

# Stability control of High Speed Train using Robust PID Controller

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**Abstract**-This paper presents control applications to high speed train. Aim of this study is to improve the ride quality and to minimize the semi-active suspension vibrations within tolerable limits by means of applying control technique. The system model considered here is of 17 DOF. This multi input and multi output model has so many difficulties to tackle on. Earlier so many control techniques including Linear Quadratic Regulator have been used to improve the system quality. Here robust PID controller is used to control vibration for high speed train. Simulation studies has been carried out using Matlab/Simulink.

**Keywords:** Robust PID, High speed train, Control, stability

## I. INTRODUCTION

If we talk about transportation, railways play an important role and it is the most convenient method for transportation. Stability, control, safety, time are the prime factors in high speed train. Hence lightweight design of train system is recommended in high speed train, because of lightweight, it may lead to a lot of vibrations which is harmful for passengers. So suspension systems are used to provide ride comfort and stability suspension systems reduce vibration of the system. High speed train consists following suspension system [14].

### a) Primary suspension system:

It is basically a mechanical translational system which consist of mass, spring, and damper elements which is used as a structural suspension system of the train. This system is situated between the bogie and the axle box in every bogie.

### b) Secondary suspension system:

The secondary suspension interconnects the car body and bogie. It has the purpose of isolating the car body from excitations transmitted from track inconsistencies through bogie frames and wheel sets. This is primarily to aid comfort the passengers travelling.

Suspension is divided in general as active and passive suspension system [1, 2]. Passive suspension systems consist of fixed parameters like damper, springs etc. These are fixed parameters because these parameters do not change when external disturbance act on it. Pneumatic, hydraulic & electro-mechanical actuators are the part of actuator suspension system

to provide control forces. Actuator suspension system [14, 15] improves the system performance in different operating conditions. Semi active suspension system has an advantage that system parameter can be monitored and can be adjusted in real time. It has variable damping, variable stiffness, and smooth spring. Few control techniques [13] have already been applied on railway vehicle model. Extended Kalman filter technique is applied on damping controller to improve the performance (ride equality).

Control technique based on sky-hook approach had proposed to minimize the coefficient of derailment in railway vehicles by restricting the vibration of wheel-set and bogie [4]. A combination of Kalman estimator and LQR Controller had been used to enhance the ride quality performance of high speed railway vehicle. Robust control method had used to minimize the vibrations in high speed car body portion [3]. Hybrid control techniques like sliding mode based adaptive technique have also been used in performance improvement of high speed train.

Vibrations or swaying motion of a railway vehicle [11,12] causes coning action. It is the result of the interaction of inertial forces and adhesive forces. These two forces play a vital role in the motion control in a railway system. When speed increases, oscillations may occur due to the difference in magnitude of inertial and adhesion forces this hunting may also create problems such as track damages, wheel damages, derailment etc. The primary suspension system is proposed to improve the hunting stability.

To overcome the difficulty of track irregularities, secondary suspension system has been recommended. Hence track irregularities are problem for stability so it leads to instability which is not desirable for any system (railway system). There are many experiments done to check the stability of high-speed trains. There are always vibrations when the speed of train gets high. Gatiman express is the fastest train of India which has the speed up to 160km/hr.

We are using primary and secondary suspension simultaneously to upgrade the performances like hunting and ride quality of the high-speed trains. Secondary suspension can

only improve ride quality, so we use primary suspension for hunting stability. PID controller is used to improve performances of high-speed train because it is simple and efficient technique and can be implemented easily.

## II. SYSTEM DYNAMICS

The railway vehicle system consists of four wheel-sets, car body and two bogies. The front and rear bogies to the car body are connected with the help of secondary suspension system. With the help of primary suspension system [10], two wheel sets are also connected to each of the two bogies. System dynamics equations for the railway vehicle system are given below.

### a) Car body dynamics

$$\begin{aligned} m_c \ddot{y}_c &= -k_{sy}(2y_c - 2h_{cs}\theta_c - y_{t1} - y_{t2}(1) \\ &- h_{ts}\theta_{t1} - h_{ts}\theta_{t2}) - c_{sv}(2\dot{y}_c - 2h_{cs}\dot{\theta}_c - \dot{y}_{t1} \\ &- \dot{y}_{t2} - h_{ts}\dot{\theta}_{t1} - h_{ts}\dot{\theta}_{t2}) + f_{s1} + f_{s2}. \\ I_{cz}\ddot{\phi}_c &= -k_{sx}(2\phi_c - \phi_{t1} - \phi_{t2})d_s^2 \quad (2) \\ &- c_{sx}(2\dot{\phi}_c - \dot{\phi}_{t1} - \dot{\phi}_{t2})d_s^2 - k_{sy}(2l\phi_c - y_{t1} + y_{t2} \\ &- h_{ts}\theta_{t1} + h_{ts}\theta_{t2})l - c_{sy}(2l\dot{\phi}_c - \dot{y}_{t1} + \dot{y}_{t2} \\ &- h_{ts}\dot{\theta}_{t1} + h_{ts}\dot{\theta}_{t2})l + (f_{s1} - f_{s2})l. \end{aligned}$$

