback controller.

6. Conclusion

In this paper, we have presented a solution of designing event-triggered and self-triggered H_∞ output tracking controller for discrete LPV systems with network-induced delay. We have constructed the augmented system model through combining with states of the control system and reference system. The relative ETM has been given with the parameter-dependent weigh matrices. Based on above results, we have derived the solution conditions such that the control system can asymptotically track the output of the reference system, and proposed the design algorithm to obtain the control gains and event-triggering parameters simultaneously. Moreover, we have extended the event-triggering condition to self-triggering mechanism, which avoids the additional hardware requirements for the event-triggered control. Finally, by a numerical example, we have demonstrated that the

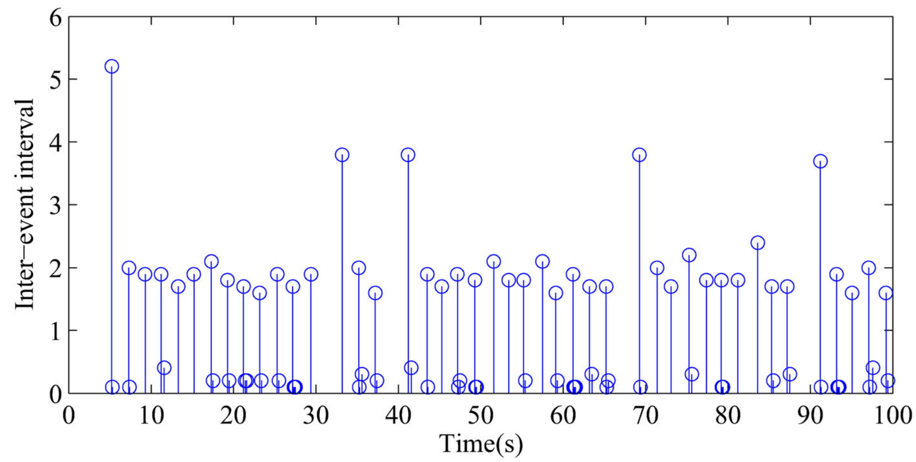

Figure 14. Inter-event interval of ETM.

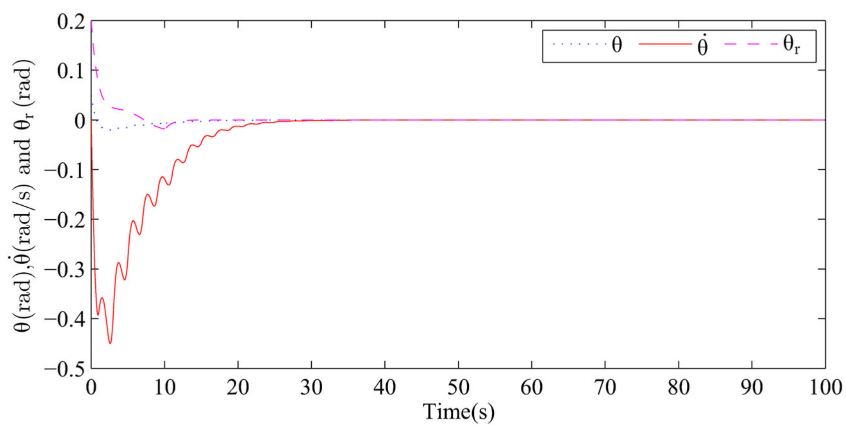

Figure 15. The closed-loop responses for networked angular positioning system.

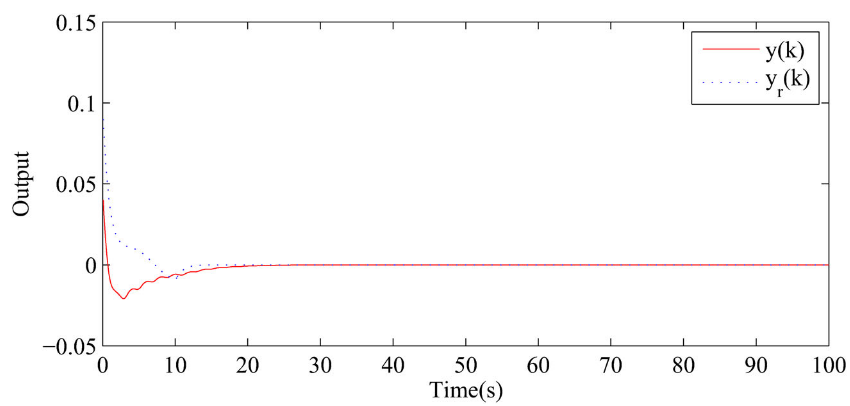

Figure 16. The outputs of the control system and the given reference system.

proposed method can guarantee tracking performance well in H_∞ sense, and reduce the update rate of the control signal to a certain degree. The simulation results have convinced us that the proposed method is effective. How to present a more reasonable event-triggering mechanism based on Lyapunov–Krasovskii functional, and jointly design of event-triggering mechanism and controller to achieve non static error tracking are left for our future study.

Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendix. Proof of Theorem 3.3

Proof: Firstly, we investigate stability of the augmented closed-loop system (7) with $\bar{w}(k) = 0$. Choose the parameter-dependent Lyapunov functional candidates as (A1):

$$V(\xi,\theta,k) = V_1(\xi,\theta,k) + V_2(\xi,k) + V_3(\xi,k), \quad k \in \mathrm{I}_a \tag{A1}$$

$$V_1(\xi,\theta,k) = \xi^T(k)P(\theta_k)\xi(k)$$

$$V_2(\xi,k) = \sum_{l=k-\eta_k}^{k-1} \xi^T(l)Q\xi(l)$$

$$V_3(\xi,k) = \sum_{\theta=-\bar{\eta}}^{-1}\sum_{l=k+\theta}^{k-1} z^T(l)Nz(l) + \sum_{\theta=-\bar{\eta}+1}^{-\underline{\eta}}\sum_{l=k+\theta}^{k-1} \xi^T(l)Q\xi(l)$$

where $P(\theta_k) = \sum_{i=1}^{N}\alpha_i(\theta_k)P_i$, $z(l) = \xi(l+1) - \xi(l)$, P_i, Q, N are positive definite symmetric matrices with appropriate dimensions. When $\bar{w}(k) = 0$ and note that $\xi(k-\eta_k) = \xi(k) - \sum_{l=k-\eta_k}^{k-1} z(l)$, combining (7), it yields (A2):

$$\begin{aligned} 0 = \hat{A}(\theta_k)\xi(k) - z(k) - \bar{B}(\theta_k)\bar{K}(\theta_k)\sum_{l=k-\eta_k}^{k-1} z(l) \\ + \bar{B}(\theta_k)\bar{K}(\theta_k)e(k-\eta_k) \end{aligned} \tag{A2}$$

where $\hat{A}(\theta_k) = \bar{A}(\theta_k) + \bar{B}(\theta_k)\bar{K}(\theta_k) - I$.

$$\begin{aligned} &\Delta V_1(\xi,\theta,k) \\ &= (\xi(k) + z(k))^T P(\theta_{k+1})(\xi(k) + z(k)) \\ &\quad - \xi^T(k)P(\theta_k)\xi(k) \\ &\quad + 2\chi^T(k)U\Bigg(\hat{A}(\theta_k)\xi(k) - z(k) \\ &\quad - \bar{B}(\theta_k)\bar{K}(\theta_k)\sum_{l=k-\eta_k}^{k-1} z(l) + \bar{B}(\theta_k)\bar{K}(\theta_k)e(k-\eta_k)\Bigg) \end{aligned}$$

where $P(\theta_{k+1}) = \sum_{j=1}^{N}\alpha_j(\theta_{k+1})P_j$, $U = \mathrm{col}\{U_1, U_2, U_3\}$, $\chi(k) \triangleq \mathrm{col}\{\xi(k), \xi(k-\eta_k), z(k)\}$. By Lemma 3.2 and ETM (4), we have

$$\begin{aligned} &2\chi^T(k)U\bar{B}(\theta_k)\bar{K}(\theta_k)e(k-\eta_k) \\ &\quad \leqslant \chi^T(k)U\bar{B}(\theta_k)\bar{K}(\theta_k)M_1^{-1}(\theta_k)\bar{K}^T(\theta_k)\bar{B}^T(\theta_k)U^T\chi(k) \\ &\qquad + e^T(k-\eta_k)M_1(\theta_k)e(k-\eta_k) \\ &\quad \leqslant \chi^T(k)U\bar{B}(\theta_k)\bar{K}(\theta_k)M_1^{-1}(\theta_k)\bar{K}^T(\theta_k)\bar{B}^T(\theta_k)U^T\chi(k) \\ &\qquad + \delta\xi^T(k-\eta_k)M_2(\theta_k)\xi(k-\eta_k) \end{aligned}$$

$$\begin{aligned} \Delta V_2(\xi,k) &= \sum_{l=k+1-\eta_{k+1}}^{k} \xi^T(l)Q\xi(l) - \sum_{l=k-\eta_k}^{k-1} \xi^T(l)Q\xi(l) \\ &= \xi^T(k)Q\xi(k) + \sum_{l=k+1-\eta_{k+1}}^{k-1} \xi^T(l)Q\xi(l) \\ &\quad - \xi^T(k-\eta_k)Q\xi(k-\eta_k) \\ &\quad - \sum_{l=k-\eta_k+1}^{k-1} \xi^T(l)Q\xi(l) \end{aligned}$$

