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Brief paper

An adaptive controller for uncertain nonlinear systems with multiple time delays☆

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ABSTRACT

This paper leverages the predictor-based model reference adaptive control (PMRAC) architecture to develop an adaptive compensation scheme for uncertain nonlinear systems with multiple input and state delays. The controller is composed of a state predictor, an auxiliary system, and adaptive laws. The adaptive laws are designed through a Lyapunov function in such a way that the predictor state and the auxiliary state asymptotically converge to the system state given that a stability condition holds. Satisfying this delay-dependent stability condition, formulated in the form of a linear matrix inequality (LMI), also ensures the input-to-state stability of the closed-loop control system. Numerical case studies with a standard F-16 aircraft model are discussed to illustrate the efficacy of the proposed control framework.

1. Introduction

Designing a control scheme capable of compensating for time delays that destabilize closed-loop systems is a notoriously challenging problem. The key issue is that when the feedback signals go through the control channels, they are lagging behind in time with respect to the system state due to the delays in the control inputs and/or feedback channels. As a result, the feedback signals do not have the stabilizing effect they are designed to achieve (Fridman, 2014; Hale & Lunel, 2013). To overcome the challenge, many standard methods have been developed. For example, the classical Smith predictor was proposed in Smith (1959) for stable delay systems with its notable modifications in Astrom, Hang, and Lim (1994), Matausek and Micic (1996) and Watanabe and Ito (1981) which produce improved performances for systems with an integral mode. Other extensions in Paor (1985) and Majhi and Atherton (2000) enable the stabilization of unstable processes with delays. Subsequent classical methods include the Artstein reduction scheme, which transforms a system with an input delay into one without a delay, the Padé approximants (Lam, 1993) and Hankel operators (Curtain & Zwart, 2012), which approximate the delay terms by rational transfer functions, sliding mode control (Roh & Oh, 1999), and $\mathcal{H}_\infty$ control via a descriptor system (Fridman & Shaked, 2002).

One of the most common methods to compensate for an input delay is to include linear feedback in terms of a predicted state. The predicted states are usually formulated in the form of an integral over the past values of the control input. These predictor feedback algorithms are often referred to as finite-spectrum assignment controllers, in which the desired placement of a finite number of eigenvalues can be achieved by designing the feedback gain matrices (see Kwon & Pearson, 1980; Manitius & Olbrot, 1979; Mondie & Michiels, 2003).

The predictor feedback methods provide the basic structures for subsequent innovations in delay compensation. Specifically, recent seminal developments in Krstic (2009, 2010), Tsubakino, Krstic, and Oliveira (2016) and Zhu, Krstic, and Su (2017) transform a delay system into a cascade PDE-ODE structure, where boundary control techniques are employed for stabilization. Notable boundary controllers include the backstepping design that leads to an output feedback predictor scheme for LTI systems with input delays in Cacace and Germani (2017), with both state and input delays in Kharitonov (2017), and an observer-predictor controller in Mazenc and Malisoff (2017). Work in Bekiaris-Liberis and Krstic (2010) developed a predictor-feedback algorithm to compensate for unknown state and input delays in linear feedforward systems. A local-stabilization PDE boundary control technique is formulated in Zhu, Krstic, and Su (2018b) to mitigate the effects of distinct and unknown input delays in LTI systems. This technique was then modified in Zhu, Krstic, and Su (2018a) assuming that the actuator state is measurable to achieve the stronger global stabilization. Work in Zhu, Krstic, and Su (2019) extended this framework to cover output feedback linear systems.

Recent work in Liu and Zhou (2016, 2018) and Zhou (2014) proposes a class of modifications, named nested prediction, to

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Table 1
Comparison between the related papers with respect to the present paper.

	Nonlinear	Uncertain	I-D	S-D
Tsubakino et al. (2016) and Zhu et al. (2018b)	No	No	Multiple	No
Zhu et al. (2017)	No	Yes	Single	No
Cacace and Germani (2017), Kharitonov (2017) and Liu and Zhou (2018)	No	No	Multiple	Yes
Bekiaris-Liberis and Krstic (2010) and Mazenc and Malisoff (2017)	No	No	Single	No
Zhu et al. (2018a) and Zhu et al. (2019)	No	Yes	Multiple	No
Liu and Zhou (2016) and Zhou (2014)	No	No	Single	Yes
Bekiaris-Liberis and Krstic (2012), Bresch-Pietri and Krstic (2014) and Karafyllis and Krstic (2016)	Yes	No	Single	No
Bekiaris-Liberis and Krstic (2013), Deng, Yao, and Ma (2017) and Sharma, Bhasin, Wang, and Dixon (2011)	Yes	No	Single	No
Bekiaris-Liberis and Krstic (2016)	Yes	No	Multiple	No
Chakrabarty, Fridman, Zak, and Buzzard (2018)	Yes	No	No	Yes
Nguyen (2018)	Yes	Yes	Single	No
This paper	Yes	Yes	Multiple	Yes

the integral that results in the predicted state. The goal is to deal with delays in both state and control input, which the traditional predictor-feedback controllers are not designed to handle when the input delay is larger than the state delay. Several recent papers addressing nonlinear systems include the predictor-feedback schemes in Bekiaris-Liberis and Krstic (2012, 2016) and Bresch-Pietri and Krstic (2014) with a delay estimation algorithm, in Karafyllis and Krstic (2016) for systems with a compact absorbing set, in Sharma et al. (2011) for an Euler–Lagrange system, in Bekiaris-Liberis and Krstic (2013) for an state-dependent input delay, in Deng et al. (2017) for output-feedback systems with additive disturbance, and in Chakrabarty et al. (2018) for delayed-measurement systems. However, these schemes were designed to compensate for a single input delay. Furthermore, all mentioned controllers for nonlinear systems require that the governing dynamical equations (the plants) are known to the controllers.

One critical property of predictor-feedback schemes is that their control implementation requires approximating the integral terms by a finite sum. As shown in Engelborghs, Dambrine, and Roose (2001), this approximation may cause stability loss. Furthermore, when the number of discretization points is large to improve accuracy, the implementation requires excessive computation time. For example, as reported in Table 1 in Liu and Zhou (2018), it takes 432 s to complete a 30-s simulation with 40 discretization points used to approximate the predictor-feedback integral. This raises issues regarding the practicality of the discretization schemes for predictor-feedback approaches in actual applications since the computation of the controls must be completed within a loop, which is usually sub-millisecond.

In this paper, we develop a control framework capable of compensating for multiple delays in the states and control inputs of an uncertain nonlinear system while maintaining desirable transient and steady-state performances. Systems with multiple delays in the control inputs and states have different structures as compared to the single delay counterparts. Furthermore, when the input delay is larger than the state delay, the computation of the state predictor requires the future state, hence is not implementable (Zhou, 2014).

