Dynamic Matrix Control Used in Stabilizing Aircraft Landing

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***Abstract*—This paper analyzes the possibility of an improved landing stabilization by using Dynamic Matrix Control. In this purpose a new control structure is proposed based on the DMC algorithm. The model used to illustrate the structure is that of a stabilized aircraft during its last phase of landing. Simulations are performed for a variety of design parameters (prediction horizons, control horizons and control action weighting factors) for a desired landing reference trajectory and also for a descending speed reference trajectory. One of the key advantages of the structure is that it ensures a smooth transition of the control surfaces involved in the aircraft movement and provides protection for the corresponding actuators. Overall, the proposed structure ensures an improved aircraft control during landing while providing increased safety for the actuators and control surfaces.**

Keywords— Model predictive control, aircraft attitude control, DMC, flight control

I. Introduction

The Dynamic Matrix Control (DMC) algorithm is one of the most popular MPC algorithms. Invented by Cutler and Ramaker [2], it has been used for a large number of applications over the years. DMC uses a linear step response model and an objective function given in quadratic form which is minimized over a given prediction horizon in order to compute the output of the optimal controller.

Numerous versions of the original DMC formulations have been proposed in the last few years. For example, in [3] an adaptive control strategy based on multiple DMC models is proposed. Another illustrative example is the extension of the classical DMC formulation to nonlinear systems as can be seen in [5].

More recent applications of DMC include adaptive DMC with interpolated parameters [7] and applications regarding embedded real-time systems [10].

There have also been detailed studies about the prediction horizon influence in DMC as can be seen in [4].

Regarding applications in aircraft control, DMC has been applied in a limited number of cases. For example, an application regarding guidance navigation and control can be found in [8]. A more complex application implies self-tuning DMC of two-axis autopilot for small airplanes as can be seen in [9].

Also, an alternative GPC control structure applied with regard to the aircraft automated flight control system can be seen in [1].

Given the limited number of DMC applications in aircraft control, in this paper we propose a new DMC based control structure for landing stabilization. The purpose of this structure is to ensure an improved control of the aircraft landing and to provide a smoother transition of the aircraft control surfaces and the corresponding actuators.

With this in mind, the proposed control problem is studied on the model of an aircraft during the last phase of flight.

II. Models and Methods

A. The Model of the Aircraft during its Last Phase of Landing

As described before, the purpose of this paper is to design a DMC based structure to stabilize the motion of the aircraft during landing.

In this purpose, the model used is that of an aircraft during the last phase of landing as presented in [6] which can be seen in Fig. 1:

$$\begin{cases} \dot{\omega}_y(t) = -0.6\omega_y(t) - 0.76\Theta(t) + 0.003w(t) + 2.34u_p(t) \\ \dot{\Theta}(t) = \omega_y(t) \\ \dot{w}(t) = 102.4\Theta(t) - 0.4w(t) \end{cases} \quad (1)$$

Where the following notations were introduced:

- $\omega_y(t)$ - the pitch angle rate
- $\Theta(t)$ - the theta angle
- w(t) – the altitude rate
- $u_p(t)$ - the actuator command on the horizontal rudder

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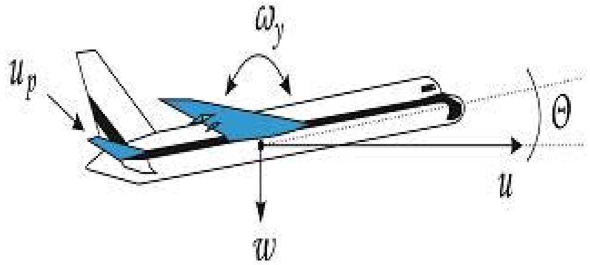

Fig.1. The aircraft during the last phase of landing

The mathematical model includes the actuator used in the control of the horizontal rudder. For a correct landing procedure, the flight instructions must respect the following principles with regard to the type of the aircraft which is considered, [6]:

- The plane is at an altitude of 30 m and has a horizontal speed of 77 m/s

- The roll and yaw movements are stabilized

- The initial descending speed (until the altitude of 30 m) is $w_0 = -6m/s$

- The Θ angle must not exceed 2°.

- The descending speed must not exceed 0.6 m/s at the exact moment in which the plane touches the runway, otherwise the landing gear can be damaged.

