Proceedings of the 40th Chinese Control Conference
July 26-28, 2021, Shanghai, China

Observer-based Optimal Control for a Class of Impulsive Switched Systems

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Abstract: Optimal switch-time control is the study that investigates how best to switch between different modes. In this paper, we solve an optimal switch-time control problem for a class of impulsive switched systems based on observer design. The considered system undergoes jumps at the switching times and the state is only partially known through the outputs. The control variables consist of the impulse times and a set of scalars which determine the jump amplitudes. We present a method that both guarantees that the current control variables remain optimal as the state estimates evolve, and that can be applied to real-time applications. Moreover, the optimal impulsive control strategy can be found using some numerical gradient descent algorithms. Finally, the viability of the proposed method is illustrated through a numerical example.

Key Words: Optimal control, Impulsive switched systems, Observer, Calculus of variation

1 Introduction

Switched systems are a particular class of hybrid system that consist of continuous dynamics, described by a series of continuous-time or discrete-time subsystems, and discrete dynamics, called switching law specifying the activated subsystem at a certain interval of time. Such systems arise in many engineering fields [1-3]. As its wide applications, the research has penetrated into various branched. The optimal control problem of such systems has also received great attention [4-7].

One special class switched systems, called impulsive switched systems, i.e., the switched systems with impulsive effects, are encountered in a wide range of disciplines, such as orbital transfer of satellites [8], neural networks [9], and many other problems arising in areas such as engineering, economics, and biology [10-12]. For example, in some circuit systems, the circuits' units switching is one of the main factors that can cause abrupt changes of system states in the transmission of signals. The electrical current changes produced by faulty circuit elements can be regarded as impulse events. Therefore, it also has great signification to study the optimal control problem of such systems.

An approach is proposed to solve the optimal timing control problem for autonomous switched systems with pre-specified sequences of active subsystems [13]. In that work, the gradient formula of the cost with respect to the switching times is derived and can be applied in various non-linear programming algorithms to locate the optimal switching instants. Further, based on calculus of variation, a similar problem is considered, and an especially simpler gradient formula is developed, which lends itself to be directly used in some gradient descent algorithms [14]. An optimal impulsive control problem for a switched autonomous delay system is studied [15]. The control variables are the strengths of the impulses and their timing. It is worth noting that this optimal impulsive control problem is related to the optimal switching problem [14, 16]. On the other hand, the problem formulation and results, presented in [15], resemble with the previous work given in [17]. While the former brings more generality to the problem, where the system considered does not require a refractory period, in the sense that once an action is taken, it takes a non-infinitesimal amount of time before a subsequent action can be taken.

However, the problems considered above are all for complete information about the state of the system. For the only partially information known case, the optimal switch-time control problem for single-switch, linear systems is considered [18]. The authors further expand the results to the nonlinear systems case [19]. In this paper, we study the optimal switch-time control problem for the switched impulsive autonomous systems, where the state of the system is only partially known through the outputs. Here, the system also does not require a refractory period. For such systems, the control consists only of a sequence of modulated impulses, i.e., control variables are the impulse times and their magnitudes. Firstly, for the complete state information case, we present a gradient descent-based algorithm for solving the optimal switch-time control problem. Then, we use observer-based control strategy to deal with for the partial state information case.

The remainder of the paper is as follows: In Section 2, the problem is formulated, and the notations are presented. In Section 3, the solution to the complete state information problem is given, and a gradient descent-based algorithm is produced. Moreover, for the partial information case, the strategy of observer-based optimal impulsive control is provided. In Section 4, control variables update and the solution to the observer-based problem are presented. In Section 5, a simulation results are given. Finally, Section 6 states the conclusions and discussions.

2 Problem Formulation

In this paper, the discussed system is modelled by the following autonomous system

*This work is supported by National Natural Science Foundation (NNSF) of China under Grant 61803238 and 61873151.

