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Nonlinear Control with High Gain Extended State Observer for Position Tracking of Electro-hydraulic Systems

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***Abstract*—In this paper, a method for nonlinear control is proposed using an extended state observer for position tracking of an electro-hydraulic system (EHS) with only position feedback. The proposed method consists of a high gain extended state observer (HGESO) and a nonlinear controller. The EHS model is transformed into the normal form to lump a system function and a external disturbance into a disturbance. The HGESO observer is accordingly designed to estimate the full state and disturbance. The nonlinear controller is then developed using backstepping to suppress the position tracking error using the input-to-state stability (ISS) property. The key innovation of the proposed method is the design of the nonlinear gain to suppress the position tracking error according to the disturbance estimation error. Thus the use of the high observer gain to accurately estimate the disturbance can be avoided. The stability of the closed loop including the EHS, controller, and observer is then mathematically demonstrated without the need for an approximation of the 'sgn' function in the EHS dynamics. The proposed method is verified using simulated and experimental testing.**

***Index Terms*—Electro-hydraulic system, Observer, Position control**

I. Introduction

Electro-hydraulic systems (EHSs) have been widely used in industrial automation applications such as robots, aircraft actuators, and rolling mills due to their high power-to-weight ratio and their ability to rapidly generate high forces [1]–[3]. Importantly, EHSs have nonlinearities in their dynamics due to their use of hydraulic fluids and the complex flow properties of servo valves. In order to improve the position/force tracking performances of EHSs, various control methods have been proposed by academic and industrial researchers to compensate for these nonlinearities. In [1], linear control theory was applied to EHSs in order to guarantee local stability. A robust, adaptive, and repetitive digital tracking control method was presented as a way to provide disturbance rejection in [4]. Variable structure control methods were developed to control EHSs in [5]. Furthermore, several input-output linearization control methods have been proposed for the control and compensation of EHS nonlinearities [6], [7]. Backstepping control methods relying upon the dynamic properties of an EHS have also been proposed [8]–[10]. A Barrier Lyapunov function based on a nonlinear control method has been introduced to provide a position tracking error constraint for EHSs [11], and a flatness-based control method was developed to improve position tracking performance and reduce the control input ripple [12]. These control methods all require complete system information for the EHS model; however, in practice, it is impossible to acquire all system information necessary to provide perfect cancellation. Singular perturbation theory-based nonlinear control methods have accordingly been presented to provide robustness to the uncertainties in the effective bulk modulus [13], [14]. To reduce the necessity of feedback control efforts and to further improve tracking accuracy, an adaptive control method using the robust integral of the sign of the error was presented in [15], but this method can only handle matched disturbances; in practice, an EHS will contain unmatched disturbances. Recently, a robust adaptive control method was proposed that provides friction and valve dead-zone compensation [16].

All of these previously developed methods require full state feedback to successfully improve EHS position and force control performance. Unfortunately, it is not always possible to measure the full state of an EHS due to cost and space limitations. Furthermore, the pressure sensor measurements contain noise that many methods can actually act to amplify. Thus, the accurate determination of EHS state remains a critical concern in the field. To estimate state variables or disturbances, several estimation methods have been proposed [7], [17]–[21]. In [17], the observer estimated only acceleration, while a proportional integral observer was designed to estimate the full state in [18], but although the estimation performance was validated in both studies, closed-loop stability was not proven in either. An observer-based controller was proposed for position-tracking control of an EHS in which disturbances were ignored [20], [21]. A disturbance estimation method was developed for disturbance compensation in [7], [19], but it too requires full state feedback. Recently, extended state observer-based control methods were proposed to compensate for the disturbance using only output feedback [22]–[29] and they were applied to the EHS control system [30], [31]. However, their stability was proven under the assumption that the sgn function could be accurately approximated by 'atan', 'tanh' or 'sigm' functions. As disturbances vary with time and their dynamics can be unknown, it is difficult to accurately estimate them so that they can be appropriately compensated for. Furthermore, the use of

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the high observer gain to accurately estimate the disturbances may result in the amplification of the measurement noise. Therefore, a control method that is able to compensate for disturbances using only position feedback is still required.

In this paper, we accordingly propose a nonlinear control method with high gain extended state observer (HGESO) for position tracking of an EHS that uses only position feedback. The proposed method consists of a HGESO and a nonlinear controller. The EHS model is transformed into the normal form to lump a system function and a external disturbance into a disturbance. The HGESO observer is accordingly designed to estimate the full state and disturbance. The nonlinear controller is then developed using backstepping to suppress the position tracking error using the input-to-state stability (ISS) property. The key innovation of the proposed method is the design of the nonlinear gain to suppress the position tracking error according to the disturbance estimation error. Thus the use of the high observer gain to accurately estimate the disturbance can be avoided. The stability of the closed loop, including the EHS, controller, and observer, is then mathematically proven without the need for an approximation of the 'sgn' function in the EHS dynamics. The proposed control method improves position tracking performance using only position feedback under the effects of an external disturbance. The performance of the proposed method is then experimentally validated and compared to the performance of a previously proposed method. The main benefits and contributions of the proposed control method can be summarized as follows:

This paper is organized as follows. In Section II, a mathematical model of the EHS and the normal form are presented. The nonlinear controller is developed in Section III. The HGESO is designed in Section IV and the stability of the closed-loop system is studied in Section V. The simulation and experimental results are presented in Section VI. Finally, conclusions follow in Section VII.

II. Mathematical Models and Problem Formulation

The structure of EHS is shown in [12]. In many EHS applications, valve dynamics are much faster than the dynamics of the other parts in the system, such that the effects of valve dynamics are often neglected without resulting in a significant reduction in control performance [7], [13], [14], [32]. Therefore, for simplicity the following approximation of valve dynamics has been applied in our mathematical model:

$$x_v = k_v i \tag{1}$$

where x_v is the spool position of the servo-valve [m], i is the input current of the torque motor [mA], and k_v is the torque motor gain [m/mA]. With the assumption that $|P_L| < P_s$ [1], the flow control equation of the hydraulic valve giving the load flow rate can be written as

$$Q_L = C_d w x_v \sqrt{\frac{1}{\rho}(P_s - \operatorname{sgn}(x_v) P_L)} \tag{2}$$

where Q_L is the load flow rate [m^3/s], C_d is the discharge coefficient, w is the area gradient of the servo-valve spool [m], P_s is the supply pressure of the pump [N/m^2], P_L is the differential pressure between the port A pressure P_A and the port B pressure P_B and ρ is the density of the hydraulic oil [kg/m^3]. By applying the law of continuity to each actuator chamber, the continuity equation for load flow rate is given by [1]

$$Q_L = A_p \dot{x}_p + C_{tl} P_L + \frac{V_t}{4\beta_e}\dot{P}_L \tag{3}$$

where x_p is the piston position [m], A_p is the pressure area of the piston [m^2], $C_{tl} = C_{il} + \frac{C_{el}}{2}$ is the total leakage coefficient [m^5/Ns], C_{il} is the internal leakage coefficient [m^5/Ns], C_{el} is the external leakage coefficient [m^5/Ns], V_t is the total actuator volume [m^3], and β_e is the effective bulk modulus of the system [N/m^2]. Combining the flow control equation (2) and the load flow rate continuity equation (3), the fluid dynamic equation of the actuator is given by

$$\dot{P}_L = -\frac{4\beta_e A_p}{V_t}\dot{x}_p - \frac{4\beta_e C_{tl}}{V_t}P_L + \frac{4\beta_e C_d w}{V_t\sqrt{\rho}}\sqrt{(P_s - \operatorname{sgn}(x_v)P_L)}\,x_v. \tag{4}$$

Finally, by applying Newton's second law, the force balance equation of the actuator can be written as

$$m\ddot{x}_p = -kx_p - b\dot{x}_p + A_p P_L - F_L - F_F \tag{5}$$

where m is the piston mass [kg], k is the load spring constant [N/m], b is the viscous damping coefficient [N/(m/s)], F_L is the load force [N], and F_F is the friction force [N]. The goal is to make the piston position track the desired position with disturbance compensation.