To address the challenges with multiple delays, we leverage the PMRAC framework in Lavretsky, Gadient, and Gregory (2010), Nguyen (2018), Nguyen and Dankowicz (2019) and Nguyen, Li, and Dankowicz (2018) which was shown to produce improved transient characteristics as compared to the classical MRAC algorithms. In particular, the architecture proposed in this paper is composed of four key components: the auxiliary system, the state predictor, the adaptive laws, and the control input formulation. The auxiliary system and state predictor are constructed in such a way that the deviations between each of these systems and the system state are independent of the input delays. The adaptive laws are then formulated so that a Lyapunov function is negative, and hence, the deviations converge to zero asymptotically given that a delay-dependent stability condition is maintained. Finally, the control law is designed for the adaptive estimates to minimize the effects of uncertain elements in the system dynamics.

This delay-compensation structure entails several advantages. It is able to stabilize uncertain nonlinear systems with multiple input and state time delays. Every component in the control architecture is simple and easily implementable and does not require functional integrals to calculate the predicted state and controls. Hence, there is no concern regarding the integral discretization and implementation associated with predictor-feedback controllers. Furthermore, since it leverages the classical MRAC structure, the work has very low barriers to entry. Any control engineer familiar with MRAC should be able to deploy the delay-compensation schemes for specific tasks. The proposed scheme also inherits the ability to achieve excellent transient tracking performance from the original MRAC.

To emphasize the contributions of this paper to the current literature on the topic, Table 1 provides an exhaustive comparison between the recent closely related papers with respect to the present paper in terms of whether or not the system of interest is nonlinear, uncertain, in the presence of input delays (I-D) and state delays (S-D).

The limitations of this work are: (i) It assumes the coefficients of the control inputs are known; (ii) There is an upper bound for the input delays, beyond which the closed-loop control system is destabilized; and (iii) The norm of the nonlinearity must grow linearly with the norm of the state.

2. Model definition and problem statement

We investigate an efficient compensation scheme for multiple delays in an uncertain nonlinear system of the form:

$$\dot{x}_p = A_p x_p + B_p\left(J_p^\top x_p(t-h) + d(x_p)\right) + \sum_{i=1}^{m_c} B_{pi} u_i(t-\tau_i),\ \ x_p(t) = x_{p0}(t)\ \forall t \in [-h, 0] \quad (1)$$

and $y = C_p x_p$, where $x_{p0}(t)$ is a continuous function, $x_p \in \mathbb{R}^{n_p}$, $y \in \mathbb{R}^m$, and $u = \begin{bmatrix} u_1 & u_2 & \dots & u_{m_c}\end{bmatrix} \in \mathbb{R}^m$ represent the state, output, and control input of the dynamical system, respectively, $u(t) = 0\ \forall t \in [-\max\{\tau_i\}, 0]$, for $i = 1, \dots, m_c$, $A_p \in \mathbb{R}^{n_p \times n_p}$, $J_p \in \mathbb{R}^{n_p \times m}$, and $B_p = \begin{bmatrix} B_{p1} & B_{p2} & \dots & B_{pm_c}\end{bmatrix} \in \mathbb{R}^{n_p \times m}$. Here, B_p and C_p are *known* constant matrices, while A_p and J_p are *unknown* constant matrices. In addition, the nonnegative constant h is the *known* time delay in the state, the quantities $\tau_i \geq 0$, $i = 1, 2, \dots, m_c$ denote *known* constant time delays in the control inputs $u_i(t) \in \mathbb{R}^{m_c}$.

The nonlinear function, $d(x_p)$, is *uncertain* and can be parameterized as $d(x_p) = \Theta^\top \Phi(x_p)$, where $\Theta \in \mathbb{R}^{N \times m}$ is a matrix that contains *unknown* parameters, and the state-dependent nonlinear regressor $\Phi(x_p) \in \mathbb{R}^N$ satisfies the following assumption.

Assumption 1. The nonlinearity $\Phi(x_p)$ satisfies

$$\|\Phi(x_p)\|_2 \leq b_\Phi \|x_p\|_2, \tag{2}$$

where b_Φ is a positive constant.

Indeed, the parameterization of the nonlinearity is a standard property and can be applied to all Lagrangian systems. It has been used extensively in control and robotics literature, especially adaptive control. See Craig, Hsu, and Sastry (1987) and Slotine and Li (1987), and the vast literature therein, for robotics applications and Lavretsky and Wise (2013) for aerospace applications.

The control objective is for $y(t)$ to track a desired reference trajectory $r(t)$ despite the destabilizing effects of multiple state and input delays. Inspired by Lavretsky et al. (2010), we define a new state variable

$$x^\top := \left[\int_0^t \Big(y(\lambda) - r(\lambda)\Big)^\top \mathrm{d}\lambda \quad x_p^\top\right], \tag{3}$$

where $x \in \mathbb{R}^n$ with $n = n_p + m$. The reason for defining this new state variable is to incorporate the integrated tracking error into the extended system state vector in (3). This helps reduce not only the tracking error itself, i.e. $y(t) - r(t)$, but also over the duration of the error, i.e. the integral operation. As a result, this integral term accelerates the movement of the output $y(t)$ toward the reference $r(t)$ and eliminate the residual steady-state error, This is similar to the action of adding an integral term to a proportional controller to form a PI controller.

By taking (3) into account, Eq. (1) is equivalently reformulated into the following augmented system:

$$\begin{aligned}\dot{x} &= Ax + B\Big(J^\top x_h + \Theta^\top \Phi(x)\Big) \\ &\quad + \sum_{i=1}^{m_c} B_i u_i(t-\tau_i) + B_c r, \quad x(t) = x_0(t)\ \forall t \in [-h, 0]\end{aligned} \tag{4}$$

and $y = Cx$, where $x_h := x(t-h)$, $x_0(t)$ is a continuous function, and

$$\begin{aligned} A &= \begin{bmatrix} 0_{m\times m} & C_p \\ 0_{n_p\times m} & A_p \end{bmatrix} \in \mathbb{R}^{n\times n},\ B_c = \begin{bmatrix} -\mathbb{I}_{m\times m} \\ 0_{n_p\times m} \end{bmatrix} \in \mathbb{R}^{n\times m}, \\ J &= \begin{bmatrix} 0_{m\times m} \\ J_p \end{bmatrix} \in \mathbb{R}^{n\times m},\ C = \begin{bmatrix} 0_{m\times m} & C_p \end{bmatrix} \in \mathbb{R}^{m\times n}, \\ B_i &= \begin{bmatrix} 0_{m\times m} \\ B_{pi} \end{bmatrix},\ B = \begin{bmatrix} B_1 & B_2 & \dots & B_{m_c} \end{bmatrix} \in \mathbb{R}^{n\times m}. \end{aligned} \tag{5}$$

We assume that these matrices satisfy the following matching condition.

Assumption 2. The system is stabilizable, i.e. there exists a matrix K such that $A_m = A + BK^\top$, where A_m is a Hurwitz matrix.

The next section presents a strategy to compensate for the state and input delays to drive $y(t)$ toward $r(t)$.

