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MIMO PID tuning for nonminimum phase systems: setting attainable limits for a stable behaviour

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Abstract: Nonminimum phase systems are presented in many industrial processes bringing some difficulties in their control scheme. In this paper we propose a methodology to tune multi-input-multi-output proportional-integral-derivative controllers that deals this particularity. The controller design employed here is based in an attainable trajectory determination, for the nonminimum phase system, that considers performance and robustness metrics. Based on that, two different tuning procedures are proposed: an optimization problem, in the frequency domain, that involves nonconvex quadratic matrix inequalities with a linear matrix inequality restriction and a time domain optimization of the simulated system. The methods are an alternative for tuning this kind of system leading the process to the best operational scenario presenting stability, robustness and limited control actions.

Keywords: PID MIMO, PID tuning, quadratic matrix inequalities, nonminimum phase behavior, optimization

1. INTRODUCTION

Nonminimum phase systems are called due to their specific phase response produced by right half-plane (RHP) transmission zero(s) and/or pure time delay. Such systems are seen in a wide processing units, such a drum boiler and distillation columns and their synthesis procedure becomes complex and challenging (Sun, Zhang, Zhang, & Li, 2014). The system who presents this features when disturbed in the direction of this RHP-zero(s) presents inverse response that is an additional difficulty for the controller stability during its operation.

Single-Input-Single-Output (SISO) Proportional-Integral-Derivative (PID) controllers are widespread in industry due to their robustness and simplicity (presenting three adjustable parameters which need to be properly chosen according to the process dynamics). Tuning rules as Ziegler-Nichols, Cohen-Coon and ITAE can provide these parameters for SISO systems, or MIMO PIDs (decentralized SISO PIDs) each at time in successive loop closure (Sung, Lee, & Lee, 1998). An alternative to multi-loop SISO PIDs is to design a MIMO PID controller which uses matrix coefficients and all sensors are used to drive all actuators, with the disadvantages related to the tuning, which require the specification of these three matrices (Boyd, Hast, & Åström, 2008) .

The tuning procedures for MIMO PID controllers are lesser reported than SISO ones, and are basically based on genetic algorithms, evolutionary algorithms, etc. presenting multi-agent based optimization, where the computational times are proportional to the number of these agents, requiring heavy computational time (Ahmad, Azuma, & Sugie, 2014). Besides that, the presence of nonminimum phase systems is still a gap among these techniques, due to its particularities and stability requirements even in SISO PID controllers.

Based on that, it is proposed here, before the tuning procedures, to estimate an (stable) attainable trajectory which the system can follows, with robustness and performance factors embedded. This attainable trajectory will be used as a limit for a MIMO PID tuning via iterated LMI restriction and a MIMO PID tuning for a simulated system.

The paper is divided as follows: in Section 2 is presented how is determined the attainable trajectory; in Section 3 are described the tuning procedures; in Section 4 are shown the case study and the simulations which corroborate the presented techniques and, in Section 5, the conclusions are drawn.

2. ATTAINABLE TRAJECTORY DETERMINATION

The stabilization of a system and the improvement of performance in the presence of uncertainty (in the model and the signals) are the main reasons for the introduction of feedback control (Trierweiler, 1997). Thus, the controller should be tuned in order to attenuate the effect of disturbances providing a good servo/regulatory tracking in a robustly way. A one degree-of-freedom (1-DOF) control system is presented in Fig. 1.

In Fig. 1, K is the controller, G is the plant model, G_d is the disturbance model, r are the reference inputs, u are the control signals, y are the plant outputs, n are the noise to the outputs, d are the disturbances and y_m are the measured outputs.

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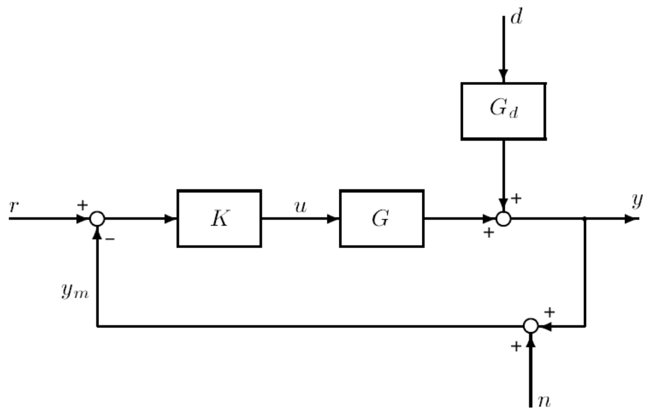

Fig. 1. Block diagram of 1-DOF feedback control system (Skogestad & Postlethwaite, 2005).