Combining (1) - (5) with the system uncertainties, Δf_2 and Δf_3, the dynamics of the EHS can be reformulated as the following state space representation:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -a_1 x_1 - a_2 x_2 + a_3 x_3 \underbrace{-\frac{F_L}{m} - \frac{F_F}{m} + \Delta f_2}_{d_2} \\
\dot{x}_3 &= -h_2 x_2 - h_3 x_3 + b_u\sqrt{P_s - \operatorname{sgn}(u)\,x_3}\,u + \underbrace{\Delta f_3}_{d_3} \\
y &= x_1
\end{aligned} \tag{6}$$

where $x = [x_1\ x_2\ x_3]^T$ is the states, $x_1 = x_p$ is the position of the piston [m], $x_2 = \dot{x}_p$ is the velocity of the piston [m/s], $x_3 = P_L$ is the pressure difference between chambers A and B [N/m^2], $u = i$ is the current input, i [mA], y is the output, and $a_1 := \frac{k}{m}$, $a_2 := \frac{b}{m}$, $a_3 := \frac{A_p}{m}$, $h_2 := \frac{4\beta_e A_p}{V_t}$, $h_3 := \frac{4\beta_e C_{tl}}{V_t}$, $b_u := \frac{4\beta_e C_d w k_v}{V_t\sqrt{\rho}}$.

Remark 1: The port A pressure P_A and the port B pressure P_B are generated from the supply pressure P_s. Because P_L is the differential pressure between the port A pressure P_A and the port B pressure P_B, this condition $|P_L| < P_s$ is always satisfied in the EHS [1]. Thus, $\sqrt{P_s - \operatorname{sgn}(u)\,x_3}$ is always positive. ♢

To transform the EHS dynamics (6) into a normal form, we define the new state variables as

$$\begin{aligned}
z_1 &= x_1 \\
z_2 &= x_2 \\
z_3 &= -a_1 x_1 - a_2 x_2 + a_3 x_3 + d_2.
\end{aligned} \tag{7}$$

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From (6) and (7), the EHS dynamics (6) can be written as

$$\begin{aligned}\dot z_1 &= z_2\\ \dot z_2 &= z_3\\ \dot z_3 &= \alpha(z)+\beta(z,u)u+d\\ y &= z_1\end{aligned}\tag{8}$$

where

$$\begin{aligned}\alpha(z) =& -a_1h_3z_1-(a_1+a_2h_3+a_3h_2)z_2-(a_2+h_3)z_3\\ \beta(z,u) =& a_3b_u\sqrt{P_s-\operatorname{sgn}(u)\left(\frac{a_1z_1+a_2z_2+z_3}{a_3}\right)}\\ d =& h_3d_2+\dot d_2+a_3d_3-\beta(z,u)\\ &+a_3b_u\sqrt{P_s-\operatorname{sgn}(u)\left(\frac{a_1z_1+a_2z_2+z_3-d_2}{a_3}\right)}\end{aligned}\tag{9}$$

and d is the total disturbance consisting of the external disturbances and the system uncertainties. In this paper, the goal is to make position $z_1=x_1$ track the desired position $z_{1_d}=x_{1_d}$ using only z_1 feedback under the effect of external disturbances and system uncertainties.

III. Nonlinear Controller

In this section, the design of the nonlinear controller is presented. The tracking error $e=\begin{bmatrix}e_1 & e_2 & e_3\end{bmatrix}^T$ is defined as

$$\begin{aligned}e_1 =& z_1-z_{1_d}\\ e_2 =& z_2-z_{2_d}\\ e_3 =& z_3-z_{3_d}\end{aligned}\tag{10}$$

where $z_{1_d}=x_{1_d}$ and $x_{i_d},\ i\in[2,\ 3]$ are yet to be designed. The tracking error dynamics are given by

$$\begin{aligned}\dot e_1 =& e_2+z_{2_d}-\dot z_{1_d}\\ \dot e_2 =& e_3+z_{3_d}-\dot z_{2_d}\\ \dot e_3 =& \alpha(z)+\beta(z,u)u+d-\dot z_{3_d}.\end{aligned}\tag{11}$$

In order to guarantee the boundedness of e_1, the controller is designed as

$$\begin{aligned}z_{2_d} =& -k_1e_1+\dot z_{1_d}\\ z_{3_d} =& -k_2e_2+\dot z_{2_d}\\ u =& \underbrace{\frac{1}{\beta(z,u)}\left(-\alpha(z)-k_3e_3+\dot z_{3_d}-\hat d\right)}_{u_a}\\ &+\underbrace{\frac{1}{\beta(z,u)}\left(-\left(k_4\sqrt{\hat d^2+\nu}\right)e_3\right)}_{u_b}.\end{aligned}\tag{12}$$

where control gains, k_1, k_2, k_3, k_4, and ν are all positive, $\hat d$ is the disturbance estimated by the observer that has yet to be designed (in Section 4 it will be proven that $\hat d$ is bounded by the observer), and $\tilde d=d-\hat d$ is defined as the disturbance estimation error.

Remark 2: In the control input u in (12), u_a provides the stabilization while u_b is a nonlinear gain term to suppress e_1. The position tracking performance depends on the estimation performance of the disturbance. However, in reality, it is difficult to precisely estimate d when including external disturbances, system function, and input gain uncertainty. Generally, as d increases, both $\hat d$ and $\tilde d$ increase, and when $\tilde d$ increases, the position tracking error e_1 will increase as long as the stability is satisfied by u_a. The nonlinear gain term u_b is designed to indirectly suppress the effect of $\tilde d$ in e_1 when $\hat d$ increases. ♢

The nonlinear gain $k_d(\hat d)$ is defined as

$$k_d(\hat d)=\left(k_4\sqrt{\hat d^2+\nu}\right).\tag{13}$$

Remark 3: Note that $\beta(z,u)$ of (12) cannot be computed 'as it is', because it contains the control input, u, on both sides of the equation and contains the $\operatorname{sgn}(\cdot)$ function. We can see that sign of u is determined by the numerator $-\alpha(z)-k_3e_3+\dot z_{3_d}-\hat d-k_d(\hat d)e_3$ because $\beta(z,u)$ is physically always greater than 0. ♢

The modified control law (12) thus takes the following form:

$$\begin{aligned}z_{2_d} =& -k_1e_1+\dot z_{1_d}\\ z_{3_d} =& -k_2e_2+\dot z_{2_d}\\ u =& \frac{1}{\beta_u}\left(-\alpha(z)-k_3e_3+\dot z_{3_d}-\hat d-k_d(\hat d)e_3\right)\end{aligned}\tag{14}$$

where

$$\begin{aligned}\beta_u =& a_3b_u\sqrt{P_s-\operatorname{sgn}(u_n)\left(\frac{a_1z_1+a_2z_2+z_3}{a_3}\right)}\\ u_n =& -\alpha(z)-k_3e_3+\dot z_{3_d}-\hat d-k_d(\hat d)e_3.\end{aligned}\tag{15}$$

With (12), the tracking error dynamics (11) become

$$\begin{aligned}\dot e_1 =& -k_1e_1+e_2\\ \dot e_2 =& -k_2e_2+e_3\\ \dot e_3 =& -k_3e_3-k_d(\hat d)e_3+\tilde d\end{aligned}\tag{16}$$

where $\tilde d=d-\hat d$ is the disturbance's estimation error.