$$\begin{aligned} I_{cx}\ddot{\theta}_c &= k_{sy}(2y_c - 2h_{cs}\theta_c - y_{t1} - y_{t2} \quad (3) \\ &- h_{ts}\theta_{t1} - h_{ts}\theta_{t2})h_{cs} + c_{sy}(2\dot{y}_c - 2h_{cs}\dot{\theta}_c \\ &- \dot{y}_{t1} - \dot{y}_{t2} - h_{ts}\dot{\theta}_{t1} - h_{ts}\dot{\theta}_{t2})\theta_c h_{cs} \\ &- k_{sz}(2\theta_c - \theta_{t1} - \theta_{t2})d_s^2 - c_{sz}(2\dot{\theta}_c - \dot{\theta}_{t1} - \dot{\theta}_{t2})d_s^2 \\ &- (f_{s1} + f_{s2})h_{cs}. \end{aligned}$$

### b) Bogie dynamics

$$\begin{aligned} m_t \ddot{y}_{ti} &= k_{sy}(y_c - (-1)^i l \phi_c - h_{cs}\theta_c - y_{ti} - h_{ts}\theta_{ti}) \quad (4) \\ &+ c_{sy}(\dot{y}_c - (-1)^i l \dot{\phi}_c - h_{cs}\dot{\theta}_c - \dot{y}_{ti} - h_{ts}\dot{\theta}_{ti}) \\ &- k_{py}(2y_{ti} - 2h_{tp}\theta_{ti} - y_{w(2i-1)} - y_{w(2i)}) \\ &- c_{py}(2\dot{y}_{ti} - 2h_{tp}\dot{\theta}_{ti} - \dot{y}_{w(2i-1)} - \dot{y}_{w(2i)}) - f_{si} + (f_{p(2i-1)} + f_{p(2i)}). \\ I_{tz}\ddot{\phi}_{ti} &= k_{sx}(\phi_c - \phi_{ti})d_s^2 + c_{sx}(\dot{\phi}_c - \dot{\phi}_{ti})d_s^2 \quad (5) \\ &- k_{px}(2\phi_{ti} - \phi_{w(2i-1)} - \phi_{w(2i)})d_p^2 \end{aligned}$$

$$\begin{aligned} &- k_{py}b(2b\phi_{ti} - y_{w(2i-1)} + y_{w(2i)}) - \\ &c_{py}b(2b\dot{\phi}_{ti} - \dot{y}_{w(2i-1)} + \dot{y}_{w(2i)}) + (f_{p(2i-1)} - f_{p(2i)})b. \end{aligned}$$

$$\begin{aligned} I_{tx}\ddot{\theta}_{ti} &= k_{sy}h_{ts}(y_c - (-1)^i l \phi_c - h_{cs}\theta_c - y_{ti} - h_{ts}\theta_{ti}) \quad (6) \\ &+ c_{sy}h_{ts}(\dot{y}_c - (-1)^i l \dot{\phi}_c - h_{cs}\dot{\theta}_c - \dot{y}_{ti} - h_{ts}\dot{\theta}_{ti}) \\ &+ k_{sz}(\theta_c - \theta_{ti}) + c_{sz}(\dot{\theta}_c - \dot{\theta}_{ti})d_s^2 \\ &+ k_{py}h_{tp}(2y_{ti} - 2h_{tp}\theta_{ti} - y_{w(2i-1)} - y_{w(2i)}) \\ &+ c_{py}h_{tp}(2\dot{y}_{ti} - 2h_{tp}\dot{\theta}_{ti} - \dot{y}_{w(2i-1)} - \dot{y}_{w(2i)}) \\ &- 2k_{pz}d_p^2\theta_{ti} - 2c_{pz}d_p^2\dot{\theta}_{ti} - f_{ti}h_{ts} - (f_{p(2i-1)} + f_{p(2i)})h_{tp}. \end{aligned}$$