Considering $\eta_k < \bar{\eta}$, we have

$$\begin{aligned} &\sum_{l=k+1-\eta_{k+1}}^{k-1} \xi^T(l)Q\xi(l) \\ &\quad = \sum_{l=k+1-\underline{\eta}}^{k-1} \xi^T(l)Q\xi(l) + \sum_{l=k+1-\eta_{k+1}}^{k-\underline{\eta}} \xi^T(l)Q\xi(l) \\ &\quad \leqslant \sum_{l=k+1-\eta_k}^{k-1} \xi^T(l)Q\xi(l) + \sum_{l=k+1-\bar{\eta}}^{k-\underline{\eta}} \xi^T(l)Q\xi(l) \end{aligned}$$

So, $\Delta V_2(\xi,k) \leqslant \xi^T(k)Q\xi(k) - \xi^T(k-\eta_k)Q\xi(k-\eta_k) + \sum_{l=k+1-\bar{\eta}}^{k-\underline{\eta}} \xi^T(l)Q\xi(l)$

$$\begin{aligned} \Delta V_3(\xi,k) &= \left(\sum_{\theta=-\eta}^{-1}\sum_{l=k+1+\theta}^{k} z^T(l)Nz(l) - \sum_{\theta=-\eta}^{-1}\sum_{l=k+\theta}^{k-1} z^T(l)Nz(l)\right) \\ &\quad + \left(\sum_{\theta=-\bar{\eta}+1}^{-\underline{\eta}}\sum_{l=k+1+\theta}^{k} \xi^T(l)Q\xi(l) - \sum_{\theta=-\bar{\eta}+1}^{-\underline{\eta}}\sum_{l=k+\theta}^{k-1} \xi^T(l)Q\xi(l)\right) \\ &= \Bigg((-1-(-\bar{\eta})+1)\, z^T(k)Nz(k) \\ &\quad - \sum_{l=k-\bar{\eta}}^{k-1} z^T(l)Nz(l)\Bigg) \end{aligned}$$

$$
\begin{aligned}
&+\left(\left(-\underline{\eta}-(-\bar{\eta}+1)\right)\xi^T(k)Q\xi(k)\right.\\
&\left.-\sum_{l=k+1-\bar{\eta}}^{k-1}\xi^T(l)Q\xi(l)+\sum_{l=k+1-\underline{\eta}}^{k}\xi^T(l)Q\xi(l)\right)\\
&=\bar{\eta}z^T(k)Nz(k)-\sum_{l=k-\bar{\eta}}^{k-1}z^T(l)Nz(l)\\
&+\left(\bar{\eta}-\underline{\eta}\right)\xi^T(k)Q\xi(k)-\sum_{l=k+1-\bar{\eta}}^{k-\underline{\eta}}\xi^T(l)Q\xi(l)
\end{aligned}
$$

$$
\begin{aligned}
&\Delta V(\xi,\theta,k)\\
&\leqslant(\xi(k)+z(k))^TP(\theta_{k+1})(\xi(k)+z(k))\\
&\quad-\xi^T(k)P(\theta_k)\xi(k)\\
&\quad+2\chi^T(k)U\left(\hat{A}(\theta_k)\xi(k)-z(k)\right)\\
&\quad+\bar{\eta}\chi^T(k)R\chi(k)+\sum_{l=k-\bar{\eta}}^{k-1}z^T(l)Nz(l)\\
&\quad+2\chi^T(k)\left(S-U\bar{B}(\theta_k)\bar{K}(\theta_k)\right)(\xi(k)-\xi(k-\eta_k))\\
&\quad+\chi^T(k)U\bar{B}(\theta_k)\bar{K}(\theta_k)M_1^{-1}(\theta_k)\bar{K}^T(\theta_k)\bar{B}^T(\theta_k)U^T\chi(k)\\
&\quad+\delta\xi^T(k-\eta_k)M_2(\theta_k)\xi(k-\eta_k)+\xi^T(k)Q\xi(k)\\
&\quad-\xi^T(k-\eta_k)Q\xi(k-\eta_k)\\
&\quad+\sum_{l=k+1-\bar{\eta}}^{k-\underline{\eta}}\xi^T(l)Q\xi(l)+\bar{\eta}z^T(k)Nz(k)\\
&\quad-\sum_{l=k-\bar{\eta}}^{k-1}z^T(l)Nz(l)+\left(\bar{\eta}-\underline{\eta}\right)\xi^T(k)Q\xi(k)\\
&\quad-\sum_{l=k+1-\bar{\eta}}^{k-\underline{\eta}}\xi^T(l)Q\xi(l)\\
&\leqslant\chi^T(k)\Xi\chi(k)
\end{aligned}
\tag{A3}
$$