Theorem 1: Suppose that $\hat d$ is bounded. The tracking error dynamics (16) have the following ISS properties:

$$\begin{aligned}|e_1(t)| \le& \exp\left(-\frac{k_1}{2}t\right)|e_1(0)|+\frac{2}{k_1}\sup_{0\le\tau\le t}|e_2(\tau)|\\ |e_2(t)| \le& \exp\left(-\frac{k_2}{2}t\right)|e_2(0)|+\frac{2}{k_2}\sup_{0\le\tau\le t}|e_3(\tau)|\\ |e_3(t)| \le& \exp\left(-\frac{k_3}{2}t\right)|e_3(0)|+\sup_{0\le\tau\le t}\sigma(\tau)\end{aligned}\tag{17}$$

where

$$\sigma=\frac{|\tilde d|}{0.5k_3+k_d(\hat d)}.\tag{18}$$

♢

Proof: From (16), the dynamics of e_3^2 can be written as

$$\begin{aligned}\frac{d}{dt}\left(\frac{e_3^2}{2}\right) =& -k_3e_3^2-k_d(\hat d)e_3^2+e_3\tilde d\\ \le& -\frac{k_3}{2}e_3^2-\left(\frac{k_3}{2}+k_d(\hat e_1,\hat d)\right)|e_3|(|e_3|-\sigma).\end{aligned}\tag{19}$$

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For $|e_3| \geq \sigma$,

$$\frac{d}{dt}\left(e_3^2\right) \leq -k_3 e_3^2. \tag{20}$$

Thus, with Lemma 6.20 and Theorem C.2 in [33], we know that

$$|e_3(t)| \leq \exp\left(-\frac{k_3}{2}t\right)|e_3(0)| + \sup_{0\leq\tau\leq t} \sigma(\tau). \tag{21}$$

From (21), the relationship between e_3 and σ is known to have the ISS property. From (16), the dynamics of e_1^2 and e_2^2 are obtained as

$$\begin{aligned}\frac{d}{dt}\left(\frac{e_1^2}{2}\right) &= -k_1 e_1^2 + e_1 e_2 \\ &\leq -\frac{k_1}{2}e_1^2 - \frac{k_1}{2}|e_1|\left(|e_1| - \frac{2}{k_1}|e_2|\right) \\ \frac{d}{dt}\left(\frac{e_2^2}{2}\right) &= -k_2 e_2^2 + e_2 e_3 \\ &\leq -\frac{k_2}{2}e_2^2 - \frac{k_2}{2}|e_2|\left(|e_2| - \frac{2}{k_2}|e_3|\right).\end{aligned} \tag{22}$$

For $|e_1| \geq \frac{2}{k_1}|e_2|$ and $|e_2| \geq \frac{2}{k_2}|e_3|$,

$$\begin{aligned}\frac{d}{dt}\left(e_1^2\right) &\leq -k_1 e_1^2 \\ \frac{d}{dt}\left(e_2^2\right) &\leq -k_2 e_2^2.\end{aligned} \tag{23}$$

Therefore,

$$\begin{aligned}|e_1(t)| &\leq \exp\left(-\frac{k_1}{2}t\right)|e_1(0)| + \frac{2}{k_1}\sup_{0\leq\tau\leq t}|e_2(\tau)| \\ |e_2(t)| &\leq \exp\left(-\frac{k_2}{2}t\right)|e_2(0)| + \frac{2}{k_2}\sup_{0\leq\tau\leq t}|e_3(\tau)|.\end{aligned} \tag{24}$$

From (24), the relationships between e_1 and e_2, and e_2 and e_3 can be seen to have the ISS property, and from (21) and (24), the tracking error dynamics (16) can be seen to have the ISS properties. ■

Remark 4: The stability of the error dynamics (16) is guaranteed regardless of the nonlinear gain (13). The main role of the nonlinear gain (13) is to suppress the effect of $|\tilde{d}|$ on e_1. Increasing $\hat{d}$ in the denominator of σ (18) results in a decreasing σ. The nonlinear gain term u_b in (12) increases such that the effect of $|\tilde{d}|$ on e_1 can be sufficiently suppressed. From (17), as $t \to \infty$,

$$|e_1(\infty)| \leq \frac{4}{k_1 k_2}\sup_{0\leq\tau\leq\infty} \sigma(\tau). \tag{25}$$

Consequently, the tracking error e_1 can be sufficiently suppressed even though $|\tilde{d}|$ is not small. ♢

Note that e_1 asymptotically converges to zero when the disturbance d disappear.

Now we provide the procedure for the controller gain tuning. Because z_3 dynamics are much faster than the mechanical dynamics for z_1 and z_2, k_1 and k_2 are chosen first, then, k_3 is chosen.

- **Step 1 - Mechanical error dynamics**
  Practically, the mechanical tracking error dynamics can be represented as
  $$\begin{aligned}\dot{e}_1 &= -k_1 e_1 + e_2 \\ \dot{e}_2 &= -k_2 e_2 + e_3.\end{aligned} \tag{26}$$
- **Step 2 - Sensitivity function**
  Using a Laplace transform, we can obtain the sensitivity function from e_3 to e_1 as
  $$\frac{E_1(s)}{E_3(s)} = \frac{1}{s^2 + (k_1+k_2)s + k_1 k_2}. \tag{27}$$
  where s is the Laplace operator, $E_1(s)$ and $E_3(s)$ are the frequency domain representations of e_1 and e_3, respectively.
- **Step 3 - Eigenvalue analysis of sensitivity function**
  By eigenvalue analysis for 2^{nd} order system, we can now derive the equations
  $$\begin{aligned}k_1 k_2 &= \omega_n^2 \\ k_1 + k_2 &= 2\zeta\omega_n\end{aligned} \tag{28}$$
  where ω_n and ζ are the natural frequency and damping ratio of the sensitivity function (27), respectively.
- **Step 4 - ω_n selection**
  Based on (28), a high $k_1 k_2$ means that the magnitude of the sensitivity function decreases for low frequencies. For example, if the frequency of the input perturbation is 10 Hz, ω_n should be set to be greater than $62.8(\simeq 2*pi*10)$. Thus, $k_1 k_2$ is also greater than $3943 \simeq (62.8^2)$.
- **Step 5 - ζ selection**
  Using the pre-designed gain ω_n , we can derive another condition for k_1 as follows:
  $$k_1 + k_2 = 2\zeta\sqrt{k_1 k_2} \tag{29}$$
  Since a small damping ratio ζ increases the amplitude of the mechanical tracking error e_ω, high ζ is preferred (overdamped response).
- **Step 6 - k_3, k_4, and ν selection**
  Although z_3 dynamics are faster than the mechanical dynamics, the large value of k_3 is not required as compared to k_1 and k_2 due to the use of the nonlinear gain. When the estimated disturbance is large, the nonlinear gain gets larger. Thus, only small k_4, and ν are enough to suppress the tracking error.

From the above procedure, we can analytically determine the controller gains.

IV. High Gain Extended State Observer

In this section, the HGESO is designed to estimate the full state and disturbance. Let us define z_4 as

$$z_4 = d. \tag{30}$$

Further, we define the extended state z_a as:

$$z_a = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 \end{bmatrix}^T \tag{31}$$

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and the dynamics of d are defined as

$$\dot{d} = \delta \tag{32}$$

where δ is the derivative of the disturbance d and unknown.

Assumption 1: F_L, F_F, Δf_2, Δf_3, and their derivatives are also bounded. ♢

In fact, F_F is not differentiable at zero velocity because it includes the Coulomb friction. Fortunately, the actual friction force exhibits finite jumps so that the friction model can be chosen as any differentiable function that approximates the actual discontinuous Coulomb friction (for example, replacing $\mathrm{sgn}(x_2)$ in the Coulomb friction modelling with the smooth function $\tanh(x_2)$) [34] as follows:

$$F_F(z_2) = f_v z_2 + \tanh(z_2)\left[f_c + (f_s - f_c)e^{-(|\frac{z_2}{c_s}|)}\right] \tag{33}$$

where f_v is the viscous coefficient, f_c is the parameter for Coulomb friction, and f_s and c_s are the parameters for static and Stribeck effect coefficients, respectively. Their values are as follows: $f_v = 121.5$, $f_c = 51.3$, $f_s = 621.4$, and $c_s = 0.07$. Thus, Assumption 1 is physically reasonable [34]. In point of fact, all of the states of the EHS are physically bounded, i.e., $\|x\|_2 < x_{\max}$ where $x_{\max}$ is positive and constant [35]. Thus, Δf_2 and Δf_3 are also bounded. In (6), $\dot{x}_2$ and $\dot{x}_3$ consist of the state variables, F_L, and F_F. As a result, the derivatives of $\dot{x}_2$ and $\dot{x}_3$ are also bounded. Consequently, the constant $\delta_{\max}$ exits such that

$$\delta_{\max} \geq \sup_t |\delta(t)|. \tag{34}$$

With the extended state z_a, the EHS dynamics (8) can be expressed as

$$\begin{aligned} \dot{z}_a &= A_a z_a + A_1(z_a) + \Delta + \varphi(z_a, u) \\ y &= C_a z_a \end{aligned} \tag{35}$$

where

$$\begin{aligned} A_a &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ A_1(z_a) &= \begin{bmatrix} 0 & 0 & \alpha(z_a) & 0 \end{bmatrix}^T \\ \varphi(z_a, u) &= \begin{bmatrix} 0 & 0 & \beta(z_a, u)u & 0 \end{bmatrix}^T \\ \Delta &= \begin{bmatrix} 0 & 0 & 0 & \delta \end{bmatrix}^T \\ C_a &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \\ \alpha(z_a) &= -a_1 h_3 z_1 - (a_1 + a_2 h_3 + a_3 h_2) z_2 - (a_2 + h_3) z_3 \\ \beta(z_a, u) &= a_3 b_u \sqrt{P_s - \mathrm{sgn}(u)\left(\frac{a_1 z_1 + a_2 z_2 + z_3}{a_3}\right)}. \end{aligned} \tag{36}$$