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$$\dot{x}(t) = f\left(x(t), \xi(t)\right),$$

where $\xi(t)$ is a discrete state, counting the number of impulses.

Let the effect of the impulsive control be modelled by

$$\begin{aligned}\Delta x(T_i) &= x\left(T_i^+\right) - x\left(T_i^-\right) \\ &= G_i\left(x\left(T_i^-\right), u_i, T_i\right), i = \{1, \cdots, N-1\}\end{aligned}, \quad (1)$$

where $N-1$ is the number of jumps, $\{G_i\}_{i=1}^{N-1}$ is a set of given amplitude functions, and the amplitude parameters u_i and the discrete instants T_i are the control variable to be chosen.

As the system dynamics may change because of the impulsive inputs, we let x be given by the following differential equation

$$\dot{x}(t) = f_i\left(x(t)\right), t \in \left(T_{i-1}, T_i\right), i \in \{1, \cdots, N\}, \quad (2)$$

where $x \in \mathbb{R}^n$ is the state of system, $\{f_i\}_{i=1}^{N}$ is the given autonomous vector fields, T_0 and T_N are given initial and final times, and the initial condition is $x(T_0) = x_0$.

Now, equation (2) can be written in a more general form

$$\dot{x}(t) = f_{\xi(t)}\left(x(t)\right), \quad \forall t \in \left(T_0, T_N\right), \quad (3)$$

where discrete state $\xi(t) = i$, if $T_{i-1} \le t \le T_i$.

Then, equations (1) - (3) are the final considered impulsive switched systems with $N-1$ impulsive in this paper. Finally, our objective is to find the control variables parameters $\{T_i\}_{i=1}^{N-1}$ and $\{u_i\}_{i=1}^{N-1}$ such that the performance cost

$$J = \sum_{i=1}^{N} \int_{T_{i-1}}^{T_i} L_i\left(x(t)\right) dt + \sum_{i=1}^{N-1} K_i\left(x\left(T_i^-\right), u_i, T_i\right) + \Phi\left(x\left(T_N\right)\right) \quad (4)$$

is optimized. Here, the added generality of a running cost L may be of more interest. The K_i are discrete costs associated with the control. A terminal cost Φ is also considered, which can be applied to some real control problems, such as reachability problem.

Note that, we may set

$$\Phi\left(x\left(T_N\right)\right) = K_N\left(x\left(T_N\right), 0, T_N\right), \quad (5)$$

thus including the terminal cost in the sum of the control costs.

3 Background

3.1 Complete State Information

In the above optimal switch-time control problem, the discrete instants $\{T_i\}_{i=1}^{N-1}$ and the amplitude parameters $\{u_i\}_{i=1}^{N-1}$ are the control parameters to be designed. Denote them by the $N-1$-dimensional vector $\tau = \left(T_1, \cdots, T_{N-1}\right)^T$ and $U = \left(u_1, \cdots, u_{N-1}\right)^T$ respectively, and note that the cost J is a function of τ and U via equation (3).

Firstly, for the complete state information case, consider the problem of finding control variables τ and U that solves the following optimization problem $\Sigma_1\left(T_0, x_0\right)$:

$$\min_{\tau, U} J\left(T_0, x_0, \tau, U\right) = \sum_{i=1}^{N} \int_{T_{i-1}}^{T_i} L_i\left(x(t)\right) dt + \sum_{i=1}^{N} K_i\left(x\left(T_i^-\right), u_i, T_i\right)$$

$$\text{subject to} \begin{cases} \dot{x}(t) = \begin{cases} f_1\left(x(t)\right), \ t \in \left(T_0, T_1\right) \\ f_2\left(x(t)\right), \ t \in \left(T_1, T_2\right) \\ \vdots \\ f_N\left(x(t)\right), \ t \in \left(T_{N-1}, T_N\right) \end{cases} \\ \begin{cases} \Delta x\left(T_1\right) = G_1\left(x\left(T_1^-\right), u_1, T_1\right) \\ \Delta x\left(T_2\right) = G_2\left(x\left(T_2^-\right), u_2, T_2\right) \\ \vdots \\ \Delta x\left(T_{N-1}\right) = G_{N-1}\left(x\left(T_{N-1}^-\right), u_{N-1}, T_{N-1}\right) \end{cases} \\ x\left(T_0\right) = x_0 \end{cases} \quad (6)$$