3. The delay-compensation framework

Using the matching condition in Assumption 2, Eq. (4) can be written as

$$\begin{aligned}\dot{x} &= A_m x + B\Big[-K^\top x + J^\top x_h + \Theta^\top \Phi(x)\Big] \\ &\quad + \sum_{i=1}^{m_c} B_i u_i(t-\tau_i) + B_c r.\end{aligned} \tag{6}$$

By partitioning the coefficient matrices in (6) as follows: $K = \begin{bmatrix} K_1 & K_2 & \dots & K_{m_c}\end{bmatrix}$, $J = \begin{bmatrix} J_1 & J_2 & \dots & J_{m_c}\end{bmatrix}$, $\Theta = \begin{bmatrix} \Theta_1 & \Theta_2 & \dots & \Theta_{m_c}\end{bmatrix}$, we obtain

$$\begin{aligned}\dot{x} &= A_m x + B_c r \\ &\quad + \sum_{i=1}^{m_c} B_i\Big[u_i(t-\tau_i) - K_i^\top x + J_i^\top x_h + \Theta_i^\top \Phi(x)\Big].\end{aligned} \tag{7}$$

Consider the control law

$$u_i = \hat{K}_i^\top x - \hat{J}_i^\top x_h - \hat{\Theta}_i^\top \Phi(x). \tag{8}$$

where $\hat{K}_i(t)$, $\hat{J}_i(t)$, and $\hat{\Theta}_i(t)$ are the adaptive estimates to be designed. Substituting the law in (8) into (7) yields

$$\begin{aligned}\dot{x} &= A_m x + \sum_{i=1}^{m_c} B_i\Big[\tilde{K}_i^\top x - \tilde{J}_i^\top x_h - \tilde{\Theta}_i^\top \Phi(x)\Big] \\ &\quad + f(t, X) + B_c r,\end{aligned} \tag{9}$$

where $\tilde{K}_i := \hat{K}_i - K_i$, $\tilde{J}_i := \hat{J}_i - J_i$, $\tilde{\Theta}_i := \hat{\Theta}_i - \Theta_i$, and

$$\begin{aligned} f(t, X) := \sum_{i=1}^{m_c} B_i\Big[&\hat{K}_i^\top(t-\tau_i)x(t-\tau_i) - \hat{K}_i^\top x \\ &- \hat{J}_i^\top(t-\tau_i)x(t-h-\tau_i) + \hat{J}_i^\top x(t-h) \\ &- \hat{\Theta}_i^\top(t-\tau_i)\Phi\big(x(t-\tau_i)\big) + \hat{\Theta}_i^\top \Phi(x)\Big] \end{aligned} \tag{10}$$

gathers delay related terms with $X^\top = \Big[x^\top(t),\ x^\top(t-\tau_1),\ \dots,\ x^\top(t-\tau_{mc}),\ x^\top(t-h-\tau_1),\ \dots,\ x^\top(t-h-\tau_{mc}),\ x_h^\top\Big]$. Next, we design the auxiliary system:

$$\dot{x}_a = A_m x_a + f(t, X) + B_c r,\ \ x_a(0) = x_{a0} \tag{11}$$

and $y_a = C x_a$. The motivation for including $f(t, X)$ in the design of the auxiliary system (11) is so that this term is canceled when taking the difference between (9) and (11) to get the tracking error equation as follows

$$\dot{e} = A_m e + \sum_{i=1}^{m_c} B_i\Big[\tilde{K}_i^\top x - \tilde{J}_i^\top x_h - \tilde{\Theta}_i^\top \Phi(x)\Big], \tag{12}$$

with $e(t) := x(t) - x_a(t)$, *independent of input delays.*

We next construct the state predictor as follows

$$\begin{aligned}\dot{\hat{x}} &= A_m x + A_r \hat{e} + B_c r \\ &\quad + \sum_{i=1}^{m_c} B_i\Big[u_i(t-\tau_i) - \hat{K}_i^\top x + \hat{J}_i^\top x_h + \hat{\Theta}_i^\top \Phi(x)\Big]\end{aligned} \tag{13}$$

and $\hat{y} = C\hat{x}$, where $\hat{x}(0) = \hat{x}_0$, $\hat{e} := \hat{x} - x$, and A_r is a loop-shaping Hurwitz matrix. Similarly, this design of the state predictor eliminates the input-delay related terms when taking the difference between (7) and (13) to obtain the following prediction error equation:

$$\dot{\hat{e}} = A_r \hat{e} - \sum_{i=1}^{m_c} B_i\Big[\tilde{K}_i^\top x - \tilde{J}_i^\top x_h - \tilde{\Theta}_i^\top \Phi(x)\Big]. \tag{14}$$

With the errors defined, we can now design the adaptive estimates as follows:

$$\dot{\hat{K}}_i = -\Gamma_{xi} x \Delta e B_i, \tag{15}$$

$$\dot{\hat{J}}_i = \Gamma_{hi} x_h \Delta e B_i, \tag{16}$$

$$\dot{\hat{\Theta}}_i = \Gamma_{\Theta i} \Phi(x) \Delta e B_i, \tag{17}$$

where $\Delta e = e^\top P_m - \hat{e}^\top P_r$, $\hat{K}_i(0) = \hat{K}_{i0}$, $\hat{J}_i(0) = \hat{J}_{i0}$, $\hat{\Theta}_i(0) = \hat{\Theta}_{i0}$, and Γ_{xi}, Γ_{hi}, and $\Gamma_{\Theta i}$ are the adaptive gains. In addition, the positive definite matrices P_m and P_r are the unique solutions to the Lyapunov equations:

$$A_m^\top P_m + P_m A_m = -Q_m, \tag{18}$$

$$A_r^\top P_r + P_r A_r = -Q_r, \tag{19}$$

where Q_m and Q_r are positive definite matrices.

The stability of this control framework are analyzed in the following section.

4. Stability analysis

In this section, we establish the input-to-state stability of the closed-loop system and the asymptotic stability of the tracking and prediction errors under a delay-dependent stability condition.