The estimated state $\hat{z}$ and estimated extended state $\hat{z}_a$ can then be defined as

$$\begin{aligned} \hat{z} &= \begin{bmatrix} \hat{z}_1 & \hat{z}_2 & \hat{z}_3 \end{bmatrix}^T \\ \hat{z}_a &= \begin{bmatrix} \hat{z}_1 & \hat{z}_2 & \hat{z}_3 & \hat{z}_4 \end{bmatrix}^T. \end{aligned} \tag{37}$$

The HGESO is therefore represented by

$$\dot{\hat{z}}_a = A_a \hat{z}_a + A_1(\hat{z}_a) + \varphi(\hat{z}_a, u) + L(y - \hat{y}), \tag{38}$$

where $L = [\frac{l_1}{\varepsilon}, \ \frac{l_2}{\varepsilon^2}, \ \frac{l_3}{\varepsilon^3} \ \frac{l_4}{\varepsilon^4}]^T$, in which l_1, l_2, l_3, and l_4 are the observer gains, and ε is a small positive constant. The estimation error is defined as follows:

$$\tilde{z}_a = \begin{bmatrix} \tilde{z}_1 \\ \tilde{z}_2 \\ \tilde{z}_3 \\ \tilde{z}_4 \end{bmatrix} = \begin{bmatrix} z_1 - \hat{z}_1 \\ z_2 - \hat{z}_2 \\ z_3 - \hat{z}_3 \\ z_4 - \hat{z}_4 \end{bmatrix}. \tag{39}$$

The estimation error dynamics are thus given by

$$\dot{\tilde{z}}_a = A_o \tilde{z}_a + A_1(\tilde{z}_a) + \Delta + \phi(z_a, \hat{z}_a, u) \tag{40}$$

where

$$\begin{aligned} A_o = A_a - LC &= \begin{bmatrix} -\frac{l_1}{\varepsilon} & 1 & 0 & 0 \\ -\frac{l_2}{\varepsilon^2} & 0 & 1 & 0 \\ -\frac{l_3}{\varepsilon^3} & 0 & 0 & 1 \\ -\frac{l_4}{\varepsilon^4} & 0 & 0 & 0 \end{bmatrix} \\ \phi(z_a, \hat{z}_a, u) &= \varphi(z_a, u) - \varphi(\hat{z}_a, u) \\ &= \begin{bmatrix} 0 \\ 0 \\ \beta(z, u)u - \beta(\hat{z}, u)u \\ 0 \end{bmatrix}. \end{aligned} \tag{41}$$

Let us define the scaled estimation error as

$$\begin{aligned} \eta &= \begin{bmatrix} \eta_1 & \eta_2 & \eta_3 & \eta_4 \end{bmatrix}^T \\ &= \begin{bmatrix} \frac{1}{\varepsilon^3}\tilde{z}_1 & \frac{1}{\varepsilon^2}\tilde{z}_2 & \frac{1}{\varepsilon}\tilde{z}_3 & \tilde{z}_4 \end{bmatrix}^T. \end{aligned} \tag{42}$$

Using the newly defined variable η, the estimation error dynamics (40) are transformed into fast dynamics in a singularly perturbed form as

$$\varepsilon \dot{\eta} = A_\eta \eta + A_1(\eta, \varepsilon) + \varepsilon \Delta + \phi(z_a, \hat{z}_a, u) \tag{43}$$

where

$$A_\eta = \begin{bmatrix} -l_1 & 1 & 0 & 0 \\ -l_2 & 0 & 1 & 0 \\ -l_3 & 0 & 0 & 1 \\ -l_4 & 0 & 0 & 0 \end{bmatrix}$$

and

$$A_1 = \begin{bmatrix} 0 \\ 0 \\ -\varepsilon^3 a_1 h_3 \eta_1 - \varepsilon^2 (a_1 + a_2 h_3 + a_3 h_2) \eta_2 - \varepsilon (a_2 + h_3) \eta_3 \\ 0 \end{bmatrix}. \tag{44}$$

The observer gains l_1, l_2, l_3, and l_4 are chosen such that A_η is Hurwitz. A positive constant γ_{A_1} thus exists satisfying

$$\|A_1(\eta, \varepsilon)\|_2 \leq \varepsilon \gamma_{A_1} \|\eta\|_2. \tag{45}$$

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In (40), the term $\phi\left(z_a, \hat{z}_a, u\right)$ can be rewritten as

$$\begin{aligned}\phi\left(z_a, \hat{z}_a, u\right) &= \begin{bmatrix} 0 \\ 0 \\ a_3 b_u \sqrt{P_s - \phi_a(z,u)}u - a_3 b_u \sqrt{P_s - \phi_a(\hat{z},u)}u \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 0 \\ \phi_3 \\ 0 \end{bmatrix} \left(a_1\tilde{z}_1 + a_2\tilde{z}_2 + \tilde{z}_3\right) \\ &= \begin{bmatrix} 0 \\ 0 \\ \varepsilon\phi_3 \\ 0 \end{bmatrix} \left(a_1\varepsilon^2\eta_1 + a_2\varepsilon\eta_2 + \eta_3\right) \\ &= \phi_\eta\left(\varepsilon, \eta, z_a, \hat{z}_a, u\right). \end{aligned} \tag{46}$$

where

$$\begin{aligned}\phi_a(z,u) &= \operatorname{sgn}(u)\left(\frac{a_1 z_1 + a_2 z_2 + z_3}{a_3}\right) \\ \phi_a(\hat{z},u) &= \operatorname{sgn}(u)\left(\frac{a_1\hat{z}_1 + a_2\hat{z}_2 + \hat{z}_3}{a_3}\right) \\ \phi_3 &= -\frac{b_u \operatorname{sgn}(u) u}{\left(\sqrt{P_s - \phi_a(z,u)} + \sqrt{P_s - \phi_a(\hat{z},u)}\right)} \le 0. \end{aligned} \tag{47}$$

The positive constant γ_ϕ thus exists such that

$$\|\phi_\eta\left(\varepsilon, \eta, z_a, \hat{z}_a, u\right)\|_2 \le \varepsilon\gamma_\phi\|\eta\|_2. \tag{48}$$

In most cases, in order to satisfy the Lipschitz condition of the function $\phi\left(z_a, \hat{z}_a, u\right)$ for stability proof, the function $\operatorname{sgn}(u)$ is assumed to be a continuously differentiable function of the form, $\operatorname{atan}(ku)$, $\tanh(ku)$ or $\operatorname{sigm}(x) = \frac{1-e^{-ax}}{1+e^{-ax}}$. However, in this paper, from (46) and (48), it is shown that Lipschitz condition is satisfied in $\phi\left(z_a, \hat{z}_a, u\right)$. Consequently, the stability of the state observer is analyzed without approximating the nonlinearity of the EHS.

