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To cite this article: B H Chen and T L Chen 2020 *J. Phys.: Conf. Ser.* **1509** 012007

Design of Missile Mid-Course Guidance for Constrained Impact Angle

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Abstract. This research develops a mid-course guidance law, which enables a constrained impact angle at the engagement, in response to the needs of modern missile design. To achieve this goal and share the workload with the terminal guidance, the mid-course guidance has to regulate the missile to a specific target location and a specific flight angle for a specific "time-to-go." Therefore, conventional mid-course guidance laws are inadequate in this case. The proposed guidance law achieves this goal by recursively updating the target dynamics for the mid-course guidance, and applying optimal controls with frequently updated control parameters. Simulation results demonstrate the importance and effectiveness of the proposed method. In a simulation case, the interception missile is 100 kilometers away from the target missile, the flight speed of the two missiles are 1200 m/s and 800 m/s, respectively, and the requested impact angle is 150^o. The proposed method achieves the positioning error and flight-path-angle error of 791 meters and 0.56^o, at the end of mid-course guidance. When working with the previously developed terminal guidance, the miss distance and impact-angle error are 0.002m and 0.001^o at the end of the engagement.

1. Introduction

Missiles are currently the most important defence and combat weapons. After years of development, the interception techniques gradually migrate from igniting the interception missile in the region nearby the target, to a "direct-hit" on the target to maximize its ballistic effect [1]. Furthermore, the most advanced missile guidance request not only the direct-hit but also a specific impact angle to increase the lethality [2]. To cope with this trend, the missile guidance or missile trajectory controls must be modified accordingly.

In order to regulate the missile trajectory to hit a high-speed target, one needs the information of the target dynamics such as location, velocity, accelerations, etc. And, all of these information relies on the sensor system such as radar, seeker, on-board inertial sensors. The radar can output the location, velocity of both interception missile and target missile. However, its data throughput is around 2 Hz. This low data throughput is inadequate for high-speed trajectory controls. The seeker can output the line-of-sight angle and the angle change rate between the interception missile and target missile. It is accurate and has high data-throughput. However, the seeker only "sees" the target within 20 km. Due to the difference in the sensor capability, the missile guidance is often divided into two phases, which are mid-course guidance and terminal guidance [3]. During the mid-course guidance, the interception missile is far away from the target missile. Moreover, the interception missile replies on the radar and on-board inertial sensors to get close to the target. During the terminal guidance, the distance between two missiles is less than 20 km. The seeker and on-board inertial sensors are utilized for the trajectory controls. Due to the availability of sensing information, the control strategy of the mid-course guidance and terminal guidance is also different from each other.

As discussed before, conventional missile guidance requests a direct-hit but no specific impact angle. In that case, the mid-course guidance needs to regulate the missile trajectory to a specific location at a specific time-to-go [4][5]. However, when specifying an impact angle and share the workload of the terminal guidance, the mid-course guidance needs to regulate the missile trajectory to a specific position, a specific flight path angle at a specific time-to-go. Therefore, conventional guidance laws are inadequate in this case.

In this paper, we proposed novel mid-course guidance for achieving a constrained impact angle. Note that, in this research, both the mid-course and terminal guidance use normal accelerations only to change the flight path angle of the interception missile but not the longitudinal speed. Two parts in this mid-course guidance require special attention, one is to set the control goals of the mid-course guidance; the other one is to develop the control laws to achieve that goal. The proposed method is discussed and analysed in detail. Simulation results are used to verify the effectiveness of the proposed method.

2. Engagement model and state equation

In order to develop a guidance law for the missile interception, one needs to have a kinematic model that can describe the missile trajectory. Note that, these kinematics equations can be written in either Cartesian coordinate, polar coordinate, or other coordinates. Since most of the terminal guidance are developed in 2D, polar coordinates [6][7], we chose the same framework to develop the mid-course guidance. When there exists neither thrust force nor drift force for the missile dynamics, the kinematics of the missile in 2D space in polar coordinate can be written as follows.

$$\ddot{r} - r \cdot \dot{\gamma}_m^{\,2} = 0$$
$$r \cdot \ddot{\gamma}_m + 2\dot{r} \cdot \dot{\gamma}_m = a_{m\gamma_m} \tag{1}$$

where r is the distance between missile current position and its initial position, ($a_{m\gamma_m}$) is the normal acceleration that changes the flight path angle (γ_m) of the missile.