Lemma 1. *The errors $e(t)$, $\hat{e}(t)$, $\tilde{K}_i(t)$, $\tilde{J}_i(t)$ and $\tilde{\Theta}_i(t)$, for $i = 1, \ldots, m_c$, are all uniformly ultimately bounded. Here, a trajectory $z(t)$ is uniformly ultimately bounded if there exist b and c and for every $0 < a < c$, there is a $T \geq 0$ such that*

$$\|z(t_0)\| \leq a \Rightarrow \|z(t)\| \leq b, \ \forall t \geq t_0 + T. \tag{20}$$

Proof. Consider the Lyapunov function candidate:

$$V(e, \hat{e}, \tilde{K}_i, \tilde{J}_i, \tilde{\Theta}_i) = e^\top P_m e + \hat{e}^\top P_r \hat{e} + \sum_{i=1}^{m_c} \mathrm{tr}\big(\tilde{K}_i^\top \Gamma_{xi}^{-1} \tilde{K}_i + \tilde{J}_i^\top \Gamma_{hi}^{-1} \tilde{J}_i + \tilde{\Theta}_i^\top \Gamma_{\Theta i}^{-1} \tilde{\Theta}_i\big), \tag{21}$$

where $\mathrm{tr}(\cdot)$ denotes the trace of a matrix. It follows from (12), (14), (18), (19), and the time derivative of (21) that

$$\dot{V} + e^\top Q_m e + \hat{e}^\top Q_r \hat{e} = 2\sum_{i=1}^{m_c} \Delta e B_i \tilde{K}_i^\top x - 2\sum_{i=1}^{m_c} \Delta e B_i \tilde{J}_i^\top x_h - 2\sum_{i=1}^{m_c} \Delta e B_i \tilde{\Theta}_i^\top \Phi(x) + 2\sum_{i=1}^{m_c} \mathrm{tr}\big(\tilde{K}_i^\top \Gamma_{xi}^{-1} \dot{\hat{K}}_i + \tilde{J}_i^\top \Gamma_{hi}^{-1} \dot{\hat{J}}_i + \tilde{\Theta}_i^\top \Gamma_{\Theta i}^{-1} \dot{\hat{\Theta}}_i\big). \tag{22}$$

Using the property that $\mathrm{tr}(ab^\top) = b^\top a$, we have

$$\dot{V} + e^\top Q_m e + \hat{e}^\top Q_r \hat{e} = 2\sum_{i=1}^{m_c} \mathrm{tr}\Big[\tilde{K}_i^\top x \Delta e B_i\Big] - 2\sum_{i=1}^{m_c} \mathrm{tr}\Big[\tilde{J}_i^\top x_h \Delta e B_i\Big] - 2\sum_{i=1}^{m_c} \mathrm{tr}\Big[\tilde{\Theta}_i^\top \Phi(x) \Delta e B_i\Big] + 2\sum_{i=1}^{m_c} \mathrm{tr}\big(\tilde{K}_i^\top \Gamma_{xi}^{-1} \dot{\hat{K}}_i + \tilde{J}_i^\top \Gamma_{hi}^{-1} \dot{\hat{J}}_i + \tilde{\Theta}_i^\top \Gamma_{\Theta i}^{-1} \dot{\hat{\Theta}}_i\big). \tag{23}$$

By rearranging relevant terms, we get

$$\dot{V} + e^\top Q_m e + \hat{e}^\top Q_r \hat{e} = 2\sum_{i=1}^{m_c} \mathrm{tr}\Big[\tilde{K}_i^\top \Big(x \Delta e B_i + \Gamma_{xi}^{-1} \dot{\hat{K}}_i\Big)\Big] - 2\sum_{i=1}^{m_c} \mathrm{tr}\Big[\tilde{J}_i^\top \Big(x_h \Delta e B_i - \Gamma_{hi}^{-1} \dot{\hat{J}}_i\Big)\Big] - 2\sum_{i=1}^{m_c} \mathrm{tr}\Big[\tilde{\Theta}_i^\top \Big(\Phi(x) \Delta e B_i - \Gamma_{\Theta i}^{-1} \dot{\hat{\Theta}}_i\Big)\Big]. \tag{24}$$

Now substituting (15), (16), and (17) in (24) to eliminate its right-hand side leads to

$$\dot{V} = -e^\top Q_m e - \hat{e}^\top Q_r \hat{e} \leq 0. \tag{25}$$

This implies the uniform ultimate boundedness of e, $\hat{e}$, $\tilde{K}_i$, $\tilde{J}_i$, and $\tilde{\Theta}_i$, for $i = 1, \ldots, m_c$. □

It is noted that to this point, the boundedness of the errors and the adaptive estimates is independent of the input-delay values since their governing equations in (12), (14), (15), (16), and (17) are all free of the input delays. The dependence on the input delays only arises as a sufficient condition for the input-to-state stability of the closed-loop adaptive system in the following lemma.

Lemma 2. *Suppose that there exist matrices P_1, P_2, scalars λ and ϵ, and R_i, $i = 1, \ldots, 2m_c + 1$, that satisfy the delay-dependent LMI*

$$W \leq 0, \tag{26}$$

where W is a symmetric matrix whose components are

$$W_{ii} = -R_i \mathrm{e}^{-2\epsilon_1 \tau_i} + \lambda b_g \mathbb{I}, \text{ for } i = 1, \ldots, 2m_c + 1$$
$$W_{i(2m_c+5)} = R_i \mathrm{e}^{-2\epsilon_1 \tau_i}, \text{ for } i = 1, \ldots, 2m_c + 1$$
$$W_{(2m_c+2)(2m_c+2)} = -P_2^\top - P_2 + \sum_{i=1}^{2m_c+1} \tau_i^2 R_i$$
$$W_{(2m_c+2)(2m_c+3)} = P_2^\top, \ W_{(2m_c+2)(2m_c+4)} = P_2^\top B_c$$
$$W_{(2m_c+2)(2m_c+5)} = -P_1 + P_2^\top A_m$$
$$W_{(2m_c+3)(2m_c+3)} = -\lambda \mathbb{I}, \ W_{(2m_c+3)(2m_c+5)} = P_m + P_1$$
$$W_{(2m_c+4)(2m_c+4)} = -\epsilon_2 \mathbb{I}$$
$$W_{(2m_c+4)(2m_c+5)} = B_c^\top (P_m + P_1)$$
$$W_{(2m_c+5)(2m_c+5)} = -Q_m - \sum_{i=1}^{2m_c+1} R_i \mathrm{e}^{-2\epsilon_1 \tau_i} + 2\epsilon_1 P_m + \lambda b_g \mathbb{I} + P_1^\top A_m + A_m^\top P_1,$$

and all other elements are zero. The closed-loop system in (6) is input-to-state stable with respect to $r(t)$ for all τ_i small enough. In other words, there exist a class $\mathcal{KL}$ function β and a positive constant c such that (cf. Fridman, 2014)

$$\|x(s)\|_2 \leq \beta(x_0, s) + c \int_0^s \|r(t)\|_2^2 \mathrm{d}t, \ \forall s > 0.$$