- The landing time is 20 sec

- The landing trajectory (in a vertical frame) is:

$$h(t) = \begin{cases} 30e^{-t/5}, 0 \le t \le 15 \\ 6 - t, 15 < t \le 20 \end{cases} \quad (2)$$

- The descending speed reference is:

$$w(t) = \dot{h}(t) = \begin{cases} -6e^{-t/5}, 0 \le t \le 15 \\ -1, 15 < t \le 20 \end{cases} \quad (3)$$

The mathematical model describing the aircraft during this flight phase reveals an unstable object, fact which is demonstrated by the eigenvalues:

$$\begin{aligned} \lambda_1 &= -0.5016 + j0.8670 \\ \lambda_2 &= -0.5016 - j0.8670 \\ \lambda_3 &= 0.0032 \end{aligned} \quad (4)$$

In the control structure proposed in [6] a partial state feedback is introduced (ω_y and Θ) and a feedback for the output w. This is shown in Fig. 2 below.

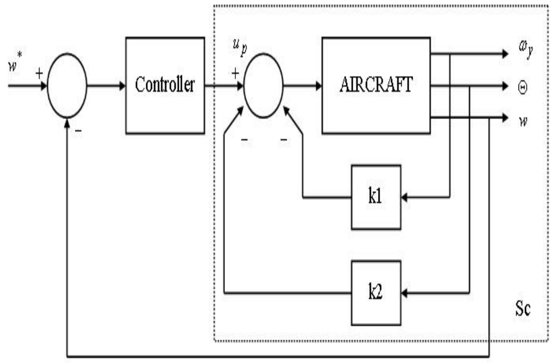

Fig.2. Control system with a partial state feedback (ω_y and Θ) and a feedback loop for the w output with S_c representing the compensated system as presented in [6]

A feedback loop was introduced with regards to the state variables, with the feedback matrix K being calculated with the LQR algorithm, [6]:

$$J = \int_0^\infty \left[x^T(t)Qx(t) + u^T(t)Vu(t) \right] dt \quad (5)$$

$$K = V^{-1}B^T P \quad (6)$$

Where the P matrix represents the solution of the Ricatti algebraic equation:

$$PA + A^T P - PBV^{-1}B^T P + Q = 0 \quad (7)$$

Imposing the eigenvalues of the compensated system to be:

$$\begin{aligned} \lambda_1^* &= \text{-}8.1072 + j7.4184 \\ \lambda_2^* &= \text{-}8.1072 - j7.4184 \\ \lambda_3^* &= -0.3973 \end{aligned} \quad (8)$$

The feedback matrix K has the following form:

$$K = [6.5761 \quad 50.5295 \quad 0] \quad (9)$$

The state space representation is:

$$A = \begin{bmatrix} -0.6 & -0.76 & 0.003 \\ 1 & 0 & 0 \\ 0 & 102.4 & -0.4 \end{bmatrix} \quad (10)$$

$$B = \begin{bmatrix} 2.374 \\ 0 \\ 0 \end{bmatrix} \quad C = [0 \quad 0 \quad 1] \quad D = 0$$

Given the state space representation, an equivalent model can be calculated for the compensated system:

$$A_e = A - BK \tag{11}$$

This equivalent model is used in the calculations for the proposed control problem.

B. The Proposed Control Structure and Simulation Results

The proposed control structure, using the DMC controller is shown in Fig. 3:

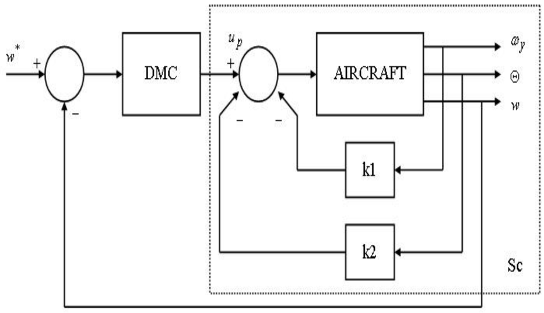

Fig. 3. Control system with a partial state feedback (ω_y and Θ) and a feedback loop for the w output with S_c representing the compensated system using the DMC controller

To analyze the effects of the proposed structure, a series of simulations are carried out to show the advantages of applying this type of control structure and the influence of the MPC parameters.

Since one of the purposes of the control structure is to ensure a more stable command of the aircraft control surfaces, we analyze the theta angle response for a step variation of the reference in Fig. 4:

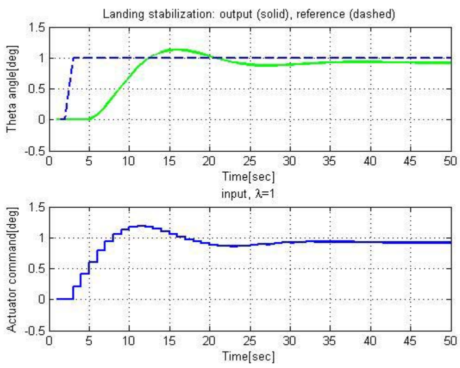

Fig. 4. The theta angle using the landing stabilization DMC structure proposed for $N_p = 5$, $N_c = 1$, given a step variation of the reference

The response can be slower, but prevents the apparition of strong oscillations which can damage the control surfaces.