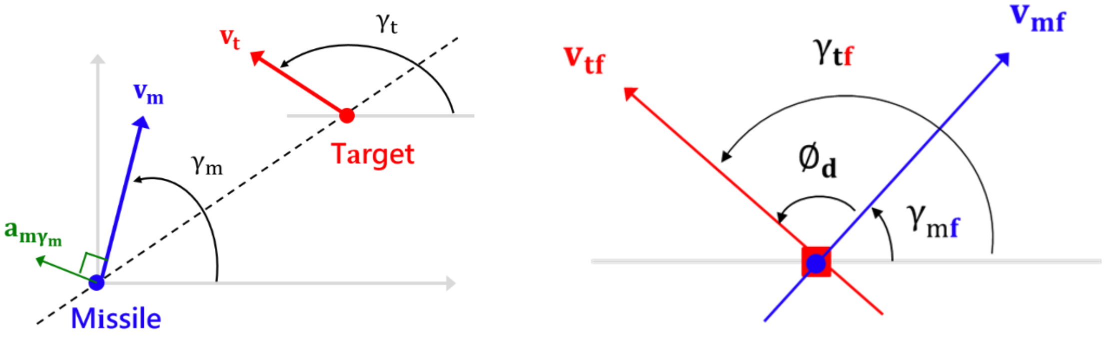

Figure 1. A schematic of the missile interception and hit-angle.

Figure 1 shows the schematics of the missile interception. ($\vec{V}_{tf}$) and (γ_{tf}) are the respective target missile velocity ($\vec{V}_t$) and flight path angle (γ_t) at the interception, ($\vec{V}_{mf}$) and (γ_{mf}) are the respective interception missile velocity ($\vec{V}_m$) and flight path angle (γ_m) at the interception. The hit-angle ($\emptyset_d$) is defined as the angle difference between the respective fly path angle of interception missile and target missile as shown in equation (2).

$$\emptyset_d = \gamma_{tf} - \gamma_{mf} \tag{2}$$

3. Mid-course guidance law for the constrained impact angle

In order to reduce the workload of the angle-constrained terminal guidance, the goal of the mid-course guidance is to control the missile trajectory so that the missile can enter the terminal-guidance mode without sharply changing its target position and flight path angle. And, this work is done by determining the requested missile dynamics at the end of the mid-course guidance, and by developing control methods to achieve those ideal dynamics.

3.1. Determination the target position and fly path angle of the mid-course guidance

Figure 2 shows the proposed engagement model for the missile interception. Assuming that, at the time "0", the position of the interception missile is ($\vec{x}_{m0}$), the target missile position is ($\vec{x}_{t0}$), and the distance between two missiles is (R). The velocity of the interception missile ($\vec{V}_m$) is known and constant and its initial fly path angle is (γ_{m0}), the velocity of the target missile ($\vec{V}_t$) and flight path angle (γ_t) are both known and constant. If the designated impact angle is $\emptyset_d$, we can calculate the location of the predicted-interception-point ($\overrightarrow{PIP}$) and time-to-go (t_{go}) by the following equations.

$$\|R\|^2 = \left\|\vec{V}_m \cdot t_{go}\right\|^2 + \left\|\vec{V}_t \cdot t_{go}\right\|^2 - 2 \cdot \left\|\vec{V}_m \cdot t_{go}\right\|\left\|\vec{V}_t \cdot t_{go}\right\| \cos\emptyset_d \tag{3}$$

$$\overrightarrow{PIP} = \vec{x}_t + \vec{V}_t \cdot t_{go} \tag{4}$$

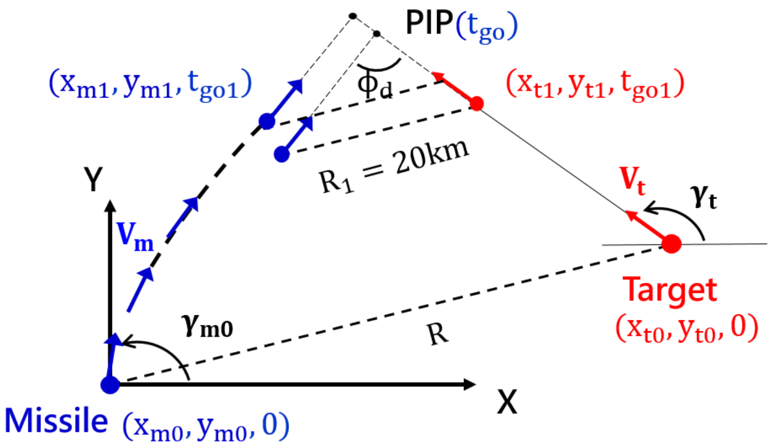

Figure 2. The proposed engagement model for the missile interception.