Proof. By substituting (8) in (6), the closed-loop adaptive system becomes the following delay-differential equation:

$$\dot{x} = A_m x + B_c r + g(t, X), \tag{27}$$

where $g(t, X) := \sum_{i=1}^{m_c} B_i \Big[\hat{K}_i^\top (t - \tau_i) x(t - \tau_i) - \hat{J}_i^\top (t - \tau_i) x(t - h - \tau_i) - \hat{\Theta}_i^\top (t - \tau_i) \Phi\big(x(t - \tau_i)\big) + J_i^\top x_h - K_i^\top x + \Theta_i^\top \Phi(x)\Big]$. Because of the property of $\Phi(x)$ stated in Assumption 1 and since $\hat{K}_i(t)$, $\hat{J}_i(t)$, and $\hat{\Theta}_i(t)$, for $i = 1, \ldots, m_c$, are uniformly bounded as per Lemma 1, there exists $b_g > 0$ such that:

$$\|g\|_2^2 \leq b_g X^\top X. \tag{28}$$

To investigate the stability of the delay system in (27), consider the following Lyapunov-Krasovskii functional

$$V(t) = x^\top(t) P_m x(t) + \sum_{i=1}^{2m_c+1} \underbrace{\tau_i \int_{-\tau_i}^{0} \int_{t+\theta}^{t} \mathrm{e}^{-2\epsilon_1 (t-s)} \dot{x}^\top(s) R_i \dot{x}(s) \mathrm{d}s \mathrm{d}\theta}_{V_i(t)} \tag{29}$$

for some positive definite matrix R_i and some scalar constant $\epsilon_1 > 0$. Here, $\tau_{m_c+1} := h + \tau_1, \ldots, \tau_{2m_c} := h + \tau_{m_c}$, and $\tau_{2m_c+1} := h$, for convenience. Applying integration by part with

$$v = \int_{t+\theta}^{t} \mathrm{e}^{-2\epsilon_1 (t-s)} \dot{x}^\top(s) R_i \dot{x}(s) \mathrm{d}s$$
$$\Rightarrow \mathrm{d}v = -\mathrm{e}^{2\epsilon_1 \theta} \dot{x}^\top(t + \theta) R_i \dot{x}(t + \theta) \mathrm{d}\theta \tag{30}$$

and $w = \theta$, which means $\mathrm{d}w = \mathrm{d}\theta$, leads to

$$V_i(t) = \tau_i \int_{-\tau_i}^{0} v\mathrm{d}w = \tau_i \big[vw\big]_{\theta=-\tau_i}^{\theta=0} - \tau_i \int_{-\tau_i}^{0} w\mathrm{d}v$$
$$= \tau_i \int_{t-\tau_i}^{t} \tau_i \mathrm{e}^{-2\epsilon_1(t-s)}\dot{x}^\top(s)R_i\dot{x}(s)\mathrm{d}s$$
$$+ \tau_i \int_{-\tau_i}^{0} \theta \mathrm{e}^{2\epsilon_1\theta}\dot{x}^\top(t+\theta)R_i\dot{x}(t+\theta)\mathrm{d}\theta \quad (31)$$
$$= \tau_i \int_{t-\tau_i}^{t} (\tau_i + s - t)\mathrm{e}^{-2\epsilon_1(t-s)}\dot{x}^\top(s)R_i\dot{x}(s)\mathrm{d}s.$$

Here, a change in variables with $s = t + \theta$ is used. It then follows from the Leibniz integral rule that

$$\dot{V}_i(t) = \tau_i^2\dot{x}^\top(t)R_i\dot{x}(t) - 2\epsilon_1 V_i(t)$$
$$- \tau_i \int_{t-\tau_i}^{t} \mathrm{e}^{-2\epsilon_1(t-s)}\dot{x}^\top(s)R_i\dot{x}(s)\mathrm{d}s. \quad (32)$$

As $\mathrm{e}^{-2\epsilon_1(t-s)} \geq \mathrm{e}^{-2\epsilon_1\tau_i}$ for $s \in [t-\tau_i, t]$, we have

$$\dot{V}_i(t) \leq \tau_i^2\dot{x}^\top(t)R_i\dot{x}(t) - 2\epsilon_1 V_i(t)$$
$$- \tau_i \mathrm{e}^{-2\epsilon_1\tau_i} \int_{t-\tau_i}^{t} \dot{x}^\top(s)R_i\dot{x}(s)\mathrm{d}s. \quad (33)$$

Keeping this in mind, we obtain:

$$\dot{V}(t) \leq 2x^\top(t)P_m\dot{x}(t) + \sum_{i=1}^{2m_c+1} \tau_i^2\dot{x}^\top(t)R_i\dot{x}(t)$$
$$- 2\epsilon_1\Big(V(t) - x^\top(t)P_m x(t)\Big)$$
$$- \sum_{i=1}^{2m_c+1} \tau_i \mathrm{e}^{-2\epsilon_1\tau_i} \int_{t-\tau_i}^{t} \dot{x}^\top(s)R_i\dot{x}(s)\mathrm{d}s. \quad (34)$$

Therefore,

$$\dot{V}(t) + 2\epsilon_1 V(t) - \epsilon_2\|r(t)\|_2^2 \leq 2x^\top(t)P_m\dot{x}(t)$$
$$+ \sum_{i=1}^{2m_c+1} \tau_i^2\dot{x}^\top(t)R_i\dot{x}(t) + 2\epsilon_1 x^\top(t)P_m x(t)$$
$$- \sum_{i=1}^{2m_c+1} \tau_i \mathrm{e}^{-2\epsilon_1\tau_i} \int_{t-\tau_i}^{t} \dot{x}^\top(s)R_i\dot{x}(s)\mathrm{d}s - \epsilon_2\|r(t)\|_2^2$$
$$\underbrace{+\lambda\left[b_g X^\top X - \|g\|_2^2\right]}_{\geq 0 \text{ due to (28)}} + 2\Big[x^\top(t)P_1 + \dot{x}^\top(t)P_2^\top\Big]$$
$$\cdot \underbrace{\Big[A_m x(t) + g + B_c r(t) - \dot{x}(t)\Big]}_{= 0 \text{ due to (27)}}. \quad (35)$$

It follows from Jensen's inequality that (see Fridman, 2014, p. 87)

$$\tau_i \int_{t-\tau_i}^{t} \dot{x}^\top(s)R_i\dot{x}(s)\mathrm{d}s \geq \int_{t-\tau_i}^{t} \dot{x}^\top(s)\mathrm{d}s R_i \int_{t-\tau_i}^{t} \dot{x}(s)\mathrm{d}s$$
$$= \big[x(t) - x(t-\tau_i)\big]^\top R_i \big[x(t) - x(t-\tau_i)\big]. \quad (36)$$