A less satisfactory response can be seen in Fig. 5. due to increased control and prediction horizons ($N_p = 6$ and $N_c = 2$):

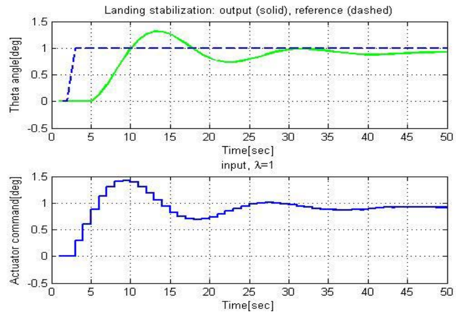

Fig. 5. The theta angle using the landing stabilization DMC structure proposed for $N_p = 6$, $N_c = 2$, given a step variation of the reference

For comparison, a classical PI controller was designed with parameters selected in order to compensate the λ_3^* pole, for which the time response is presented in Fig. 6:

$$H(s) = \frac{0.9(s + 0.3973)}{s} \tag{12}$$

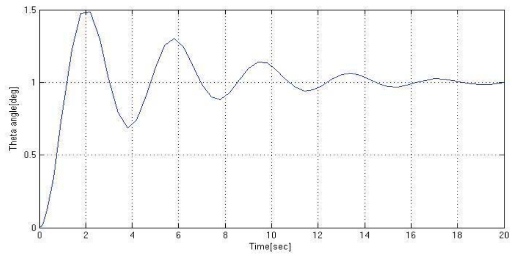

Fig. 6 Step response for the pitch attitude control system using a classical PID controller

Given the landing trajectory in a vertical frame from (2), the following simulations are performed for the proposed DMC structure, as per Table I:

TABLE I. SIMULATION PARAMETERS- LANDING REFERENCE TRAJECTORY

	Prediction Horizon	***Control Horizon***	***λ - control action weight***
1.	$N_p = 5$	$N_c = 1$	1
2.	$N_p = 4$	$N_c = 1$	1
3.	$N_p = 4$	$N_c = 1$	0.7
4.	$N_p = 6$	$N_c = 2$	1
5.	$N_p = 6$	$N_c = 2$	1
6.	$N_p = 6$	$N_c = 1$	0.7

For lines 2 and 5 of table 1, the DMC algorithm can predict also the future reference.

Analyzing the landing stabilization structure proposed we can see the influence of the design parameters. For example, in Fig. 7, for $N_p = 5$ and $N_c = 1$, the proposed structure ensures the accurate control of the aircraft during the last phase of landing.

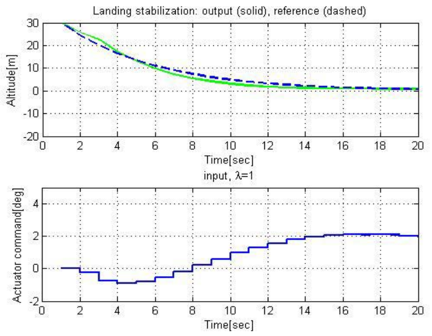

Fig. 7. The last phase of landing using the proposed DMC structure for $N_p = 5$ and $N_c = 1$

In Fig. 8, the prediction and control horizons are $N_p = 4$ and $N_c = 1$ and the DMC algorithm also predicts the future reference.

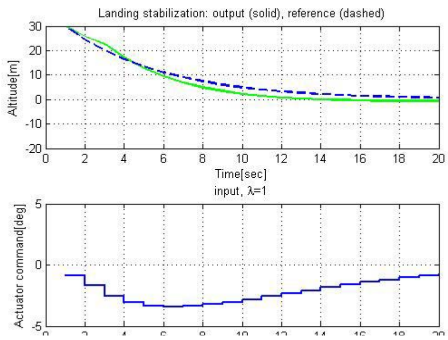

Fig. 8. The last phase of landing using the proposed DMC structure for $N_p = 4$ and $N_c = 1$ and the prediction of the future reference

Modifying the weighting factor of the control action, $\lambda = 0.7$, for a prediction and control horizon of $N_p = 4$ and respectively $N_c = 1$, we obtain the response from Fig. 9:

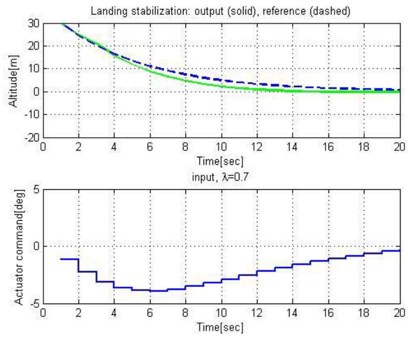

Fig. 9. The last phase of landing using the proposed DMC structure for $N_p = 4$, $N_c = 1$ and $\lambda = 0.7$

Increasing the control and prediction horizons, we obtain the less satisfactory responses from Fig. 10 ($N_p = 6$ and $N_c = 2$), Fig. 11 ($N_p = 6$ and $N_c = 2$ with reference prediction) and respectively Fig. 12 ($N_p = 6$, $N_c = 1$ and $\lambda = 0.7$):

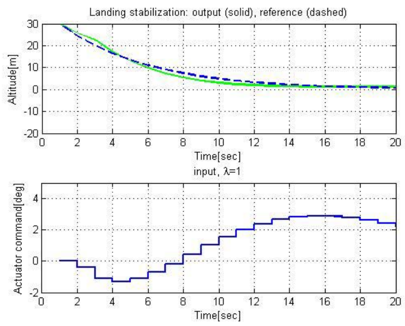

Fig. 10. The last phase of landing using the proposed DMC structure for $N_p = 6$, $N_c = 2$

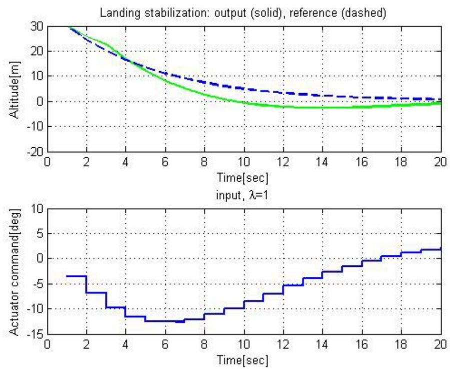

Fig. 11. The last phase of landing using the proposed DMC structure for $N_p = 6$, $N_c = 2$ and prediction of the future reference

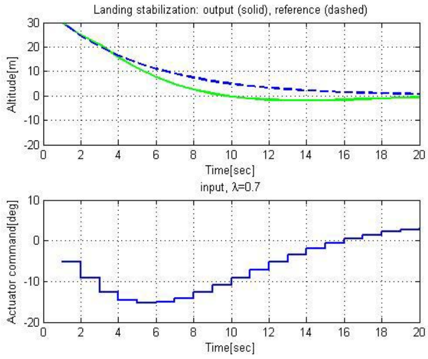

Fig. 12. The last phase of landing using the proposed DMC structure for $N_p = 6$, $N_c = 1$ and $\lambda = 0.7$

Given the descending speed reference from (3), the following simulations were performed, as per Table 2:

TABLE II. SIMULATION PARAMETERS- DESCENDING SPEED REFERENCE

	Prediction Horizon	***Control Horizon***	***λ - control action weight***
1.	$N_p = 5$	$N_c = 1$	1
2.	$N_p = 4$	$N_c = 1$	1
3.	$N_p = 4$	$N_c = 1$	0.8
4.	$N_p = 6$	$N_c = 2$	1
5.	$N_p = 6$	$N_c = 1$	1
6.	$N_p = 6$	$N_c = 2$	0.8

For lines 2 and 5 from table II, the future reference is also predicted.

First an analysis of the satisfactory situations of the descending speed reference tracking can be seen in Fig. 13 (with $N_p = 5$ and $N_c = 1$), Fig. 14 ($N_p = 4$, $N_c = 1$ and prediction of the future reference) and Fig. 15 ($N_p = 4$, $N_c = 1$ and $\lambda = 0.8$).