Based on Fig. 1, are derived some closed-loop transfer functions:

Sensitivity function (S). This function represents the closed-loop transfer function from the reference signal r to error e and also, for this control system, the function from the output disturbances d to the outputs y, defined as $S = (I + GK)^{-1}$.

Complementary Sensitivity function (T). Represents the closed-loop transfer function from the reference signals r to the outputs y, defined as $T = GK(I + GK)^{-1}$.

Q-function. Transfer function from the reference signal r to control signals u, defined as $Q = K(I + GK)^{-1}$.

One of the main issues in designing feedback controllers is the stability (Skogestad & Postlethwaite, 2005). The benefits from high gain are crucial and increase the danger of loop instability, actuator saturation and sensor noise amplification.

The desired performance of the system can be specified on the desired (output) complementary sensitivity function T_d, that relates the outputs y with the reference signal r. For the SISO case, these specifications can be casted in a second order transfer function (1).

$$T_d \triangleq \frac{1-\varepsilon_\infty}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\zeta s}{\omega_n} + 1} \Leftrightarrow T_d\big(rt, M_p[\%], \varepsilon_\infty\big) \quad (1)$$

where rt is the rise time, M_p is the maximal overshoot, and ε_∞ are the permitted offset (steady-state error), easily calculated from the specifications in the time domain. For the MIMO case, an extension of such specification is to describe the system as a decoupled (or almost decoupled) response, i.e., $T_d = diag\big(T_{d,1}, \dots, T_{d,no}\big)$ (Trierweiler & Farina, 2003).

To compute this function are proposed by Santos, Trierweiler, Farenzena (2017) to find the smallest value for the rise time, respecting the control actions constraints and considering maximal sensitivity robustness criterion. Thus, the following optimization problem is employed:

$$\min_{\Omega} \phi(\Omega) = \min_{\Omega} \sum_{i=1}^{n} (rt_i)^2 \quad (2)$$

subject to

$$rRPN(G) \leq 1.0$$
$$Ms(G) \leq 2.2$$
$$Mk(G) \leq 10.0$$

where $\Omega = [rt_1, \dots, rt_i]$ with $i = 1 \dots n$ being the number of outputs of the complete transfer matrix, $rRPN(G)$ is the relative Robust Performance Number, $Ms(G)$ corresponds the maximal sensitivity value and $Mk(G)$ the maximal value of the Q-function. Considering the value of $rRPN(G) \leq 1$, it is granted that the system correspond to an acceptable Robust Performance Number, that is a measure of a system controllability (Trierweiler, 2002). $Mk(G) \leq 10.0$ ensures that no abrupt control actions are sent to the plant and, the maximal sensitivity function $(Ms \leq 2.0)$ ensures a robustness compromise (according to (Aström & Hägglund, 1995)). Besides that, are considered, for the design of the complementary sensitivity function that is no allowed steady-state error ($\varepsilon_\infty = 0$) and a maximal overshoot of $M_p = 5\%$.

For systems with RHP-poles, RHP-zeros and dead time there are additional performance limitations which result from internal stability conditions. These constraints consider that the transfer function T must preserve the RHP-zero (z) with the same direction as $G(s)$. In a similar way the RHP-poles (p) and the dead time (θ) must not be cancelled by the controller K. These constraints are satisfied through the Blaschke factorization providing an attainable transfer function, T_{alc} (more details about this factorization could be found in Trierweiler (1997)):

$$T_{alc}(s) = B_{O,z}(s) B_{O,z}^{\dagger}(0) T_d(s) \quad (3)$$

where $B_{O,z}(s)$ is the zero output Blaschke factorization, $B_{O,z}^{\dagger}(0)$ denotes the pseudoinverse of $B_{O,z}$ and, T_d is the desired closed-loop performance function. The factorization over the poles and dead time were omitted in this representation.