Hence,

$$\dot{V}(t) + 2\epsilon_1 V(t) - \epsilon_2\|r(t)\|_2^2 \leq$$
$$2x(t)^\top P_m\Big[A_m x(t) + g + B_c r(t)\Big]$$
$$- \sum_{i=1}^{2m_c+1} \big[x(t) - x(t-\tau_i)\big]^\top R_i \mathrm{e}^{-2\epsilon_1\tau_i}\big[x(t) - x(t-\tau_i)\big]$$
$$+ \sum_{i=1}^{2m_c+1} \tau_i^2\dot{x}^\top(t)R_i\dot{x}(t) + 2\epsilon_1 x^\top(t)P_m x(t) - \epsilon_2\|r(t)\|_2^2$$
$$+ \lambda\left[b_g X^\top X - \|g\|_2^2\right] + 2\Big[x^\top(t)P_1^\top + \dot{x}^\top(t)P_2^\top\Big]$$
$$\cdot\Big[A_m x(t) + g + B_c r(t) - \dot{x}(t)\Big] \leq \zeta^\top(t)W\zeta(t), \quad (37)$$

where ζ is a column vector of $x(t-\tau_1), \ldots, x(t-\tau_{m_c})$, $x(t-h-\tau_1)$, $\ldots$, $x(t-h-\tau_{m_c})$, x_h, $\dot{x}$, g, r, and x, in this particular order, and W is defined below (26). Therefore, if there exist matrices R_i, P_1, P_2, and scalars λ and ϵ such that $W \leq 0$, by multiplying $\mathrm{e}^{2\epsilon_1(t-s)}$ to both sides of (37), we have:

$$\mathrm{e}^{2\epsilon_1(t-s)}\Big[\dot{V}(t) + 2\epsilon_1 V(t) - \epsilon_2\|r(t)\|_2^2\Big] \leq 0. \quad (38)$$

Integrating both sides from 0 to s leads to

$$\int_0^s \frac{\mathrm{d}}{\mathrm{d}t}\left[\mathrm{e}^{2\epsilon_1(t-s)}V(t)\right]\mathrm{d}t \leq \epsilon_2 \int_0^s \mathrm{e}^{2\epsilon_1(t-s)}\|r(t)\|_2^2\mathrm{d}t.$$
$$\Rightarrow V(s) - \mathrm{e}^{-2\epsilon_1 s}V(0) \leq \epsilon_2 \int_0^s \mathrm{e}^{2\epsilon_1(t-s)}\mathrm{d}t \int_0^s \|r(t)\|_2^2\mathrm{d}t.$$

Therefore,

$$\lambda_{\min}(P_m)\|x\|_2 \leq x^\top(s)P_m x(s) \leq V(s) \quad (39)$$
$$\leq \mathrm{e}^{-2\epsilon_1 s}V(0) + \frac{\epsilon_2}{2\epsilon_1}\left(1 - \mathrm{e}^{-2\epsilon_1 s}\right)\int_0^s \|r(t)\|_2^2\mathrm{d}t.$$

Thus, the closed-loop system in (6) is input-to-state stable with respect to $r(t)$. □

Remark 1. For a given set of parameters that satisfy the delay-dependent stability condition (26) for $\tau_i = 0$, all eigenvalues of W are negative. When the delay is increased, the eigenvalues gradually move closer to the 0 axis because of the continuous dependence of the LMI on the delays (Fridman, 2014). Beyond some critical delay $\max\{\tau_i\} > \tau_m$, they start crossing the 0 axis and W is no longer negative definite. Hence, the stability condition (26) is satisfied with the input delays small enough.

To numerically check the feasibility of the LMI (26) given a system, one will need to know the bound b_g. It follows from (28) that the bound b_g depends on the bounds on the adaptive estimates $\hat{K}_i(t)$, $\hat{\Theta}_i(t)$, and $\hat{J}_i(t)$, among other terms. Though Lemma 1 proves the uniform ultimate boundedness, i.e. the existence of bounds, of $\hat{K}_i(t)$, $\hat{\Theta}_i(t)$, and $\hat{J}_i(t)$, the theory does not provide a closed-form expression to compute the bounds. Therefore, numerically checking the LMI feasibility here is not a practical task. Nonetheless, if b_g is known, solving the LMI is possible, though not trivial, with the help of optimization toolboxes, such as YALMIP (Lofberg, 2004), together with an efficient solver, e.g. MOSEK. The illustrative example below demonstrates how checking of the LMI feasibility may be done using these computational tools.

Illustrative example: Consider a simple closed-loop adaptive control system in which $m_c = 1$, $B_c = 1$, $b_g = 0.1$, and the baseline control parameters $A_m = -1$, $Q_m = 1$, and $P_m = 0.5$. The variables in the LMI problem in (26) are λ, ϵ_2, R_1, R_2, R_3, P_1, and P_2. To avoid solving a nonlinear LMI problem, which is unnecessarily complicated and most tools are not capable of solving, we set $\epsilon_1 = 0.01$ instead of considering it a variable.

In the optimization toolbox YALMIP, we use the `sdpvar` to define the decision variables as symbolic scalars or matrices. We employ `blkvar` to define the block matrix W with the symbolic variables as shown in (26). The LMI problem can be then defined by `LMIs=[W<=0, λ>=0, ϵ2>=0, R1>=0, R2>=0, R3>=0]`, the same way a semidefinite programming optimization problem is set up. While YALMIP is a very efficient parsing tool to construct the LMI problem into an appropriate structure, it requires a solver to actually solve the problem. The off-the-shelf solver MOSEK can be declared in the setting: `options=sdpsettings('solver','mosek')`. The problem is then solved using the command `optimize(LMIs,[],options)`. The output of this function will indicate whether or not the LMI is feasible.

For instance, for the particular problem in this example, the outcome of this process indicates that the LMI in (26) is feasible for $\tau_1 = 0.25$ s and $h = 1.7$ s. A specific solution is when $\lambda = 0.50781$, $\epsilon_2 = 33.0902$, $R_1 = 0.20426$, $R_2 = 0.071516$, $R_3 = 0.074088$, $P_1 = 0.020647$, and $P_2 = 0.50067$. For validation, with these values, the eigenvalues of W are -33.1067, -1.9235, -0.4679, -0.0976, -0.0194, -0.0007, and -7.1999×10^{-5}, showing that W is indeed a negative definite matrix.

The results of Lemmas 1 and 2 lead to the following theorem on the asymptotic stability of the errors.

Theorem 1. *Suppose the delay-dependent stability condition in Lemma 2 is satisfied. The tracking and prediction errors $e(t)$ and $\hat{e}(t)$ are asymptotically stable. Here, a trajectory $z(t)$ is asymptotically stable if there exists $a > 0$ and for any $b > 0$, there is a $T \geq 0$ such that*

$$\|z(t_0)\| \leq a \Rightarrow \|z(t)\| \leq b, \ \forall t \geq t_0 + T. \tag{40}$$

Proof. From (22) we have

$$\ddot{V}(t) = -2e^\top Q_m \dot{e} - 2\hat{e}^\top Q_r \dot{\hat{e}}. \tag{41}$$

Since the errors $e(t)$ and $\hat{e}(t)$, estimates $\hat{K}(t)$, $\hat{J}(t)$, and $\hat{\Theta}(t)$, and system state $x(t)$ are all bounded according to Lemmas 1 and 2, it follows from (12) and (14) that the derivatives $\dot{e}(t)$ and $\dot{\hat{e}}(t)$ are both bounded. Hence, $\ddot{V}(t)$ is bounded and, consequently, $\dot{V}(t)$ is uniformly continuous. In addition, as $V(t)$ is lower bounded and $\dot{V}(t) \leq 0$, we have $\dot{V}(t) \to 0$ as $t \to \infty$ in light of the Barbalat lemma (Khalil, 1992, p.186). The claim then follows. □