$$
\begin{aligned}
\Xi=&\begin{bmatrix}P(\theta_{k+1})-P(\theta_k)+\left(1+\bar{\eta}+\underline{\eta}\right) & 0 & P(\theta_{k+1})\\ * & \delta M_2(\theta_k)-Q & 0\\ * & * & \bar{\eta}N+P(\theta_{k+1})\end{bmatrix}\\
&+\mathrm{sym}\{U\mathfrak{A}(\theta_k)\}+\mathrm{sym}\{S\Gamma\}+\bar{\eta}R\\
&+U\bar{B}(\theta_k)\bar{K}(\theta_k)M_1^{-1}(\theta_k)\bar{K}^T(\theta_k)\bar{B}^T(\theta_k)U^T
\end{aligned}
$$

where $\mathfrak{A}(\theta_k)=[\bar{A}(\theta_k)-I\ \ \bar{B}(\theta_k)\bar{K}(\theta_k)\ \ -I]$, $\Gamma=[I\ \ -I\ \ 0]$.

Next, $\Xi<-\delta_1^{-1}I$ will be proved. For (8), using Schur complement Lemma, we obtain $\Psi+\delta_1^{-1}F^TF<0$, $F=\mathrm{col}\{F_1,F_2,F_3,F_4\}$, where F is the matrix with appropriate dimension. Hence, it further yields that $\Psi+\delta_1^{-1}I<0$, where $\Psi=\begin{bmatrix}\Xi & U\bar{E}\\ * & -\gamma I\end{bmatrix}+\gamma^{-1}[\hat{C}\ \bar{D}]^{\mathrm{T}}[\hat{C}\ \bar{D}]$, $\hat{C}=[\bar{C}\ 0\ 0]$.

Using Schur complement Lemma again, we have $\Xi+\gamma^{-1}U\bar{E}\bar{E}^TL+\gamma^{-1}[\hat{C}\ \bar{D}]^{\mathrm{T}}[\hat{C}\ \bar{D}]+\delta_1^{-1}I<0$. When $\bar{w}(k)=0$, it yields $\Xi+\delta_1^{-1}I<0$. Taking norm, inequality (A3) then yields

$$
\begin{aligned}
\Delta V(\xi,\theta,k)&\leqslant\chi^T(k)\Xi\chi(k)\\
&\leqslant-\delta_1^{-1}\|\chi(k)\|^2\triangleq-\alpha(\|\chi(k)\|)<0
\end{aligned}
\tag{A4}
$$

When $\bar{w}(k)=0$, (A4) satisfies condition (3) in Lemma 2 in F. Li et al. (2015); S. B. Li et al. (2015), and by parameter-dependent Lyapunov functional (A1), (A4) obviously satisfies condition (1) and (2) in Lemma 2 in F. Li et al. (2015) and S. B. Li et al. (2015). Hence, closed-loop system (7) is global asymptotically stable.

When $\bar{w}(k)\neq 0$ and note that $\bar{\chi}(k)=\mathrm{col}\{\chi(k)\ \bar{w}(k)\}$, then it yields $\|\chi(k)\|\leqslant\|\bar{\chi}(k)\|$, so inequality (A5) holds.

$$
\begin{aligned}
&\Delta V(\xi,\theta,k)+\gamma^{-1}\bar{e}^T(k)\bar{e}(k)-\gamma\bar{w}^T(k)\bar{w}(k)\\
&\leqslant\bar{\chi}^T(k)\Psi\bar{\chi}(k)\leqslant-\delta_1^{-1}\|\bar{\chi}(k)\|^2<0
\end{aligned}
\tag{A5}
$$

Considering zero initial conditions $V(\xi,\theta,k_0)=0$, it yields

$$
\sum_{k=k_0}^{m}(\gamma^{-1}\bar{e}^T(k)\bar{e}(k)-\gamma\bar{w}^T\bar{w}(k)+\Delta V(\xi,\theta,k))\leqslant 0
$$

$$
\begin{aligned}
&\sum_{k=k_0}^{m}\|\bar{e}(k)\|^2\\
&\leqslant\gamma^2\sum_{k=k_0}^{m}\|\bar{w}(k)\|^2-\gamma V(\xi,\theta,m+1)+\gamma V(\xi,\theta,k_0)\\
&\leqslant\gamma^2\sum_{k=k_0}^{m}\|\bar{w}(k)\|^2+\gamma V(\xi,\theta,k_0)\\
&\leqslant\gamma^2\sum_{k=k_0}^{m}\|\bar{w}(k)\|^2
\end{aligned}
$$

■