3. DESIGN PROBLEM

In this section are proposed two tuning procedures for a MIMO PID controller. The controller is given by:

$$C(s) = K_P + \frac{1}{s} K_I + \frac{s}{1+\tau s} K_D \quad (4)$$

where $K_P, K_I, K_D \in \boldsymbol{R}^{m \times p}$, are the proportional gain matrix, integral gain matrix, and derivative gain matrix, respectively, and p and m the number of outputs and inputs, respectively. The constant $\tau > 0$ is the derivative action time constant (assumed to be fixed and chosen as a modest fraction of the desired closed-loop response time).

3.1 MIMO PID tuning via iterated LMI restriction

This tuning procedure was proposed by Boyd, Hast, & Åström (2008) and assumes that for a plant P, $P(0)K_I$ is nonsingular, providing $S(0) = 0$, which means to have zero error for constant reference signals. The objective is to attain the best possible low-frequency sensitivity, by minimizing $\|(P(0)K_I)^{-1}\|$. Besides that, the authors propose constraints as upper limits related to the $\boldsymbol{H}_\infty$-norm of the functions S, T and Q, in the way:

$$\min_{K_P,K_I,K_D} \|(P(0)K_I)^{-1}\|$$

subject to

$$\begin{aligned} \|S\|_\infty &\leq S_{max} \\ \|T\|_\infty &\leq T_{max} \\ \|Q\|_\infty &\leq Q_{max} \end{aligned} \tag{5}$$

where $S_{max} > 1$ and reasonable values are in the range 1.1 to 1.6. The same occurs to T_{max}. Q_{max} is chosen as a multiple (3 to 10) of $1/\sigma_{\min}(P(0))$. These constraints can be expressed as sampling semi-infinite constraints, for example, $\|S(i\omega_k)\| = \|S_k\| \leq S_{max}$, one for each $\omega \geq 0$. The frequency sampled design problem (5) is then casted in a simple form which every constraint has the same quadratic matrix inequality (QMI) form:

$$Z^*Z \succcurlyeq Y^*Y \tag{6}$$

where both Z and Y are affine functions of the variables, Z^* is the (Hermitian) conjugate transpose and, the symbol $\succcurlyeq$ is used to denote matrix inequality, being $Z \succcurlyeq 0$ means that Z is Hermitian and positive semidefinite. The objective function and the constraints become:

$$\begin{aligned} (P(0)K_I)^*(P(0)K_I) &\succcurlyeq t^2 I \\ (I + P_kC_k)^*(I + P_kC_k) &\succcurlyeq (1/S_{max}^2)I \\ (I + P_kC_k)^*(I + P_kC_k) &\succcurlyeq (1/T_{max}^2)(P_kC_k)^*(P_kC_k) \\ (I + P_kC_k)^*(I + P_kC_k) &\succcurlyeq (1/Q_{max}^2)C_k \end{aligned} \tag{7}$$

In order to form a convex LMI restriction for the QMI (6), it is necessary to guarantee that the QMI is convex in Z, since it is already convex in Y. Considering an arbitrary matrix $\tilde{Z}$, are proposed the LMI (8b) that implies the QMI (6), called LMI restriction of the QMI, obtained at point $\tilde{Z}$. The optimization problem proposed is:

$$\text{maximize } t \tag{8a}$$

subject to

$$\begin{bmatrix} Z_k^* \tilde{Z}_k + \tilde{Z}_k^* Z_k - \tilde{Z}_k^* \tilde{Z}_k & Y_k^* \\ Y_k & I \end{bmatrix} \succcurlyeq 0 \tag{8b}$$

This problem has linear objective and LMI constraints, and so it is an SDP. More details about the implementation could be found in (S Boyd, El Ghaoui, Feron, & Balakrishnan, 1994; Stephen Boyd et al., 2008).