In particular, an optimal control problem for a class of switched autonomous system with single time-delay that undergoes jumps at the switching times is solved, and the optimality conditions are also presented using the classical variational methods [15]. To solve the above optimization problem $\Sigma_1\left(T_0, x_0\right)$, i.e., the case of a delay free impulsive switched systems, we borrow the main results and get the requisite necessary conditions by letting $g_i = 0$ in the theorem 3.1 presented in [15].

The necessary optimality conditions for finding the control variables $\{T_i\}_{i=1}^{N-1}$ and $\{u_i\}_{i=1}^{N-1}$ are as follows:

Lemma 1: Given smooth functions $\{f_i\}_{i=1}^{N}$ from $\mathbb{R}^n$ to $\mathbb{R}^n$, $\{L_i\}_{i=1}^{N}$ from $\mathbb{R}^n$ to $\mathbb{R}$, $\{G_i\}_{i=1}^{N}$ and $\{K_i\}_{i=1}^{N}$ from $\mathbb{R}^n \times \mathbb{R} \times \mathbb{R}$ to $\mathbb{R}^n$, a necessary condition for the impulsive switched system with dynamic equations

$$\begin{aligned}&\dot{x}(t) = f_i\left(x(t)\right), t \in \left(T_{i-1}, T_i\right), i \in \{1, \cdots, N\}, \\ &\Delta x(T_i) = G_i\left(x\left(T_i^-\right), u_i, T_i\right), i = \{1, \cdots, N-1\},\end{aligned} \quad (7)$$

to minimize the performance cost

$$J = \sum_{i=1}^{N} \int_{T_{i-1}}^{T_i} L_i\left(x(t)\right) dt + \sum_{i=1}^{N} K_i\left(x\left(T_i^-\right), u_i, T_i\right) \quad (8)$$

is that the control variables $\{T_i\}_{i=1}^{N-1}$ and $\{u_i\}_{i=1}^{N-1}$ satisfy:

Define:

$$\begin{aligned}H_i &= L_i + \lambda^T f_i(x), \\ M_i &= K_i + \mu_i^T G_i.\end{aligned} \quad (9)$$

Euler-Lagrange equations:

$$\dot{\lambda}^T(t) = -\frac{\partial L_i\left(x(t)\right)}{\partial x} - \lambda^T(t) \frac{\partial f_i\left(x(t)\right)}{\partial x}. \quad (10)$$

Boundary conditions:

$$\begin{aligned}&\lambda^T\left(T_i^-\right) = \lambda^T\left(T_i^+\right) + \frac{\partial M_i}{\partial x}, \quad i = 1, \cdots, N, \\ &\lambda^T\left(T_N\right) = 0.\end{aligned} \quad (11)$$

Multipliers:

$$\mu_i^T = \lambda^T\left(T_i^+\right), \quad i = 1, \cdots, N. \quad (12)$$

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Optimality conditions:

$$\frac{\partial J}{\partial T_i} = \frac{\partial M_i}{\partial T} + H_i\left(T_i^-\right) - H_{i+1}\left(T_i^+\right) = 0, \qquad (13)$$

$$\frac{\partial J}{\partial u_i} = \frac{\partial M_i}{\partial u} = 0. \qquad (14)$$

Analytic solutions to equations (13) - (14) may be quite hard to achieve. Instead, the expressions for the partial derivatives of the cost J can be used in some numerical gradient descent algorithms. Thus, in the case where the complete state information is available, we can easily produce the following gradient descent-based algorithm for finding the optimal control variables, e.g.:

Algorithm 1:

$T_i = T_{i0}, u_i = u_{i0}, \quad i = 1, \cdots, N-1$ (Initial guess)

repeat

solve for $x(t), t \in [T_0, T_N]$ forward in time;

solve for $\lambda(t), t \in [T_0, T_N]$ backward in time;

compute the partial derivatives $\frac{\partial J}{\partial \tau} = \left[\frac{\partial J}{\partial T_1}, \cdots, \frac{\partial J}{\partial T_N}\right]^T$ and $\frac{\partial J}{\partial U} = \left[\frac{\partial J}{\partial u_1}, \cdots, \frac{\partial J}{\partial u_N}\right]^T$ with

$$\frac{\partial J}{\partial T_i} = \frac{\partial M_i}{\partial T} + H_i\left(T_i^-\right) - H_{i+1}\left(T_i^+\right), \ \frac{\partial J}{\partial u_i} = \frac{\partial M_i}{\partial u};$$

updates $T_i := T_i - \gamma_2 \frac{\partial J}{\partial T_i}$ and $u_i := u_i - \gamma_1 \frac{\partial J}{\partial u_i}$;

until $\left\|\frac{\partial J}{\partial \tau}\right\| \le \varepsilon_2$ and $\left\|\frac{\partial J}{\partial U}\right\| \le \varepsilon_1$.

Here $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ are the termination threshold, γ_1 and γ_2 are the step length. Note that γ_1 and γ_2 could possibly be varying, e.g., using the Armijo step size [20].

3.2 Partial State Information

Then, we now turn our attention to a slightly different problem, namely the problem to find the optimal control variable $\{T_i\}_{i=1}^{N-1}$ and $\{u_i\}_{i=1}^{N-1}$ when only partial information about the state is available. By this we understand that only $y(t) \in R^p$ (and not $x(t)$) is known, where

$$y(t) = \begin{cases} h_1\left(x(t)\right), t \in \left(T_0, T_1\right) \\ h_2\left(x(t)\right), t \in \left(T_1, T_2\right) \\ \vdots \\ h_N\left(x(t)\right), t \in \left(T_{N-1}, T_N\right) \end{cases}, \qquad (15)$$

$$\begin{cases} \Delta y\left(T_1\right) = D_1\left(y\left(T_1^-\right), u_1, T_1\right) \\ \Delta y\left(T_2\right) = D_2\left(y\left(T_2^-\right), u_2, T_2\right) \\ \vdots \\ \Delta y\left(T_{N-1}\right) = D_{N-1}\left(y\left(T_{N-1}^-\right), u_{N-1}, T_{N-1}\right) \end{cases}$$

and $\{h_i\}_{i=1}^N : \mathbb{R}^n \to \mathbb{R}^p$ are the continuously differentiable output functions, and $\{D_i\}_{i=1}^{N-1}$ are the given amplitude functions.

The strategy that we will use is to guess the initial state value $\hat{x}_0$, and then solve the optimal impulsive control problem for this initial state using Algorithm 1, resulting in the optimal control variables $\hat{\tau}\left(T_0\right) = \left\{\hat{T}_{i0}\right\}_{i=1}^{N-1}$ and $\hat{U}\left(T_0\right) = \left\{\hat{u}_{i0}\right\}_{i\ 1}^{N-1}$. The idea is that this computation can be performed off-line, i.e., before the system actually starts evolving. Once this happens, we will use an observer to estimate the state.

Moreover, the main idea is to update $\hat{\tau}(t)$ and $\hat{U}(t)$ in such a way that for all times $t \in [T_0, T_N]$, $\hat{\tau}(t)$ and $\hat{U}(t)$ are optimal given the current state estimate $\hat{x}(t)$; and the time evolution of them must be computational reasonable. That is to say, we should only pay high computational price for the initial state estimate guess.