### c) Wheel set dynamics

$$\begin{aligned} m_w \ddot{y}_{wi} &= k_{py}(y_{tj} - (-1)^i b \phi_{tj} - h_{tp}\theta_{tj} - y_{wi}) \quad (7) \\ &+ c_{py}(\dot{y}_{tj} - (-1)^i b \dot{\phi}_{tj} - h_{tp}\dot{\theta}_{tj} - \dot{y}_{wi}) \\ &- 2f_{z2}[\frac{1}{V}(1 + \frac{\sigma r_0}{Va})\dot{y}_{wi} - \frac{\sigma r_0}{Va}\dot{y}_{ai} - \frac{\sigma r_0^2}{Va}\dot{\theta}_{cli} - \phi_{wi}] \\ &- k_{gy}(y_{wi} - y_{ai} - r_o \theta_{cli}) - f_{pj}. \end{aligned}$$

$$\begin{aligned} I_{wz}\ddot{\phi}_{wi} &= k_{px}(\phi_{tj} - \phi_{wi})d_p^2 + c_{px}(\dot{\phi}_{tj} - \dot{\phi}_{wi})d_p^2 \quad (8) \\ &- 2f_{l1}[\frac{\lambda e a}{r_o}(y_{wi} - y_{ai} - r_o \theta_{cli}) + \frac{a^2}{V}\dot{\phi}_{wi}] + k_{g\phi}\phi_{wi} \end{aligned}$$

Parameter values of the equations (1) to (8) are considered from reference [6]. Lateral movements of the 17-DOF railway vehicle system are considered from reference [6].

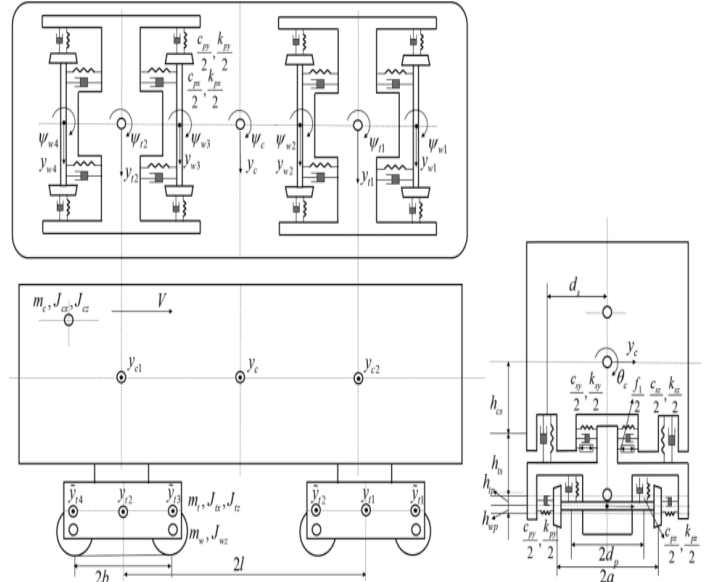


Fig. 1 model of railway vehicle combined with actuator for suspension [7]

### III. FORMULATION OF STATE SPACE MODEL

Let  $q$ ,  $d$ , and  $f$  be defined as the following vectors:

$$q = [yc, \varphi c, \theta c, yt1, \varphi t1, \theta t1, yt2, \varphi t2, \theta t2, yw1, \varphi w1, yw2, \varphi w2, yw3, \varphi w3, yw4, \varphi w4]^T$$

$$d_1 = [y_{a1}, y_{a2}, y_{a3}, y_{a4}, \theta_{cl1}, \theta_{cl2}, \theta_{cl3}, \theta_{cl4}]^T$$

$$d = [d_1, d_1]^T$$

$$f_s = [f_{s1}, f_{s2}], \quad f_p = [f_{p1}, f_{p2}, f_{p3}, f_{p4}]$$

$$f = [f_s, f_p]^T$$

Then, the equations (1) to (8) can be represented as

$$M\ddot{q} + C\dot{q} + Kq = F_f f + F_d d. \quad (9)$$

where  $M(R^{17 \times 17})$ ,  $K(R^{17 \times 17})$ , and  $C(R^{17 \times 17})$  represent the mass, stiffness, and damping matrices of the vehicle system;  $f$  represents the vector of the control forces generated by the actuators;  $d$  represents the vector representing the track irregularities functioned on wheel-sets; and  $F_d(R^{17 \times 16})$ ,  $F_f(R^{17 \times 6})$ , represent the disturbance and control coefficient matrices, respectively [6].