Theorem 2: Consider the estimation error dynamics (43). If A_η is Hurwitz and ε is designed to satisfy the condition

$$0 < \varepsilon < \frac{\theta_\eta}{2\gamma_{A_1}\lambda_{\max}(P_\eta) + 2\gamma_\phi\lambda_{\max}(P_\eta)}, \tag{49}$$

where $0 < \theta_\eta < 1 - 2\varepsilon\gamma_{A_1}\lambda_{\max}(P_\eta) - 2\varepsilon\gamma_\phi\lambda_{\max}(P_\eta)$, P_η is a positive definite matrix, and $\lambda_{\max}(P_\eta)$ is the maximum eigenvalue of the matrix P_η, then η is uniformly ultimately bounded. $\diamondsuit$

Proof: To study the convergence of the equilibrium points $\eta = 0$ of (43), we define the following Lyapunov candidate function

$$V_\eta = \varepsilon\eta^T P_\eta \eta. \tag{50}$$

Because A_η is Hurwitz, a positive definite matrix P_η exists such that

$$A_\eta^T P_\eta + P_\eta A_\eta = -I. \tag{51}$$

The derivative of V_η is

$$\begin{aligned}\dot{V}_\eta =& -\eta^T\eta + 2\eta^T P_\eta A_1(\eta,\varepsilon) + 2\eta^T P_\eta \phi_\eta\left(\varepsilon, \eta, z_a, \hat{z}_a, u\right) \\ &+ 2\varepsilon\eta^T P_\eta \Delta. \end{aligned} \tag{52}$$

Consequently, it can be seen that

$$\begin{aligned}\dot{V}_\eta \le& -\|\eta\|_2^2 + 2\varepsilon\gamma_{A_1}\lambda_{\max}(P_\eta)\|\eta\|_2^2 + 2\varepsilon\gamma_\phi\lambda_{\max}(P_\eta)\|\eta\|_2^2 \\ &+ 2\varepsilon\delta_{\max}\|P_\eta\|_2\|\eta\|_2 \\ \le& -(1 - 2\varepsilon\gamma_{A_1}\lambda_{\max}(P_\eta) - 2\varepsilon\gamma_\phi\lambda_{\max}(P_\eta))\|\eta\|_2^2 \\ &+ 2\varepsilon\delta_{\max}\|P_\eta\|_2\|\eta\|_2 \\ \le& -(1 - 2\varepsilon\gamma_{A_1}\lambda_{\max}(P_\eta) - 2\varepsilon\gamma_\phi\lambda_{\max}(P_\eta) - \theta_\eta)\|\eta\|_2^2 \\ &- \theta_\eta\|\eta\|_2^2 + 2\varepsilon\delta_{\max}\|P_\eta\|_2\|\eta\|_2. \end{aligned} \tag{53}$$

Then,

$$\dot{V}_\eta \le -(1 - 2\varepsilon\gamma_{A_1}\lambda_{\max}(P_\eta) - 2\varepsilon\gamma_\phi\lambda_{\max}(P_\eta) - \theta_\eta)\|\eta\|_2^2 \tag{54}$$

for $\|\eta\|_2 \ge \frac{2\varepsilon\delta_{\max}\|P_\eta\|_2}{\theta_\eta} = \mu_\eta$. Thus, η is uniformly ultimately bounded. ■

V. Closed-loop Stability Analysis

The controller (12) is obtained with full state feedback, but in this paper, we assume that only output z_1 is measurable. With this output feedback, the actual controller can be obtained as

$$\begin{aligned}\hat{z}_{2_d} &= -k_1\hat{e}_1 + \dot{z}_{1_d} \\ \hat{z}_{3_d} &= -k_2\hat{e}_2 + \dot{\hat{z}}_{2_d} \\ \hat{u} &= \frac{1}{\beta(\hat{z},\hat{u})}\left(-\alpha(\hat{z}) - k_3\hat{e}_3 + \dot{\hat{z}}_{3_d} - \hat{d} - k_d(\hat{d})\hat{e}_3\right). \end{aligned} \tag{55}$$

where control gains, k_1, k_2, k_3, k_d, and ν are positive. Equation (17) shows the cascade property of the tracking errors. The actual control input $\hat{u}$ (55) is injected into the tracking error dynamics instead of the control input u (12). Thus, the closed-loop system is

$$\begin{aligned}\dot{e} &= A_e(\hat{d})e + B_e\zeta \\ \varepsilon\dot{\eta} &= A_\eta\eta + A_1(\eta,\varepsilon) + \varepsilon\Delta + \phi\left(z_a, \hat{z}_a, u\right) \end{aligned} \tag{56}$$

where

$$\begin{aligned} A_e(\hat{d}) &= A_k + A_d(\hat{d}) \\ A_e(\hat{d}) &= \begin{bmatrix} -k_1 & 1 & 0 \\ 0 & -k_2 & 1 \\ 0 & 0 & -k_3 - k_d(\hat{d}) \end{bmatrix} \\ A_k &= \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -k_2 & 0 \\ 0 & 0 & -k_3 \end{bmatrix} \\ A_d(\hat{d}) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -k_d(\hat{d}) \end{bmatrix} \\ B_e &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \\ \zeta &= \tilde{z}_4 - \beta(z,u)u + \beta(\hat{z},\hat{u})\hat{u}. \end{aligned} \tag{57}$$

In (56), the perturbation term ζ appears in the e dynamics due to the estimation error. We will show the boundedness of $e(t)$ for $t \ge 0$.

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Theorem 3: Consider the closed-loop system (56). If the condition (49) is satisfied, η is uniformly ultimately bounded. Then, if the observer gains, l_i and ε are chosen such that η enters the bounded ball $B_\mu = \{\eta \ : \ \|\eta\|_2 = \mu_\eta\}$ at $t = T_0$ and the condition (69) is satisfied, e is also uniformly ultimately bounded. ◊

Proof: To show the boundedness of $e(t)$ for $t \geq 0$, we define a number $R > 0$ such that $e \in B_R$ where B_R denotes the closed ball of radius R. In Section III, it was proven that the origin of the unperturbed system $\dot{e} = A_e(\hat{d})e$ is globally asymptotically stable. Thus, a Lyapunov function $V_e(e)$ exists satisfying

$$\frac{\partial V_e}{\partial e} A_e(\hat{d})e \leq -\alpha_e(\|e\|_2) \tag{58}$$

for some class $\mathscr{K}^\infty$ function $\alpha_e(\cdot)$. A positive number, c, is chosen such that

$$\Omega_c = \{e \ : \ V_e(e) \leq c\} \supset B_R. \tag{59}$$

As will be demonstrated later, among other things, one of the purposes of the controller design is to ensure that, for all times $t \geq 0$, the vector $e(t)$ remains in Ω_{c+1}. We begin by analyzing what happens to $e(t)$ for small values of $t \geq 0$. It was proven in Section IV that η is ultimately bounded. Thus, looking at the expression of the perturbation term ζ in the upper system (56), it is obvious that a number $\zeta_{\max_0} > 0$ exists such that $|\zeta| \leq \zeta_{\max_0}$ The parameter c depends only on the choice of the number R and thus, indirectly, only on the choice of the set in which the initial conditions are established. Letting $e(0) \in B_R \subset \Omega_c$, as long as $e \in \Omega_{c+1}$,

$$\begin{aligned} \dot{V}_e(e(t)) &= \frac{\partial V_e}{\partial e}[A_e(\hat{d})e + B_e\zeta] \\ &\leq -\alpha_e(\|e\|_2) + \left|\frac{\partial V_e}{\partial e}\right| \zeta_{\max_0}. \end{aligned} \tag{60}$$

We then define M_0 as

$$M_0 = \max_{e \in \Omega_{c+1}} \left|\frac{\partial V_e}{\partial e}\right|. \tag{61}$$

Further, $\dot{V}_e \leq M_0 \zeta_{\max_0}$, which yields

$$V_e(t) - V_e(0) \leq (M_0 \zeta_{\max_0})t. \tag{62}$$

Because $V_e(0) \leq c$, it can be seen that $e(t)$ remains in Ω_{c+1} at least until the time reaches $T_0 = \frac{1}{M_0 \zeta_{\max_0}}$. The latter may be small but, it depends only on the choice of the number R and thus, indirectly, only on the choice of the set in which the initial conditions are established. Because, from Theorem 2, it is known that $|\eta(t)| \leq \mu_\eta$ is bounded, the observer gains, l_i for $i \in [1,\ 4]$ and ε can be chosen such that η enters the bounded ball $B_\mu = \{\eta \ : \ \|\eta\|_2 = \mu_\eta\}$ at $t = T_0$. Examining the motion of $e(t)$, which is in Ω_{c+1} for $t \leq T_1$, it can be supposed that, for some $T_1 > T_0$, $e(t) \in \Omega_{c+1}$ for all $t \in [T_0, T_1)$. For $t > T_0$, a number $\zeta_{\max_1} > 0$ exists such that $|\zeta| \leq \zeta_{\max_1}$. M_1 is defined as

$$M_1 = \max_{e \in \Omega_{c+1}} \left|\frac{\partial V_e}{\partial e}\right|. \tag{63}$$