5. Numerical experiments

5.1. Comparison with the delay compensator in Nguyen (2018)

For comparison, we employ the F-16 aircraft model implemented in Lavretsky et al. (2010), Nguyen (2018) and Stevens and Lewis (2003). In particular, the state $x_p = [\alpha \quad q]^\top$ contains the aircraft angle of attack, α, and the pitch rate, q. The parameters in (1) are:

$$A_p = \begin{bmatrix} -1.0189 & 0.9051 \\ 0.8223 & -1.0774 \end{bmatrix}, d(x_p) = 0.5e^{-\frac{(\alpha-\alpha_c)^2}{2\sigma^2}} - d_0,$$

$$d_0 = 0.5e^{-\frac{\alpha_c^2}{2\sigma^2}}, \ B = \begin{bmatrix} -0.0022 \\ -0.1756 \end{bmatrix}, J = \begin{bmatrix} 7 \\ 9 \end{bmatrix}, \ C_p = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

with the center of the Gaussian $\alpha_c = 2 \deg \frac{\pi}{180}$, and the width $\sigma = 0.0233$. The state delay is $h = 2$ s and the input delay is $\tau = 0.2$ s. The system is then transformed into the extended form in (3).

We design the proposed delay-compensation scheme with A_m having eigenvalues of -1.0, -0.8, and -0.7. We then use a standard pole placement algorithm to arrive at A_m and K as follows:

$$A_m = \begin{bmatrix} 0 & 0 & 1 \\ -0.0068 & -1.0292 & 0.9002 \\ -0.5441 & -0.0024 & -1.4708 \end{bmatrix}, \ K = \begin{bmatrix} 3.0986 \\ 4.6963 \\ 2.2401 \end{bmatrix}.$$

Other control parameters are $\Gamma_x = 10\mathbb{I}$, $\Gamma_\Theta = 10\mathbb{I}$, $\Gamma_h = 50\mathbb{I}$, $Q_m = \mathbb{I}$, $A_p = 10A_m$, $P_r = 5P_m$. The estimates $\hat{K}(t)$, $\hat{\Theta}(t)$, and $\hat{J}(t)$ are all initialized at zero initial conditions. The closed-loop adaptive system, the auxiliary system, and the state predictor are initialized arbitrarily as follows: $x_0 = [0 \ 0 \ 0]^\top$, $x_{a0} = [0 \ 0 \ 1]^\top$, and $\hat{x}_0 = [0 \ 0 \ -1]^\top$. The output is tasked to track a square-wave trajectory with different step amplitudes.

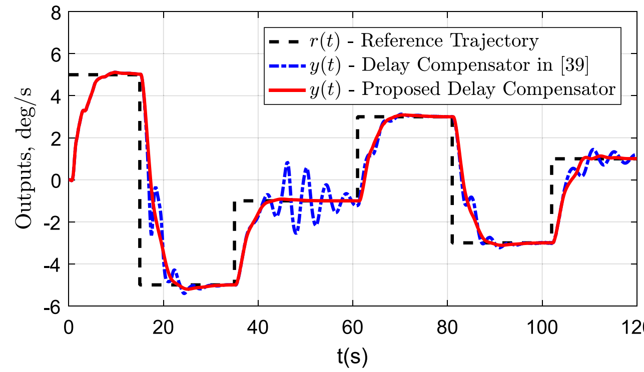

Fig. 1. Tracking performance of the proposed delay compensator as compared with the controller in Nguyen (2018). In this case, the input and state delays are $\tau = 0.2$ s and $h = 2$ s, respectively.

To illustrate the efficacy of the proposed scheme, we compare its performance with that of the controller in Nguyen (2018), which is implemented with the same values of A_m, A_p, and the adaptation gains. The comparative results are shown in Fig. 1 with the red solid line representing the output of the proposed controller and the blue dotted line representing the output of the controller in Nguyen (2018). As shown, the proposed controller is able to compensate for the input delay and the large state delay, while the effect of the state delay is evident in the output of the controller developed in Nguyen (2018).

There are several reasons we compare the performance of the controller proposed here against the controller in Nguyen (2018): (i) To show that a delay in the state may cause catastrophic damage to the control system's performance as evident by the blue dotted line in Fig. 1. Hence, this delay needs to be compensated to achieve desirable performance and this compensation is not trivial. (ii) To illustrate the effect of the components added to compensate for the state delay, which is shown to deteriorate the performance of the previous controller. (iii) As shown in Table 1, the previous controller in Nguyen (2018) is the most closely related work to the present paper.

5.2. Performance with a higher-order system

In this section, we demonstrate the performance of the proposed compensator when there are multiple input delays. For this purpose, another control channel is added to the case study in the last section. In this case, by applying the transformation in (3), we obtain the extended system (4) with the following parameters:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1.0189 & 0.9051 \\ 0 & 0 & 0.8223 & -1.0774 \end{bmatrix}, \ B_c = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{42}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ -0.0022 \\ -0.1756 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 \\ 0 \\ 0.1 \\ 0.2 \end{bmatrix}, \ J = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 6 & 7 \\ 9 & 8 \end{bmatrix}, \tag{43}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ d(x_p) = \begin{bmatrix} 0.5e^{-\frac{(\alpha-\alpha_c)^2}{2\sigma^2}} - d_0 \\ \sin(q) \end{bmatrix}.$$

The control parameters are set to

$$A_m = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4705 & 0.0404 & -1.3768 & 0.0641 \\ 0.0404 & -0.5501 & 0.0641 & -1.5032 \end{bmatrix},$$

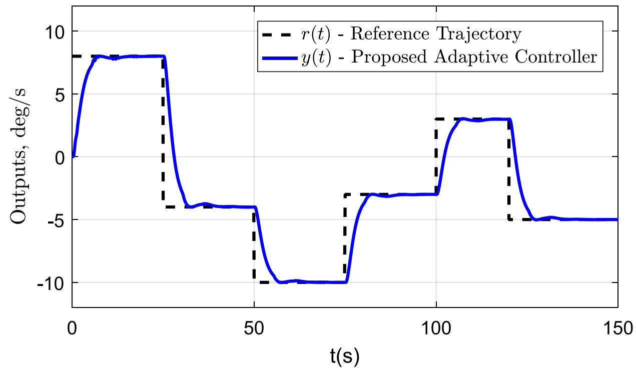

Fig. 2. Tracking performance of the proposed delay compensator. In this case, the input and state delays are $\tau_1 = 0.5$ s, $\tau_2 = 0.2$ s, and $h = 5$ s, respectively, for the system controlled by the controller proposed in this paper.

$$K = \begin{bmatrix} -5.7323 & -4.8310 \\ 3.6849 & 0.4848 \\ 0.2476 & -3.5737 \\ -7.3378 & -8.5715 \end{bmatrix}. \tag{44}$$

In addition, we set $\Gamma_x = 10\mathbb{I}$, $\Gamma_\Theta = 10\mathbb{I}$, $\Gamma_h = 50\mathbb{I}$, $Q_m = \mathbb{I}$, $A_p = 10A_m$, $P_r = 30P_m$. The estimates $\hat{K}(t)$, $\hat{\Theta}(t)$, and $\hat{J}(t)$ are all initialized at zero initial conditions. The closed-loop adaptive system, the auxiliary system, and the state predictor are initialized as follows: $x_0 = [0\ \ 0\ \ 0\ \ 0]^\top$, $x_{a0} = [0.5\ \ -0.5\ 1.5\ 1]^\top$, $\hat{x}_0 = [1.5\ 0.5\ -0.5\ -1]^\top$. The two control channels have delays of 0.5 s and 0.2 s and the state delay is 5 s.