*State space model:*

$$\dot{X}_v = A X_v + B_1 w + B_2 u, \quad (10)$$

$$Z_v = C_1 X_v + D_{11} w + D_{12} u,$$

$$Y_v = C_2 X_v + D_{21} w + D_2 u$$

$$A = \begin{bmatrix} -\frac{2c_{sy}l^2}{J_{cz}} & -\frac{2k_{sy}l^2}{J_{cz}} \\ 1 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \frac{2c_{sy}l^2}{J_{cz}} \\ -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} \frac{l^2}{J_{cz}} \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -\frac{2c_{sy}l^2}{J_{cz}} & -\frac{2k_{sy}l^2}{J_{cz}} \\ 0 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -\frac{2c_{sy}l^2}{J_{cz}} & -\frac{2k_{sy}l^2}{J_{cz}} \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} \frac{2c_{sy}l^2}{J_{cz}} \\ 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} \frac{l^2}{J_{cz}} \\ 1 \end{bmatrix}$$

$$D_{21} = \begin{bmatrix} \frac{2c_{sy}l^2}{J_{cz}} \end{bmatrix}, \quad D_{22} = \begin{bmatrix} \frac{l^2}{J_{cz}} \end{bmatrix}$$

Values of the parameters are taken from reference [7]

### IV. RANDOM TRACK IRREGULARITIES

The main reasons of the vibrations in the high speed trains are geometrical track irregularities. These irregularities comprise of lateral alignment, cross level. This can be expressed as [5]

$$y_a = \frac{y_l - y_r}{2}, \quad \theta_{cl} = \frac{z_l - z_r}{2b}, \quad (11)$$

here,  $y_l$  = lateral track irregularities of the left rail;

$z_r$  = vertical track irregularities of right rail.

$y_r$  = lateral track irregularities of the right rail;

$z_l$  = vertical track irregularities of the left rail;

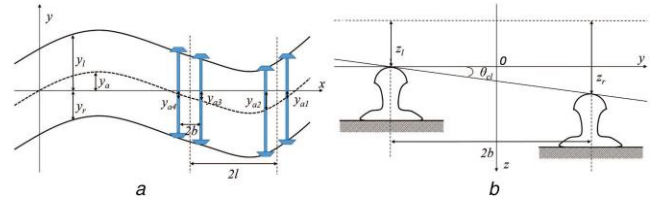


Fig2. Schematic diagram of track irregularities [13]

The random track variations can be determined by their Power Spectral Densities (PSDs), which uses measured data. The one-sided density functions of the lateral alignment and cross-level are obtained by the following equations, respectively

$$S_a(\Omega) = \frac{\Lambda_a \Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_s^2)}, \quad (12)$$

$$S_c(\Omega) = \frac{(\Lambda_v/\beta^2)\Omega_c^2\Omega^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)(\Omega^2 + \Omega_s^2)} \quad (13)$$

Here  $\Omega$  denotes spatial frequency (rad/m),  $\Omega_c$ ,  $\Omega_r$ , and  $\Omega_s$  denote wave numbers (rad/m),  $\beta$  denotes half of the reference distances between the rails,  $\Lambda_a$  and  $\Lambda_v$  denote scalar factor of track irregularities.

If a vehicle travels with a velocity  $V$ , then

$$\omega = V\Omega, \quad \Omega = \frac{2\pi}{L}; \quad (14)$$

Here  $\omega$  denotes angular frequency, then PSD functions would be

$$S_a(\omega) = \frac{\Lambda_a V^2 \Omega_c^2}{(\omega^2 + \tilde{\Omega}_r^2)(\omega^2 + \tilde{\Omega}_c^2)}, \quad (15)$$

$$S_c(\omega) = \frac{(\Lambda_v/b^2)\tilde{\Omega}_c^2\omega^2}{(\omega^2 + \tilde{\Omega}_r^2)(\omega^2 + \tilde{\Omega}_c^2)(\omega^2 + \tilde{\Omega}_s^2)} \quad (16)$$

Here  $\tilde{\Omega}_c = V\Omega_c$ ,  $\tilde{\Omega}_r = V\Omega_r$  and  $\tilde{\Omega}_s = V\Omega_s$

V. CONTROL TECHNIQUE

*Robust PID Controller*

This control technique is conventional technique to minimize the vibrations of car body. It minimizes the difference between the wheel and body displacements to provide the ride quality comfort to passengers. It provides more comfort against riding disturbances.

The PID Controller [7,8] is a linear controller which consist three parameters P, I and D .Here P stands for proportional, I stands for integral and D stands for derivative. So PID basically is proportional integral derivative controller. Its control formula is [9]

$$F=K_p e(t) + K_I \int_0^t e(t) + K_D \frac{de(t)}{dt} \tag{17}$$

Here,  $K_p$  is proportional constant,  
 $K_I$  is integral time constant,  
 $K_D$  is derivative time constant.