Thus, the strategy is given as follows:

(i) Guess an initial state value $\hat{x}_0$, using Algorithm 1, resulting in the optimal variables $\hat{\tau}\left(T_0\right)$ and $\hat{U}\left(T_0\right)$.

(ii) Use an observer to estimate the state.

4 Observer-based Optimal Impulsive Control

This section is concerned with the problem of solving $\Sigma_1\left(t, \hat{x}(t)\right)$, where $\hat{x}(t)$ is the state of the observer presented in the following.

4.1 Control Variables Update

Let us consider the dynamics of the optimal control variables for the current state estimate $\hat{x}(t)$. For this, we assume that we have been able to compute $\hat{\tau}\left(T_0\right)$ and $\hat{U}\left(T_0\right)$ as the solution to $\Sigma_1\left(T_0, \hat{x}_0\right)$ using Algorithm 1, where $\hat{x}_0$ is the initial state estimate.

In this paper, we will make the explicit assumption that $\hat{\tau}(t)$ and $\hat{U}(t)$ are a local minimum to $\Sigma_1\left(t, \hat{x}(t)\right)$ for all $t \in [T_0, T_N]$, and hence that

$$\begin{pmatrix} \frac{\partial^2 M_i}{\partial T^2} & \frac{\partial^2 M_i}{\partial u \partial T} \\ \frac{\partial^2 M_i}{\partial T \partial u} & \frac{\partial^2 M_i}{\partial u^2} \end{pmatrix} > 0 \qquad (16)$$

holds.

By computing

$$\frac{d}{dt}\left(\frac{\partial J}{\partial u_i}\right) = 0, \quad \frac{d}{dt}\left(\frac{\partial J}{\partial T_i}\right) = 0, \qquad (17)$$

we obtain

$$\dot{\hat{T}}_i(t) = -\frac{\frac{\partial^2 M_i}{\partial u^2}}{\det\begin{pmatrix} \frac{\partial^2 M_i}{\partial T^2} & \frac{\partial^2 M_i}{\partial u \partial T} \\ \frac{\partial^2 M_i}{\partial T \partial u} & \frac{\partial^2 M_i}{\partial u^2} \end{pmatrix}} \begin{pmatrix} \left(1 - \left(\frac{\frac{\partial^2 M_i}{\partial u \partial T}}{\frac{\partial^2 M_i}{\partial u^2}}\right)\right)\left(\frac{\partial^2 M_i}{\partial x \partial T}\right)^T \dot{x}\left(T_i^-\right) + \\ \left(\frac{\partial H_i}{\partial x}\right)^T \dot{x}\left(T_i^-\right) - \left(\frac{\partial H_{i+1}}{\partial x}\right)^T \dot{x}\left(T_i^+\right) \end{pmatrix}, \qquad (18)$$

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$$\dot{\hat{u}}_i(t) = -\frac{1}{\frac{\partial^2 M_i}{\partial u^2}}\left(\left(\frac{\partial^2 M_i}{\partial x \partial u}\right)^T \dot{x}\left(T_i^-\right) + \frac{\partial^2 M_i}{\partial T \partial u}\dot{\hat{T}}_i(t)\right). \quad (19)$$

As equation (16) holds, equations (18) - (19) are well-defined. From the solution process of equations (18) - (19), as long as $\partial J/\partial u_i = 0$ and $\partial J/\partial T_i = 0$ initially, it will remain zero, and both $\hat{\tau}(t)$ and $\hat{U}(t)$ will in fact remain optimal (i.e. optimality), which means that the current switch time remains optimal as the state estimates evolve.

4.2 Observers Design

The observers of this optimization strategy should provide as good estimates of the state as possible, i.e. the estimated state should quickly converge to the real system state.

Specifically, if the impulsive switched system is composed of linear subsystems, we can use a standard switched-type Luenberger observer [18]. For the impulsive switched systems with nonlinear subsystems, the Grizzle-Moraal Newton observer can be employed [21].