Assuming that the mid-course guidance switches to the terminal guidance when the distance between two missiles is less than 20 km, the time-to-go in the phase of the terminal guidance (t_{gof}) can be calculated by equation (5). Therefore, the time-to-go of the mid-course guidance (t_{go1}) can be calculated using equation (6). The requested missile position at the end of the mid-course guidance ($\vec{x}_{m1}$) can be obtained using equation (7).

$$\|20\mathrm{km}\|^2 = \left\|\vec{V}_m \cdot t_{gof}\right\|^2 + \left\|\vec{V}_t \cdot t_{gof}\right\|^2 - 2 \cdot \left\|\vec{V}_m \cdot t_{gof}\right\| \left\|\vec{V}_t \cdot t_{gof}\right\| \cos \emptyset_d \tag{5}$$

$$t_{go1} = t_{go} - t_{gof} \tag{6}$$

$$\vec{x}_{m1} = \overrightarrow{PIP} - \vec{V}_m \cdot t_{gof} \tag{7}$$

According to the above derivation, if mid-course guidance succeeds to regulate the missile trajectory and flight path angle to the designated values at the designated time-to-go, the missile can enter the terminal-guidance mode and hit the target missile at a specified impact angle without sharply changing its flight trajectory in the terminal phase. However, It is noted that the initial fly path angle of the interception missile (γ_{m0}) does not equal to the designated (γ_{mf}), calculated from equation (2), in most cases. Besides, there is no thrust force during the mid-course guidance. Consequently, it is impossible to reach the designated target position ($\vec{x}_{m1}$) at the expected time (t_{go1}), no matter which guidance law has been applied. Therefore, the proposed engagement model and requested missile dynamics at the end of mid-course guidance need to be modified for practical use. The modification methods will be discussed shortly.

3.2. Guidance law

Since it is impossible to achieve all the control goals as discussed above, we proposed using optimal controls to have the best performance as we can get. Furthermore, since both the initial conditions and the endpoint conditions are specified in this optimization problem, it is impossible to get an analytical solution for the optimal controls if the system dynamics are nonlinear. For these reasons, we linearize the missile dynamics shown in equation (1) first and work on the linear-quadratic-regulator (LQR) controls subsequently. The linearized model is shown in equation (8).

$$\boldsymbol{X} = \boldsymbol{X_o} + \boldsymbol{\delta X}$$

$$\boldsymbol{X} = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [r \quad \dot{r} \quad \gamma_m \quad \dot{\gamma}_m]^T$$

$$\begin{bmatrix} \delta\dot{x}_1 \\ \delta\dot{x}_2 \\ \delta\dot{x}_3 \\ \delta\dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ x_{4\,o}^2 & 0 & 0 & 2x_{1\,o}x_{4\,o} \\ 0 & 0 & 0 & 1 \\ \frac{2x_{2\,o}x_{4\,o}}{x_{1\,o}^2} & -\frac{2x_{4\,o}}{x_{1\,o}} & 0 & -\frac{2x_{2\,o}}{x_{1\,o}} \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \\ \delta x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{x_{1\,o}} \end{bmatrix} a_{m\gamma_m} \tag{8}$$

where $\boldsymbol{X}$ is the state values of the system dynamics, $\boldsymbol{X_o}$ is the operation point, $\boldsymbol{\delta X}$ is the small variation around that operation point. After obtaining the linear equations above, we discretized the system and chose a cost function shown below. The cost function is chosen to minimize the difference between state values at the end of the mid-course guidance and the designated missile dynamics.

$$J_{i,N} = \left(\boldsymbol{X_N} - \boldsymbol{X_{ref}}\right)^T P_N \left(\boldsymbol{X_N} - \boldsymbol{X_{ref}}\right) + \sum_{k=0}^{N-1}\left(\left(\boldsymbol{X_k} - \boldsymbol{X_{ref}}\right)^T Q \left(\boldsymbol{X_k} - \boldsymbol{X_{ref}}\right) + u_k^T R u_k\right)$$

$$\boldsymbol{X_{ref}} = \left[\left(x_{m1}^2(t_{go1}) + y_{m1}^2(t_{go1})\right)^{0.5} \quad 0 \quad \gamma_{mf} \quad 0\right]^T$$

$$N = {}^{t_{go1}}/_{\Delta t} \tag{9}$$

where $\boldsymbol{X_k}$ is state value at k-th sampling time, $\boldsymbol{X_{ref}}$ is designated missile dynamics at the end of mid-course guidance. P, Q, R are the weighting matrices which are the design parameters in this case. The optimal solution for the above system dynamics and cost function can be derived using the "dynamic programming" techniques [8]. The optimal control input can be calculated using equation (10) - (13).