From the previous discussions it is obvious that, for all $t \in [T_0, T_1)$,

$$\begin{aligned} \dot{V}_e(e(t)) &= \frac{\partial V}{\partial e}[A_e(\hat{d})e + B_e\zeta] \\ &\leq -\alpha_e(\|e\|_2) + \left|\frac{\partial V}{\partial e}\right| \zeta_{\max_1}. \end{aligned} \tag{64}$$

Further, $\dot{V}_e \leq M\zeta_{\max}$, which yields

$$V_e(t) - V_e(T_0) \leq (M_1 \zeta_{\max})(t - T_0). \tag{65}$$

Thus, it can be seen that $e(t)$ remains in Ω_{c+1} at least until the time reaches $T_1 = \frac{c+1-V_e(T_0)}{M_1 \zeta_{\max_1}} + T_0$. Picking any number $d \ll c$ and considering the annular compact set S_d^{c+1} as

$$S_d^{c+1} = \{e \ : \ d \leq V_e(e) \leq c+1\} \tag{66}$$

let $e_{\min}$ be defined as

$$e_{\min} = \min_{e \in S_d^{c+1}} \|e\|. \tag{67}$$

Then,

$$\alpha_e(\|e\|) \geq \alpha_e(e_{\min}) \tag{68}$$

for all $e \in S_d^{c+1}$. Because $e(t) \in \Omega_{c+1}$, $\zeta_{\max}$ depends on the observer gains, l_i for $i \in [1,\ 4]$ and ε. If the observer gains, l_i and ε are chosen such that

$$\zeta_{\max_1} \leq \frac{1}{2}\alpha_e(e_{\min}) \tag{69}$$

it follows that

$$\dot{V}_e(e) \leq -\frac{1}{2}\alpha_e(\|e\|) \tag{70}$$

as long as $e(t) \in S_d^{c+1}$. In turn, this implies that

$$\begin{aligned} V_e(e(t)) &\leq V_e(e(T_1)) - \frac{1}{2}\alpha_e(e_{\min})(t - T_1) \\ &\leq c + 1 - \frac{1}{2}\alpha_e(e_{\min})(t - T_1) \end{aligned} \tag{71}$$

as long as $e(t) \in S_d^{c+1}$. Clearly, there is a time $T_2 > T_1$ such that $e(t) \in \Omega_{c+1}$ for all $t \in [T_1,\ T_2]$ and $V_e(e(T_2)) = d$. Because $\dot{V}_e(e(t))$ is negative at the boundary of Ω_d, it is concluded that $e(t) \in \Omega_d$ for all $t \geq T_2$. Consequently, e is uniformly ultimately bounded for all $t \geq 0$. ■

VI. Simulations and Experiments

Simulations and experiments were performed to evaluate the position tracking and estimation performances of the proposed controller. Simulations of the proposed control method were evaluated using both sinusoidal and step reference tracking, and experimental evaluations of the proposed method and a conventional backstepping controller were conducted using sinusoidal reference tracking to provide performance comparisons. Table I lists the EHS and controller parameters used in the simulations and experiments. The magnitude of the load is 1000 N.

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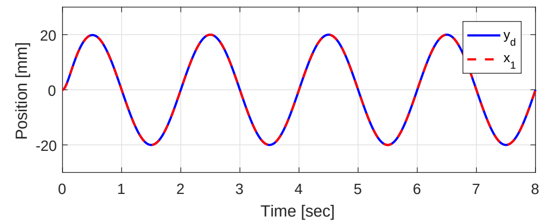
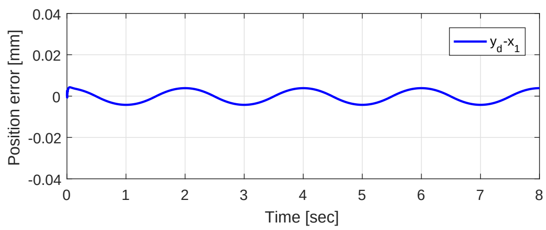
TABLE I
PARAMETERS OF EHS AND CONTROLLER

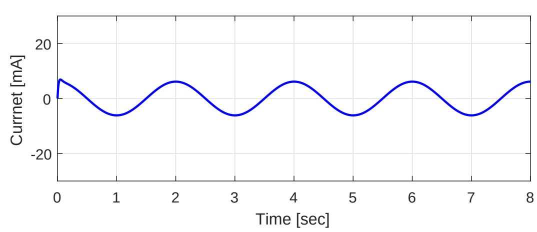
Parameter	Value	Parameter	Value
m	10	k	5000
b	1000	C_d	0.6
A_p	4.812×10^{-4}	β_e	1.8×10^{9}
V_t	6.2×10^{-5}	C_{il}	2.488×10^{-14}
C_{el}	1.666×10^{-14}	w	5.2×10^{-3}
ρ	840	k_v	1.33×10^{-5}
P_s	12.0×10^{6}	k_1	300
k_2	500	k_4	350
k_d	1	v	0.001
l_1	2.96×10^{2}	l_2	-2.66×10^{6}
l_3	-5.16×10^{8}	l_4	-2.59×10^{7}
ε	0.1		



(a) Position tracking performance

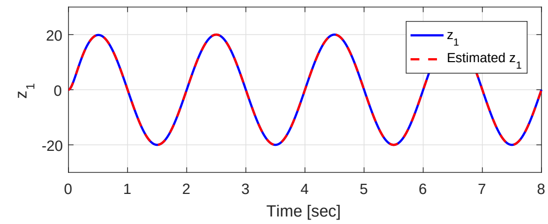


(b) Position tracking error

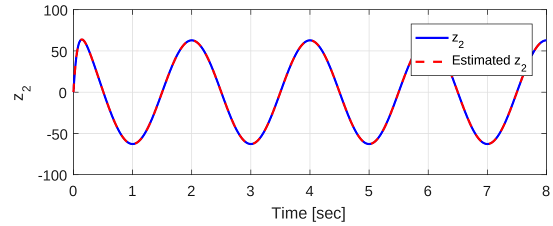


(c) Control input

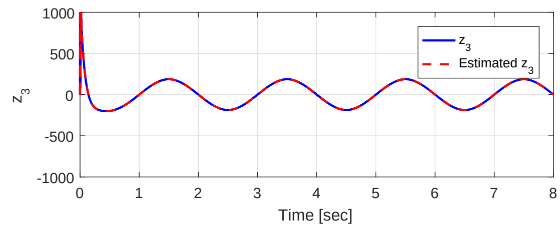
Fig. 1. Position tracking performance of the proposed method using sinusoidal reference, $x_{1_d}(t) = 0.02\sin(\pi t)$



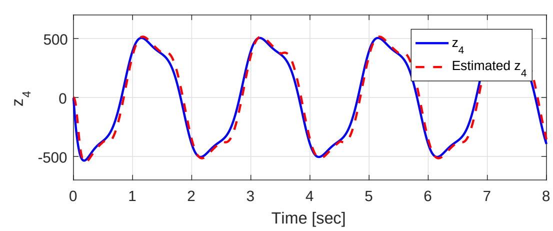
(a) Estimation of z_1



(b) Estimation of z_2



(c) Estimation of z_3



(d) Estimation of z_4

Fig. 2. Estimation results of the proposed HGESO using sinusoidal reference, $x_{1_d}(t) = 0.02\sin(\pi t)$

A. Simulations

Simulations were performed by MATLAB/Simulink in which a loading force was used as the external disturbance. In order to study the robustness of the proposed method, different parameter values were used in the EHS (6), the nonlinear controller (55), and the HGESO (38). The nominal values given in Table 1 were used in the nonlinear controller (55) and the HGESO (38). The parameter uncertainties were at most 20% of the nominal values in the EHS Eq. (6).