5 Introduction

Consider the following impulsive switched systems with one switching time defined by

$$\dot{x}(t) = \begin{cases} -x(t), t \in (0, T_1) \\ x(t), t \in (T_1, 1) \end{cases}$$

$$\Delta x(T_1) = u$$

with performance cost

$$J = \frac{1}{2}\int_0^1 x^2(t)\,dt + \frac{u^2}{T_1}.$$

For the complete state information case, the Algorithm 1 is applied to minimize the cost J. We choose the step length $\gamma_1 = \gamma_2 = 0.1$, termination values $\varepsilon_1 = \varepsilon_2 = 0.01$, initial control variables $T_{10} = 0.3, u_0 = 0$, and initial condition $x(0) = 0.55$. After 20 iterations the cost quickly converges to a minimum value. The result is shown in Fig. 1.

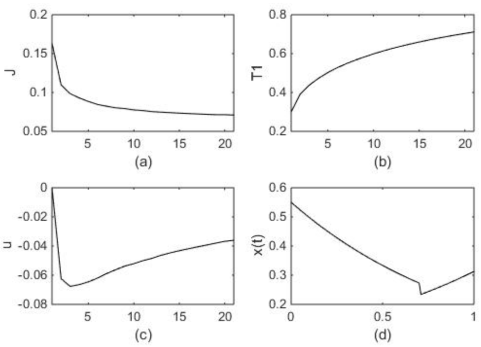

Fig. 1: (a) The performance cost J; (b) The switching time T_1; (c) The jump amplitude u; (d) The state trajectories of the systems at the last iteration of the algorithm

For the partial state information case, the output $y(t)$ is known, where

$$y(t) = \begin{cases} x(t), t \in (0, T_1) \\ x(t), t \in (T_1, 1) \end{cases}$$

$$\Delta y(T_1) = u$$

The observer used here is

$$\dot{\hat{x}}(t) = \begin{cases} -\hat{x}(t) - k_1(\hat{x}(t) - y(t)), t \in (0, T_1) \\ \hat{x}(t) - k_2(\hat{x}(t) - y(t)), t \in (T_1, 1) \end{cases}.$$

We choose observer gains k_1, k_2 such that the poles of the closed loop system are {-6, -6}. We choose initial condition $\hat{x}(0) = 0.6$, which is close to the true initial state. As the observer state approaches to the real state, the control variables converge to their true optimal values. Fig. 2 shows the state and observer trajectories.

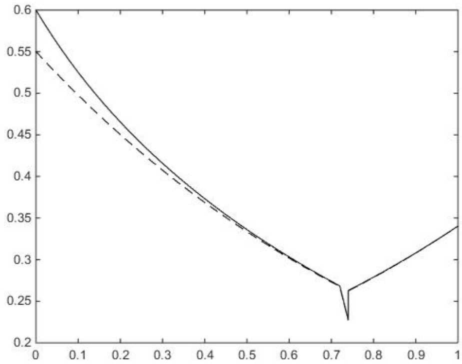

Fig. 2: State and observer trajectories (solid line and dotted line)

6 Conclusions

In this paper, when the state of the system is only partially known through the outputs, the optimal switch-time control problem for the impulsive switched system has been studied. The necessary optimality conditions were given and an algorithm for solving such problem was presented. Also, a method was given that both guarantees that the current control variables remain optimal as the state estimates evolve, and that ensures this in a computationally feasible manner. Meanwhile, the non-autonomous switched impulsive systems need considering in the future.

References

[1] D. Liberzon, *Switching in Systems and Control*, Boston: Birkhaser, 2003.

[2] P. Zhao, S. Mohan, and R. Vasudevan, Optimal control of polynomial hybrid systems via convex relaxations, *IEEE Transactions on Automatic Control*, 65(5): 2062-2077, 2020.