The assumptions of the limits for the transfer functions ($S_{max}, T_{max}, Q_{max}$) that compose the control system are that they are fixed and considered the same through the range of frequencies $[\omega_1, \dots, \omega_N]$. However for systems who present nonminimum phase dynamics, these assumptions may not be feasible or present instabilities for the obtained controller. Considering this premise together with the attainable transfer function previously determined, T_{alc}, that is stable and presents performance e robustness metrics embedded (as in Section 2) , it is then proposed to set for the range of frequencies $[\omega_1, \dots, \omega_N]$ the values:

$$\begin{aligned} S_{max,k} &= \left\|I - T_{alc_k}\right\| \\ T_{max,k} &= \left\|T_{alc_k}\right\| \\ Q_{max,k} &= \left\|P_k^{-1} T_{alc_k}\right\| \\ \text{being } k &= [j\omega_1, \dots, j\omega_N] \end{aligned} \tag{9}$$

Considering (9) as the limits in the optimization problem (8) we ensure that the obtained control system will correspond to the attainable trajectory T_{alc}.

Another two considerations could be taken for the limits in the transfer functions (9): (i) the attainable function determination does not take into account about the order of the controller, so an slack ϵ in each constraint should be considered; (ii) in order to give more flexibility to the controller C, the maximal value of each constraint are also an alternative for the parameter determination.

3.2 MIMO PID tuning via simulated system

This alternative for tuning the MIMO-PID controller C, considers again the attainable transfer function determined in Section 2, T_{alc}.

The definition of the Complementary sensitivity function analytically, $T = PC(I + PC)^{-1}$, here purposely changed G and K, for P and C, respectively, gives us the closed-loop transfer function from the reference signals to the outputs. In this way, it is proposed an optimization problem (10), in time domain, which minimizes the error between the attainable and simulated outputs, for the same reference input signals.

$$\min_{K_P,K_I,K_D} J = \min_{K_P,K_I,K_D} \left\| \underbrace{T_{alc}(s).\Delta r}_{y_{alc}} - \underbrace{T(s).\Delta r}_{\hat{y}} \right\| \tag{10}$$

where Δr is the variation of the reference signals, since the $T_{alc}(s)$ and $T(s)$ are transfer matrix.

This procedure aims at finding controller parameters that fit the output response of the original system according to the attainable response. Since the attainable response is stable and presents performance and robustness constraints, the controlled system will present the same features.

4. CASE STUDY

The case study used to exemplify these procedures is the Quadruple-Spherical-Tank (QST) system. This system, presented in Fig. 2, is composed by 4 spherical tanks, connected and fed by two different flow rates F_1 and F_2. A portion x_1 and x_2 are sent to the lower tanks while the complementary $(1 - x_1)$ and $(1 - x_2)$ are sent to the upper tanks.

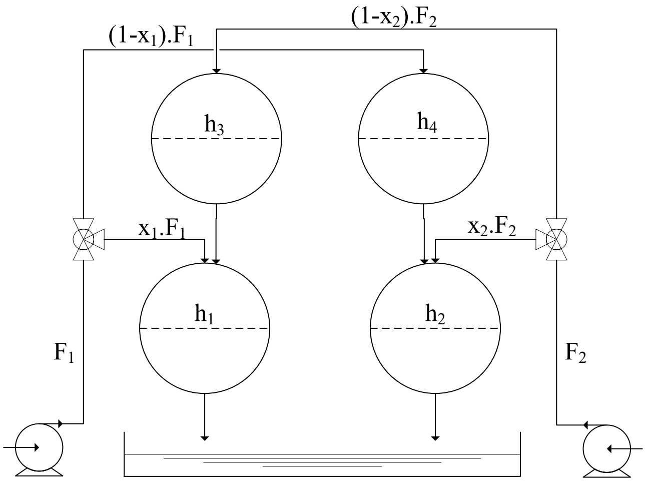

Fig. 2. The QST process.

This system, adapted from Escobar & Trierweiler (2013), presents a subset 2×2, composed by the levels h_1 and h_2 as outputs and the feed flow rates F_1 and F_2 as inputs, in which the arrangement of the fraction valves $(x_1 + x_2 < 1)$ allows the appearing of the RHP transmission zero in the system.