1) Sinusoidal Reference Tracking: Fig. 1 shows the position tracking performance of the proposed method using the sinusoidal reference $x_{1_d}(t) = 0.02\sin(\pi t)$. It can be seen that the position of the EHS matched the desired position trajectory well even though the disturbance consisting of both the parameter uncertainties and the load force was present. Fig. 2 shows the estimation performances of z_1, z_2, z_3, and z_4, in which it can be seen that the estimated state variable tracked the actual state variables quite well for z_1, z_2, and z_3. The relatively small estimation errors in z_4 were caused by the disturbance consisting of the parameter uncertainties, the load, and the friction. Despite this estimation error of z_4, the position tracking error was suppressed by the nonlinear gain effect. Consequently, using the estimated disturbance, the total disturbance including parameter uncertainties and external disturbances was successfully compensated for by the controller.

Fig. 3 shows the position tracking performance of the proposed output feedback control using the fast sinusoidal reference $x_{1_d}(t) = 0.02\sin(4\pi t)$ for validation. It can be observed that the position of the EHS matched the desired position trajectory well, although the fast reference was used and the

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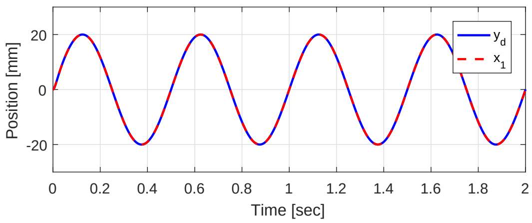
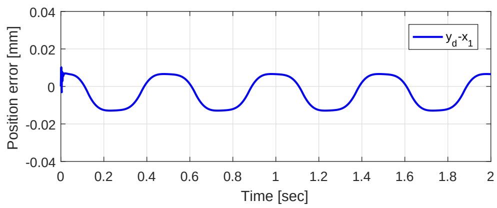
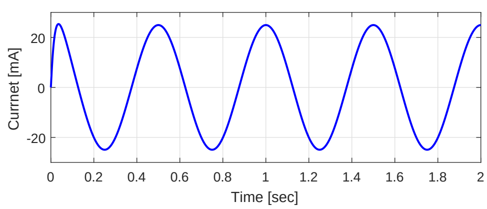


(a) Position tracking performance



(b) Position tracking error



(c) Control input

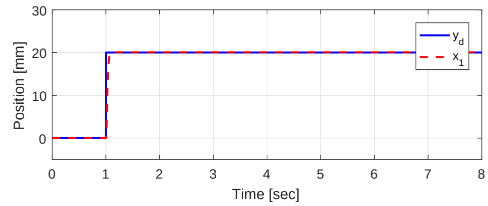
Fig. 3. Position tracking performance of proposed method using fast sinusoidal reference, $x_{1_d}(t) = 0.02\sin(4\pi t)$

total disturbance included both the parameter uncertainties and the load. The amplitude of the position error was less than 0.1% that of the reference signal. Compared to the result for sinusoidal reference $x_{1_d}(t) = 0.02\sin(\pi t)$, the use of the fast reference $x_{1_d}(t) = 0.02\sin(4\pi t)$ resulted in an increase in control input.

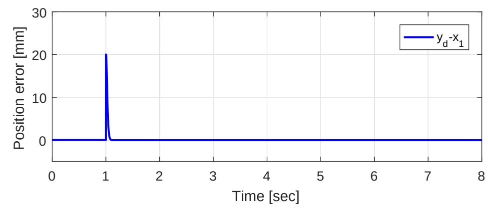
2) Step Reference Tracking: Fig. 4 shows the position tracking performance of the proposed method using step reference tracking. The position of the EHS tracked the desired position trajectory well, although the disturbance included the parameter uncertainties and the load. Fig. 5 shows the estimated values of z_1, z_2, z_3, and z_4. The estimated state variables tracked the actual state variables well. In the estimation of z_4, the relatively large estimation errors were caused by the step reference in the first second. Despite these relatively large estimation errors, the position tracking error was rapidly compensated for by the nonlinear gain effect.

B. Experiments

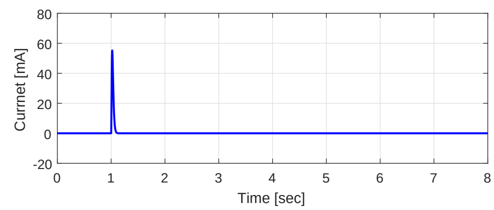
The experimental setup is depicted in Fig. 6, in which a flapper-type servo valve (M454, Star Hydraulic Ltd.) was used for the evaluation. The effective stroke length of the piston was ± 0.043 m. In order to validate the estimation performance, the position and force of the servo were measured by a linear variable differential transformer (LVDT) and a load cell, respectively. The velocity was numerically approximated by differentiating the position measurement using the forward Euler method. The control law was programmed in the C



(a) Position tracking performance



(b) Position tracking error



(c) Control input

Fig. 4. Position tracking performance of the proposed method using step reference

TABLE II
SUMMARY OF THE CONTROL PERFORMANCE COMPARISON

Item	Backstepping Contr.	Proposed method
Peak error	1.11 mm	0.39 mm
Phase lag	0.17 sec.	0.13 sec.

language with real-time implementation at a sampling rate of 1 kHz. The controller utilized a 12-bit analog-to-digital converter (ADC) and a digital-to-analog converter (DAC) with an EtherCAT interface and a dual-core 2.16-GHz processor (C6920, Intel) using the TwinCAT 3 extended automation system. The position tracking performance of the proposed method was compared to that of the following conventional backstepping controller:

$$\begin{aligned}
e_0 &= \int_0^t (x_{1_d} - x_1)d\tau \\
e_1 &= x_{1_d} - x_1 \\
x_{2_d} &= k_{1_b}e_1 + \dot{x}_{1_d} + k_{0_b}e_0 \\
e_2 &= x_{2_d} - x_2 \\
x_{3_d} &= \frac{1}{a_3}(k_{2_b}e_2 + \dot{x}_{2_d} + a_1x_1 + a_2x_2 + a_4F_L + e_1) \\
e_3 &= x_{3_d} - x_3 \\
u &= \frac{k_{3_b}e_3 + \dot{x}_{3_d} + h_1x_2 + h_2x_3 + a_3e_2}{h_3\sqrt{P_s - \mathrm{sgn}(k_{3_b}e_3 + \dot{x}_{3_d} + h_1x_2 + h_2x_3 + a_3e_2)x_3}}
\end{aligned} \tag{72}$$

where $k_{0_b} = 10$, $k_{1_b} = 1300$, $k_{2_b} = 6000$, and $k_{3_b} = 120$ are the control gains. The desired position trajectory, $x_{1_d}(t) = 0.02\sin(\pi t)$ was used to provide the comparative evaluations.

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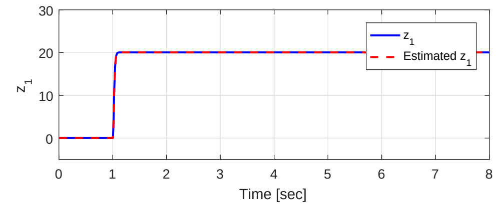
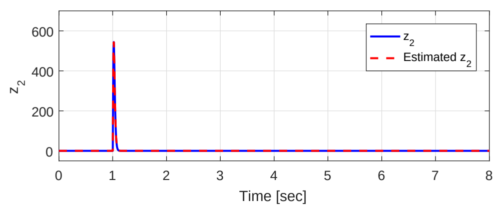
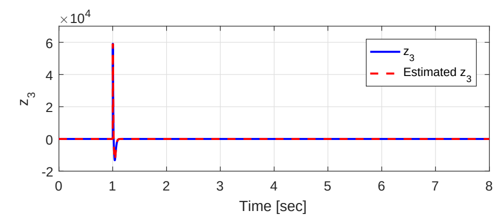