[3] B. Niu, D. Wang, H. Li, et al, A novel neural-network-based adaptive control scheme for output-constrained stochastic switched nonlinear systems, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 49(2): 418-432, 2019.

[4] W. Xu, Z. G. Feng, G. H. Lin, et al, Optimal switching of switched systems with time delay in discrete time, *Automatica*, 112: 108696, 2020.

[5] J. H. Yuan, C. Z. Wu, J. X. Ye, et al, Robust identification of nonlinear state-dependent impulsive switched system with switching duration constraints, *Nonlinear Analysis: Hybrid Systems*, 36: 100879, 2020.

Authorized licensed use limited to: Carleton University. Downloaded on March 29,2023 at 13:24:23 UTC from IEEE Xplore. Restrictions apply.

[6] X. M. Liu, S. T. Li, and K. J. Zhang, Optimal control of switching time in switched stochastic systems with multi-switching times and different costs, *International Journal of Control*, 90(8): 1604-1611, 2017.

[7] X. M. Liu, K. J. Zhang, S. T. Li, et al, Time optimization problem for switched stochastic systems with multi-switching times, *IET Control Theory and Applications*, 8(16): 1732-1740, 2014.

[8] Y. Masutani, M. Matsushita, and F. Miyazaki, Flyaround Maneuvers on a satellite orbit by impulsive thrust control, In *Proceedings of IEEE International Conference on Robotics and Automation*, Seoul, Korea, 2001: 421-426.

[9] Q. Song and J. Cao, Impulsive effects on stability of fuzzy cohen-grossberg neural networks with time-varying delays, *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 37(3): 733-741, 2007.

[10] E. I. Verriest, F. Delmotte, and M. Egerstedt, Control of epidemics by vaccination, In *Proceedings of American Control Conference*, Portland, OR, USA, 2005: 985-990.

[11] X. D. Li, P. Li, and Q. G. Wang, Input/output-to-state stability of impulsive switched systems, *Systems & Control Letters*, 116: 1-7, 2018.

[12] H. L. Xu, K. L. Teo, and X. Z. Liu, Robust stability analysis of guaranteed cost control for impulsive switched systems, *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 38(5): 1419-1422, 2008.

[13] X. P. Xu and P. J. Antsaklis, Optimal control of switched autonomous systems, In *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, Nevada USA, 2002: 4401-4406.

[14] M. Egerstedt and Y. Wardi, F, Delmotte, Optimal control of switching times in switched dynamical systems, In *Proceedings of the 42nd IEEE Conference on Decision and Control*, 2003: 2138-2143.

[15] E. I. Verriest, F. Delmotte, and M. Egerstedt, Optimal impulsive control of delay systems, *ESAIM-Control, Optimisation and Calculus of Variations*, 14(4): 767-779, 2008.

[16] E. I. Verriest, Regularization method for optimally switched and impulse systems with biomedical applications, In *Proceedings of the 42st IEEE Conference on Decision and Control*, 2003: 2156-2161.

[17] E. I. Verriest, F. Delmotte, and M. Egerstedt, Optimal impulsive control for point delay systems with refractory period, In *IFAC Workshop on Time-Delay Systems, Leuven, Belgium*, 2004.

[18] M. Egerstedt, S. Azuma, and Y. Wardi, Optimal timing control of switched linear systems based on partial information, *Nonlinear Analysis: Theory, Methods and Applications*, 65: 1736-1750, 2006.

[19] S. Azuma, M. Egerstedt, and Y. Wardi, Output-based optimal timing control of switched systems, In *Proceedings of Hybrid Systems: Computation and Control*, 2006: 64-78.

[20] L. Armijo, Minimization of functions having lipschitz continuous first-partial derivatives, *Pacific Journal of Mathematics*, 16: 1-3, 1966.

[21] P. E. Moraal and J. W. Grizzle, Observer design for nonlinear systems with discrete-time measurements, *IEEE Trans. on Automatic Control*, 40(3): 395-404, 1995.

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