The plant transfer function is:

$$P(s) = \begin{bmatrix} \dfrac{0.00073255}{(s+0.001029)} & \dfrac{2.9224 \times 10^{-6}}{(s+0.001862)(s+0.001029)} \\ \dfrac{3.6665 \times 10^{-6}}{(s+0.002639)(s+0.00129)} & \dfrac{0.00053596}{(s+0.00129)} \end{bmatrix}$$

Each entry is composed by a first-order function (main diagonal) and second-order function. The RHP-zero is $z = +0.0030$ and its value limits the performance of the entire system. When the system is disturbed in the directions of this zero, the process will present inverse response.

All simulations were made at Matlab R2012b in an Intel Core I7-4770S CPU@ 3.10 GHz with 12 GB (RAM).

The attainable trajectory determination was made, taking account the presence of the RHP-zero, robustness and performance factors.

To solve the optimization problem (8) considering the constraints (9), it was used CVX, a package for specifying and solving convex problems (Grant & Boyd, 2008, 2014). The derivative action time constant is chosen $\tau = 300$ and the semi-infinite constraints are sampled using $N = 500$ logarithmically spaced frequencies in the interval $[10^{-5}, 10^{5}]$.

For the consideration (i) it was used an slack variable ϵ in the optimization problem, which takes 220 seconds and 6 iterations to converge to

$$K_P = \begin{bmatrix} 0.7149 & 3.7516 \\ 3.0891 & 2.8821 \end{bmatrix},$$

$$K_I = \begin{bmatrix} 0.0018 & 0.0039 \\ 0.0010 & -0.0033 \end{bmatrix},$$

$$K_D = 10^3 \times \begin{bmatrix} 0.2829 & -0.1051 \\ 0.2186 & -1.3542 \end{bmatrix}$$

Fig. 3 shows the attainable trajectories estimated for the tuning procedure (T_{alc}) and the trajectory achievable by the system (T). Fig. 4 presents the step response of the obtained Q-function. The resulting closed-loop transfer function singular values are plotted in Fig. 5.

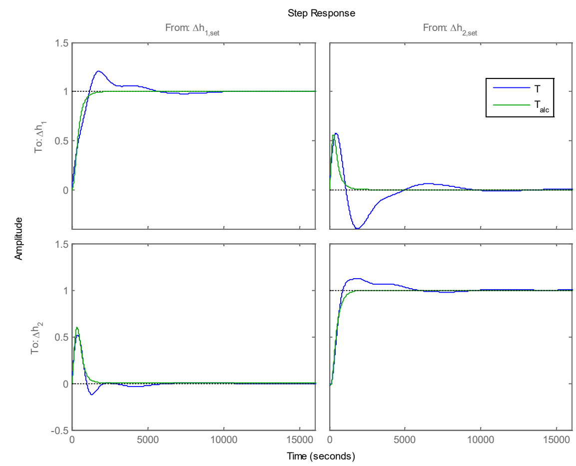

Fig.3. Estimated attainable trajectory (T_{alc}) and trajectory achievable by the system (T) for the consideration (i).

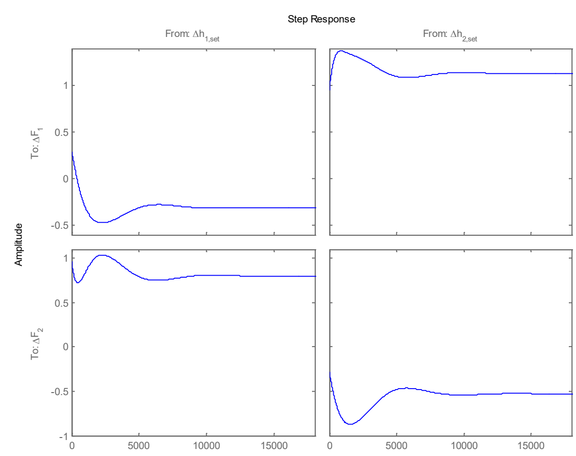

Fig. 4. Step Response of the Q-function for the consideration (i).