The value of parameters gain should be tuned at certain value. We can do manual tuning to tune the gains, but it is not a feasible method so another tuning method called Ziegler-Nichols is used to tune the value of gain. Gain values of PID are corresponding to mass and velocity given in reference [1].

So in this project, gain values of PID Controller are taken as a function of mass and velocity of high speed train are shown below in equation.

$$K_{p,I,D} = a+ bM_b + cV + dM_b^2 + eM_b V + fV^2 + gM_b^2 V + hM_b V^2 + iV^3 \tag{18}$$

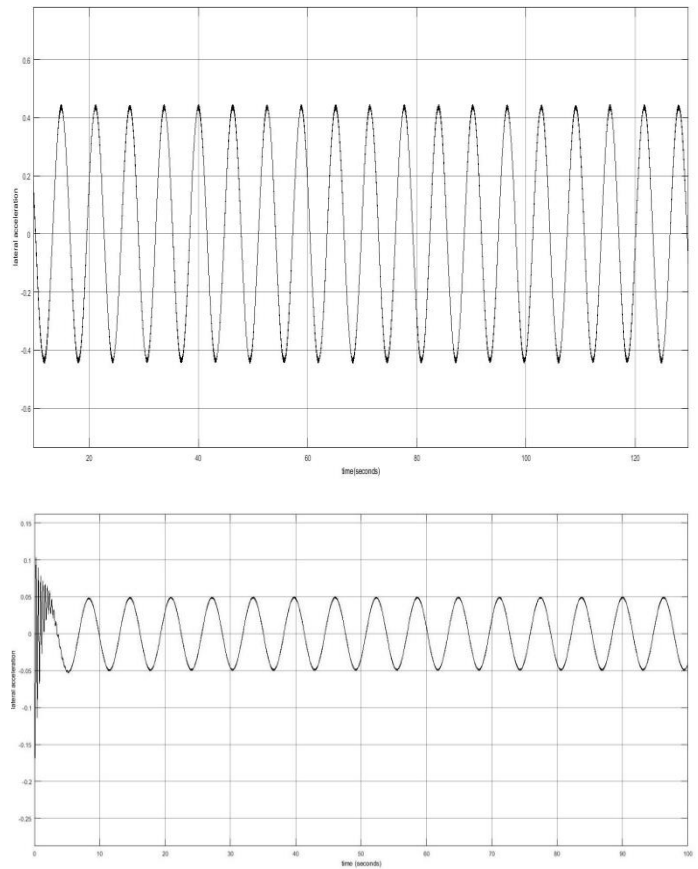
Here a,b,c,...i are constants. Values of these are taken from fitting data of mass, gains and velocity.

We are obtaining results for different values of  $K_p$ ,  $K_I$ ,  $K_D$ . This is shown in table (1).

TABLE (1)

FIGURE	$K_p$	$K_I$	$K_D$
A	1.42E5	32567	150
B	1.42E6	40000	150

Model output (without controller)



FIGURE(A)

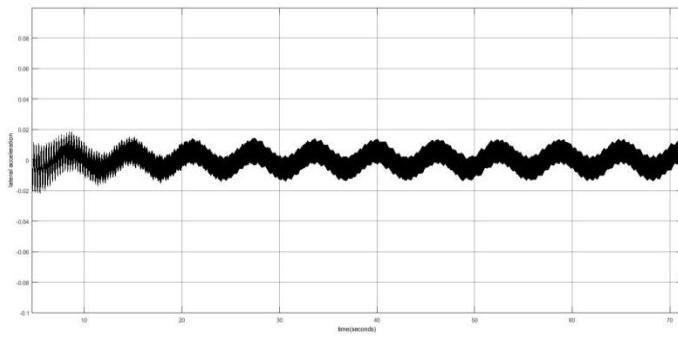


FIGURE (B)

## VI. SIMULATION RESULTS

Proposed PID controller Technique has been implemented in high speed train. It is verified with simulation using Matlab/Simulink software. Results are obtained and depicted in below figures.. Simulation results using without controller and with controller are compared. From results we can easily see that system with PID Controller gives the desirable result. From figure (A) peak overshoot is 0.05 with controller and from figure (B) peak overshoot is 0.02. While without controller peak overshoot is 0.45.

## VII. CONCLUSION

This paper focuses on dynamic modeling and control of high speed train. High speed train model is complex due to hunting stability, derailment, ride quality etc. Motion control of high speed train is a big challenge in transportation system. It is difficult to maintain smooth speed with the help of conventional controls. PID Controller is applied on motion control of high speed train. Performance analysis of high speed train (lateral acceleration of car body, front and rear body) has been carried out with help of MATLAB. These simulation results using PID are comparatively better than the system results without controller.

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