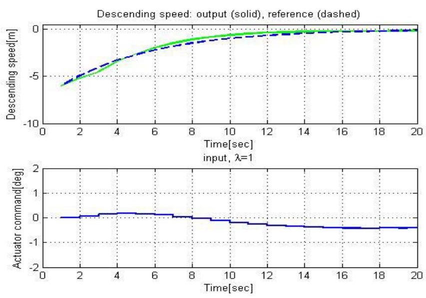

Fig. 13. The descending speed reference tracking for $N_p = 5$ and $N_c = 1$

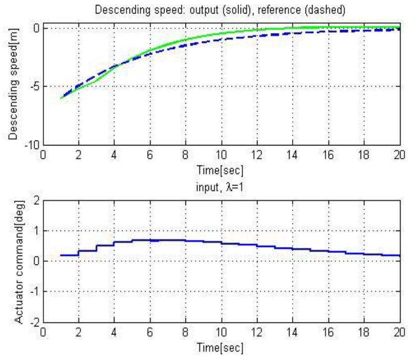

Fig. 14. The descending speed reference tracking for $N_p = 4$, $N_c = 1$ and prediction of the future reference

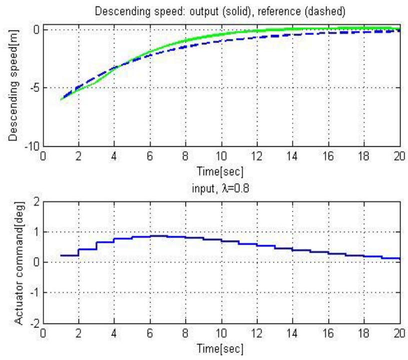

Fig. 15. The descending speed reference tracking for $N_p = 4$, $N_c = 1$ and $\lambda = 0.8$

An analysis of the less satisfactory control can be seen in Fig. 16 (with $N_p = 6$ and $N_c = 2$), Fig. 17 ($N_p = 6$, $N_c = 1$ and prediction of the future reference) and Fig. 18 ($N_p = 6$, $N_c = 1$ and $\lambda = 0.8$).

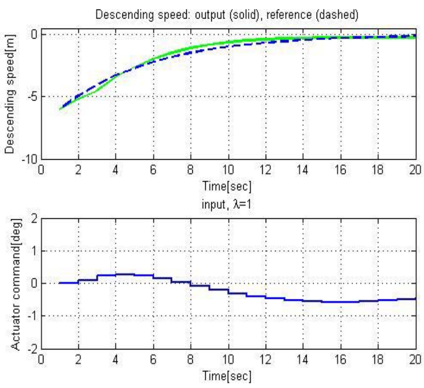

Fig. 16. The descending speed reference tracking for $N_p = 6$ and $N_c = 2$

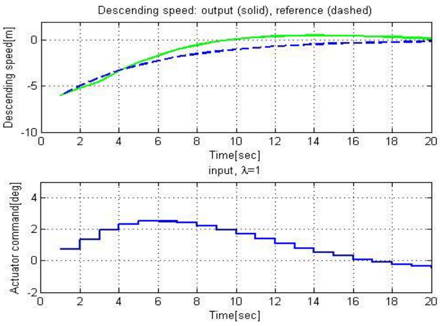

Fig 17. The descending speed reference tracking for $N_p = 6$, $N_c = 1$ and prediction of the reference trajectory

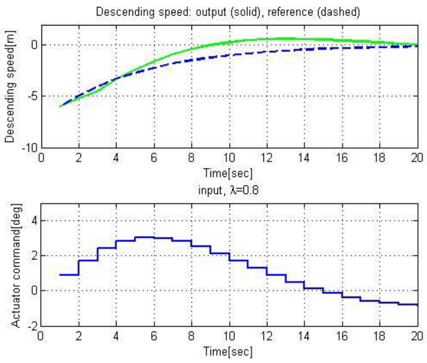

Fig. 18. The descending speed reference tracking for $N_p = 6$, $N_c = 1$ and $\lambda = 0.8$

III. Conclusions

The case study and simulations show that the proposed DMC structure can be used for aircraft landing stabilization. The performance of the proposed structure ensures also a smooth transition of the control surfaces and their corresponding actuators.

Choosing the design parameters in MPC problems is always a crucial aspect. In this regard, we have shown configurations with correctly chosen parameters, but also what happens if one or the other of those parameters is changed.

The simulations shown are treating the Θ angle response in case of a step variation of the given reference, the landing trajectory tracking and the descending speed reference tracking.

For the theta angle response, in case of a step variation we have shown a satisfactory response for $N_p = 5$ and $N_c = 1$ and a less satisfactory response when the prediction horizon is increased.

The same approach was followed for the landing trajectory tracking and the descending speed reference tracking were the best responses were obtained for $N_p = 5$, $N_c = 1$ and other simulations were performed to show the influence of increased control and prediction horizons, the prediction of the future reference and also adding weight factors to the control actions.

The step model used in DMC can contain a larger number of parameters than the methods based on transfer function models, but for a dedicated application such as this the computational cost is not an issue.

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