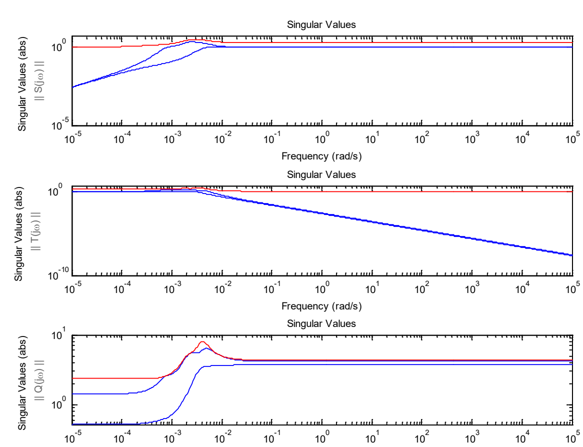

Fig. 5. Closed-loop transfer function singular values (blue line) and constraints (red line) for the consideration (i).

The same procedure was employed for the consideration (ii) that instead of evaluate for each frequency value a maximal limit for the closed-loop functions, the maximal value of each function in the range of frequencies was used. The optimization problem takes 286 seconds and 8 iterations to converge to

$$K_P = \begin{bmatrix} 0.9132 & 3.0633 \\ 2.6173 & 2.1872 \end{bmatrix},$$
$$K_I = \begin{bmatrix} 0.0021 & 0.0043 \\ 0.0010 & -0.0038 \end{bmatrix},$$
$$K_D = 10^3 \times \begin{bmatrix} 0.7720 & 0.2363 \\ 1.0447 & -1.0909 \end{bmatrix}$$

The attainable trajectories are shown in Fig. 6, the step response of the Q-function is presented in Fig.7 and the closed-loop singular values of the transfer functions are shown in Fig. 8.

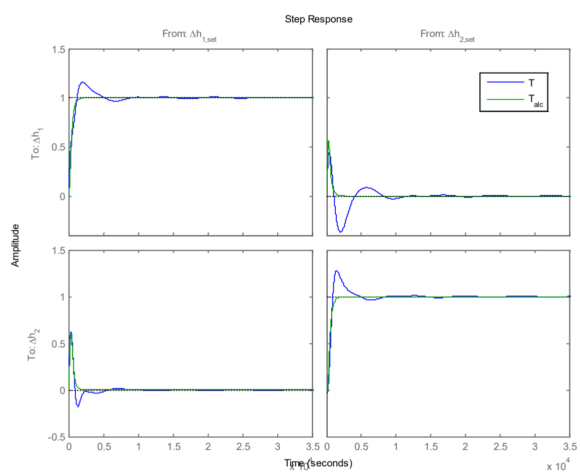

Fig. 6. Estimated attainable trajectory (T_{alc}) and trajectory achievable by the system (T) for the consideration (ii).

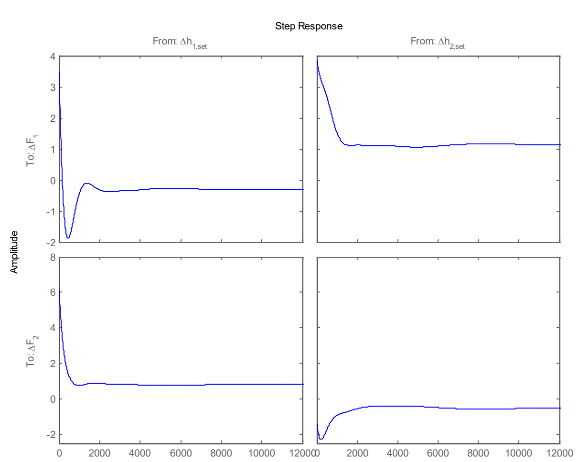

Fig. 7. Step Response of the Q-function for the consideration (ii).

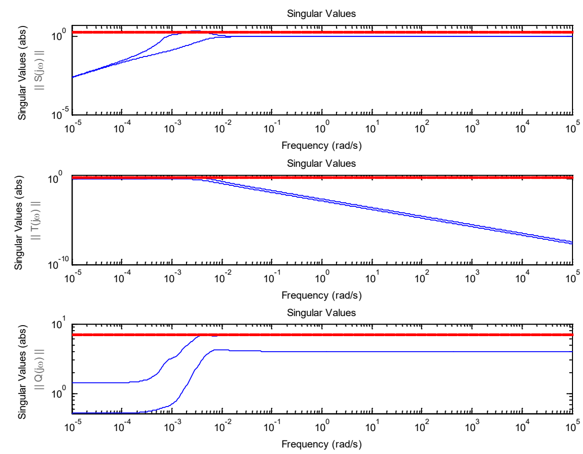

Fig. 8. Closed-loop transfer function singular values (blue line) and constraints (red line) for the consideration (ii).