Fig. 2 shows the performance of the proposed adaptive controller while tracking a square wave reference trajectory. We can see that the proposed delay compensator is able to stabilize the system in the presence of the given delays. Nonetheless, the proposed scheme, inheriting the great tracking ability of the PMRAC, exhibits a desirable tracking capability with virtually no overshoot. The proposed delay-compensation scheme achieves this performance despite knowing very little about the system and has to adapt itself to the system uncertainties and nonlinearity.

5.3. Asymptotic behaviors

In this section, we illustrate the asymptotic behaviors of the tracking error governed by (12) and prediction error governed by (14). These asymptotic behaviors are concluded in Theorem 1 in light of Lemmas 1 and 2. For this purpose, we use the same nonlinear systems and control parameters of the proposed scheme as in the last section. Here, the outputs aim to track a step reference trajectory that jumps from 0 to 5 at $t = 1$ s.

The top panel of Fig. 3 shows the four components of the tracking error $e(t)$, while the bottom panel shows the four components of the prediction error $\hat{e}(t)$. As expected, despite starting from non-zero initial conditions, all error trajectories converge to zero asymptotically in the presence of input delays of 0.5 s and 0.2 s, a state delay of 5 s, and the system nonlinearity and uncertainties.

The asymptotic stability of $\hat{e}(t)$ ensures that the uncertainty $\Theta^\top\Phi(x)$ in (4) is closely estimated and canceled. In addition, the asymptotic stability of $e(t)$ implies that the dynamics of the closed-loop adaptive system converge to the dynamics of the auxiliary system. Thus, the system output $y(t)$ closely tracks $r(t)$ since the output y_a of the auxiliary system (11) closely tracks $r(t)$. This close tracking behaviors can be seen in Figs. 1 and 2.

5.4. Limitation: Upper bound of input delays

This section demonstrates a key limitation of the proposed controller that it has an upper bound of *input delays*. When the delays are larger than this upper bound, the closed-loop control system is destabilized. For example, when we increase the input delay τ_1 to 5.7 s, the control performance is given in Fig. 4, which shows undesirable tracking with an amplitude starting to grow. Therefore, in this example, a delay of about 5.6 s is the upper bound for the input delay. Further increase in the input delay leads to instability. The controller is highly robust to the *state delay* since no stability loss is observed for any large value of the state delay.

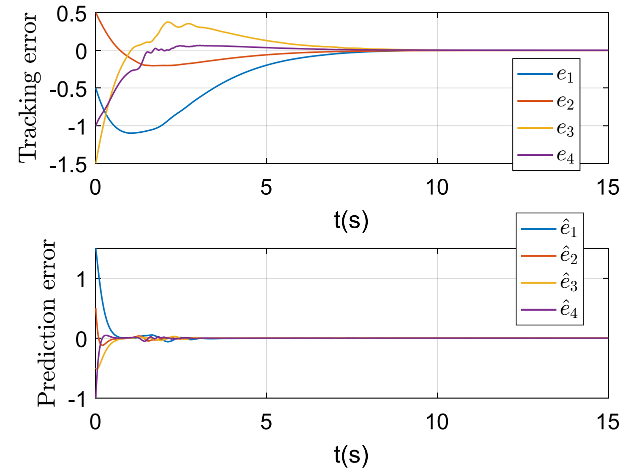

Fig. 3. The top panel shows the time histories of the tracking error $e(t) = x(t) - x_a(t)$. The bottom panel shows the time histories of the prediction error $\hat{e}(t) = \hat{x}(t) - x(t)$. In this case, the input and state delays are $\tau_1 = 0.5$ s, $\tau_2 = 0.2$ s, and $h = 5$ s, respectively, and the control system is tasked to track a step input. Asymptotic stability is evident in both tracking and prediction errors despite the large delay values.

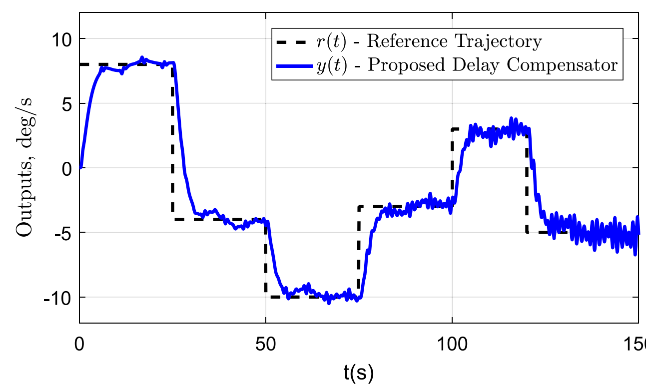

Fig. 4. Tracking performance of the proposed delay compensator. In this case, the input and state delays are $\tau_1 = 5.7$ s, $\tau_2 = 0.2$ s, and $h = 5$ s, respectively.

6. Conclusion

We have developed a new adaptive delay-compensation framework using the PMRAC architecture as the foundation. The proposed approach is able to stabilize uncertain nonlinear systems with multiple delays in the control inputs and states. A delay-dependent stability condition is formulated using a Lyapunov-Krasovskii functional which implies delay thresholds. When the delay values are below the thresholds, the asymptotic stability of the error signals and the input-to-state stability of the closed loop adaptive control system are guaranteed. In addition, unlike most other delay-compensation controllers, the proposed scheme does not require the discretization of integral terms.

Therefore, it is more computationally efficient than predictor-feedback controllers, whose control laws are formulated in terms of integrals, and does not suffer from the issue of long processing time, reported in Liu and Zhou (2018) for example. These properties are demonstrated by numerical experiments, which also include comparative results to illustrate the performance of the proposed scheme with respect to existing delay-compensation controllers.

Future research will address the limitations identified at the end of Section 1 and extend this framework to control systems governed by partial differential equations with relevant structures. Furthermore, the issues of deriving a formula for the upper bound of input delays are currently under study.

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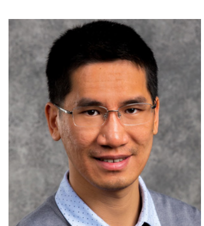

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