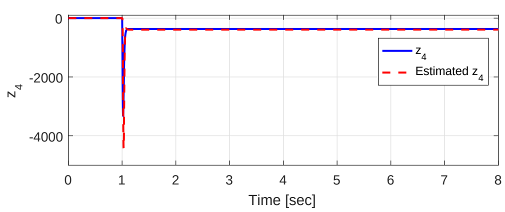
(a) Estimation of z_1



(b) Estimation of z_2



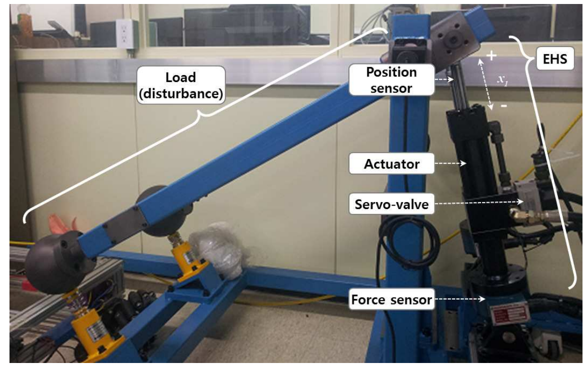
(c) Estimation of z_3



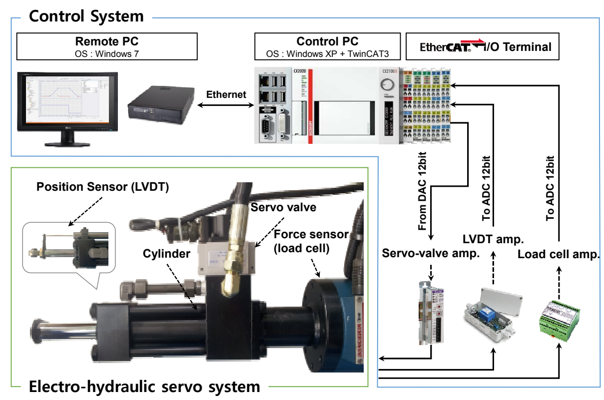
(d) Estimation of z_4

Fig. 5. Estimation results of the proposed HGESO using step reference

The position tracking performances of the conventional and proposed methods are shown in Figs. 7 and 8, respectively. It can be observed in the HGESO estimation results in Fig. 9 that the estimation performances were in good agreement with the measured signals. Notably, the measured z_3 exhibits a very noisy signal, and in both methods, as the estimated z_3 was used instead of the measured z_3, this noise in the position tracking signal was reduced. In the experiments, it was impossible to measure the total applied disturbance; however, we expected that the applied disturbance was also large because the estimated disturbance was large. Due to the effects of this large disturbance, the position tracking error of the conventional backstepping controller was larger than that of the proposed method. Furthermore, the phase lag was relatively large under the conventional backstepping controller, and its position tracking error was asymmetric because of the asymmetry of the disturbance. The main difference between the proposed method (55) and the backstepping controller (72) is the use of the nonlinear gain (13). The proposed method exhibited a relative improvement in position tracking performance as a result of the nonlinear gain, which increased with increasing $\hat{d} = \hat{z}_4$ compared to the conventional backstepping controller

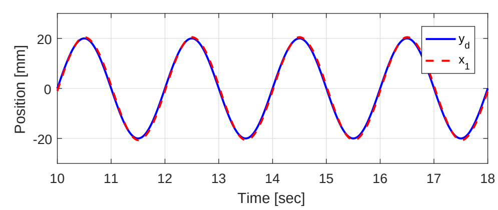


(a) Photo of experimental setup

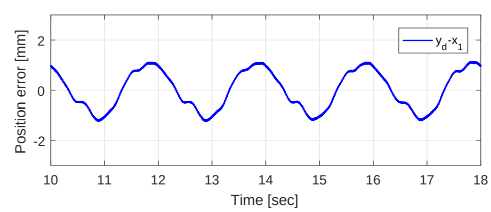


(b) Block diagram of experimental setup

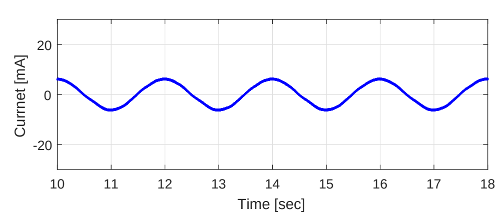
Fig. 6. Experimental setup of the EHS system



(a) Position tracking performance



(b) Position tracking error



(c) Control input

Fig. 7. Position tracking performance of backstepping controller

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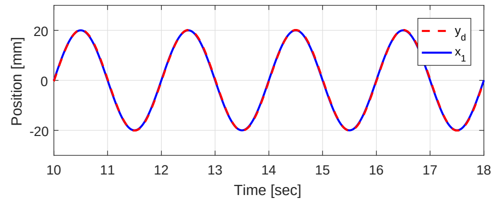
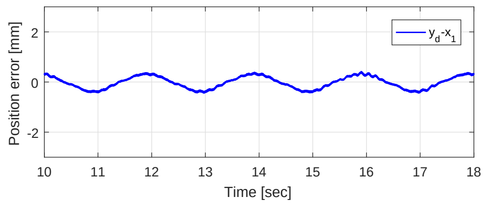
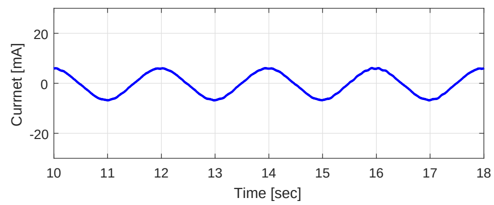


(a) Position tracking performance

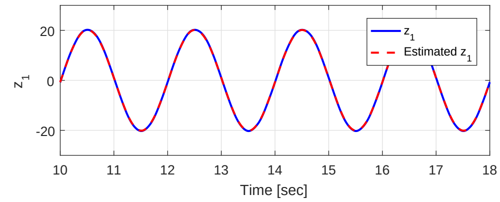


(b) Position tracking error

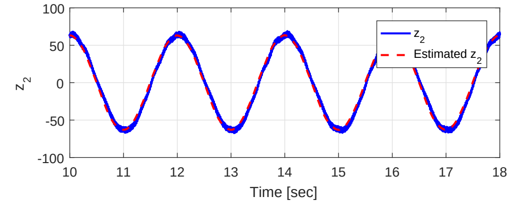


(c) Control input

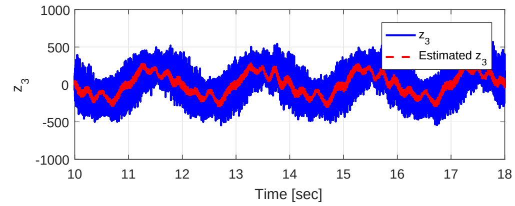
Fig. 8. Position tracking performance of proposed controller



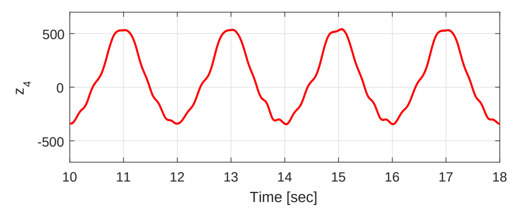
(a) Estimation of z_1



(b) Estimation of z_2



(c) Estimation of z_3



(d) Estimation of z_4

Fig. 9. Estimation results of the HGESO

(72). The position tracking error was less than 2% of the reference using the proposed method. Because the asymmetric disturbance z_4 estimated by the HGESO was compensated for, the position tracking error was symmetric. Consequently, the nonlinear gain, as well as the disturbance compensation, provided by the proposed control method facilitated the reduction of the position tracking error. The control performance comparison of the conventional backstepping controller and the proposed method is summurized in Table. II.

VII. Conclusion

A nonlinear control method was developed using an HGESO for position tracking and compensation of an EHS using only position feedback. The HGESO was designed to estimate the full system state and disturbance, including the system uncertainties and external disturbances. The proposed nonlinear controller was developed to suppress the position tracking error using the ISS property. The position tracking performance of the proposed method was then validated using both simulations and experiments, and it was shown that the proposed HGESO estimated the full state and total disturbance quite well. When a relatively large disturbance estimation error appeared, the position tracking error was suppressed by the nonlinear gain component of the proposed method. Compared to a conventional backstepping controller, the proposed method exhibited improved position tracking performance, showing promise in applications to improve the accuracy of EHS position tracking.

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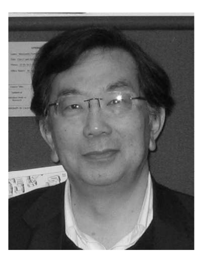

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