At the end, it was performed the tuning procedure considering the time domain optimization of the simulated system. To solve the optimization problem (10) it was used the *fminsearch* function at Matlab. The procedure takes 157 seconds and converged to

$$K_P = \begin{bmatrix} 0.0061 & 0.0048 \\ 0.0007 & -0.0004 \end{bmatrix},$$
$$K_I = \begin{bmatrix} 0.2528 & 3.2789 \\ 2.0295 & -0.6267 \end{bmatrix},$$
$$K_D = \begin{bmatrix} 2.2631 & -0.1057 \\ -6.3736 & -0.3102 \end{bmatrix}$$

The step responses of the attainable trajectories and the Q-function are presented in Fig. 9 and Fig. 10, respectively.

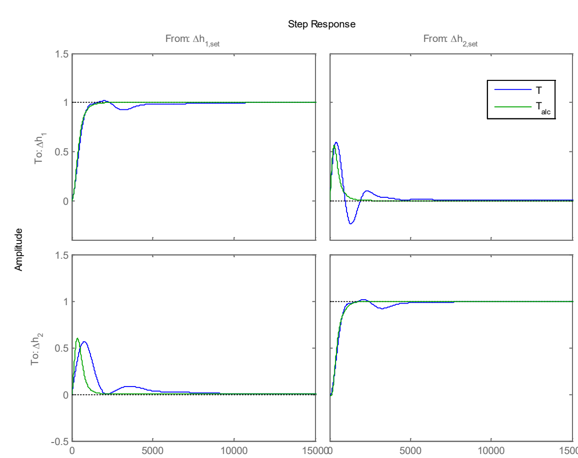

Fig. 9. Estimated attainable trajectory (T_{alc}) and trajectory achievable by the system (T) for the time domain optimization of the simulated system.

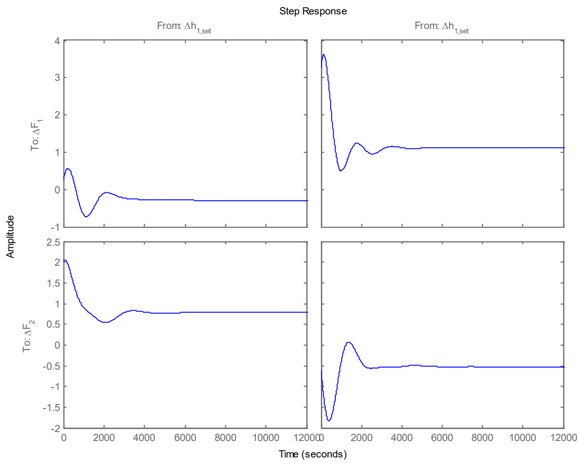

Fig. 10. Step Response of the Q-function for the time domain optimization of the simulated system.

It could be seen that the tuning procedures proposed here can guarantee that the system will remain stable what is not observed when the optimization problems considers the original values for $S_{max}, T_{max}, Q_{max}$.

The step response of the transfer function T shows that the obtained system presents a similar behaviour to T_{alc}, that is the attainable trajectory. In the same way, the step response of the Q-function, shows that for a single variation in the reference signal values the system will present finite amplitude for the control actions, which is important for the servo/regulatory tracking robustness.

5. CONCLUSIONS

In this paper it was proposed an alternative to tuning MIMO-PID controllers for systems who present nonminimum phase behaviour. This strategy was based in the determination of an attainable trajectory function that englobes robustly performance and internal stability factors.

By setting limits based on T_{alc}, in the tuning procedures, it is ensured that the originated system will remain with these characteristics. The main reason of the attainable trajectory determination is that it provides a limit for a stable behaviour.

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