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ x_{4\,o}^2 & 0 & 0 & 2x_{1\,o}x_{4\,o} \\ 0 & 0 & 0 & 1 \\ \frac{2x_{2\,o}x_{4\,o}}{x_{1\,o}^2} & -\frac{2x_{4\,o}}{x_{1\,o}} & 0 & -\frac{2x_{2\,o}}{x_{1\,o}} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{x_{1\,o}} \end{bmatrix}^T \tag{10}$$

$$K_{N-1} = \left(R + B^T P_N B\right)^{-1} B^T P_N A \tag{11}$$

$$P_{N-1} = (A - BK_{N-1})^T P_N (A - BK_{N-1}) + K_{N-1}^T R K_{N-1} + Q \tag{12}$$

$$u_k = -K_k(X_k - X_{ref}) \tag{13}$$

Unfortunately, this linearization leads to large errors because the missile dynamics vary largely during the mid-course guidance. To solve this problem, the operation point of the governing equations [equation (8)] should be frequently updated using the most recent state values. The update procedures will be discussed shortly.

3.3. Recursive LQR controls

To sum up the discussions above, there are three concerns that the aforementioned method is inadequate to achieve a good performance which are (1) the mismatch problem of the initial fly path angle stated in section 3.1, (2) the operation-point problem stated in section 3.2, (3) the target missile may have escape ability which its velocity and fly-path-angle are not constant. Since the radar would provide the information of missile dynamics every 0.5 second, we proposed updating our mid-course guidance law using the most updated missile dynamics from radar measurements.

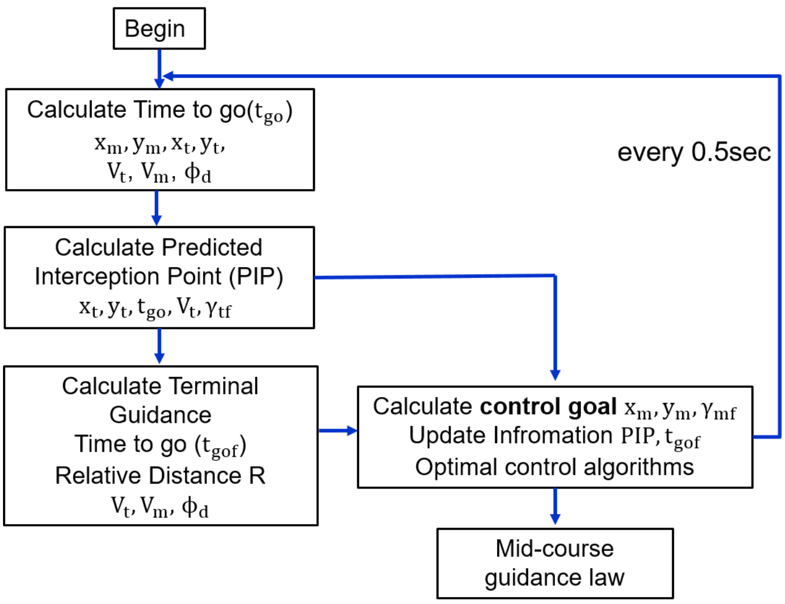

Figure 3. The proposed update procedures of the mid-course guidance.

Figure 3 shows the update procedures of the proposed mid-course guidance. At the onset of each 0.5 second, the most recent missile dynamics are acquired from radar measurements. And, these information are utilized to calculate the remaining time of the mid-course guidance (t_{go1}), and the values of the targeted position and flight path angle at the end of the mid-course guidance. On the other hand, these most recent missile dynamics along with the targeted position and flight path angle are utilized to calculate the feedback gain of the optimal controls for the coming 0.5 seconds.

4. Terminal guidance with constrained impact angle

This section introduces the proposed terminal guidance law for the constrained impact angle. This guidance law is developed based on the conventional "proportional navigation law" [9]. Although the terminal guidance is not the focus of this paper, it is presented here to help evaluate the performance of the proposed mid-course guidance. The proposed terminal guidance law is shown in equation (14) and (15), and they are theoretically proven by the Lyapunov stability theorems.

$$A_{c1} = NV_c\dot{\lambda} - \eta\frac{V_c^2(\lambda_d - \lambda)}{r} \tag{14}$$

$$\lambda_d = \tan^{-1}[\frac{V_{m_f}\sin(\gamma_{t_f}-\emptyset_d)-V_{t_f}\sin\gamma_{t_f}}{V_{m_f}\cos(\gamma_{t_f}-\emptyset_d)-V_{t_f}\cos\gamma_{t_f}}] \tag{15}$$

where r is the relative distance between two missiles, V_c is closing velocity between missiles, λ is the line-of-sight during the engagement, λ_d is the desired line-of-sight that enables the designated impact angle. N and η are the design parameters.

5. Numerical simulation and discussion

We verify the performance of the proposed mid-course guidance law using commercial software Matlab/ Simulink. In the following simulations, the interception missile is located at the inertial coordinate (10, 10) m, and the target missile is located at the coordinate (92018, 39193) m when entering the phase of mid-course guidance. The distance between these two missiles is about 100 kilometers. The initial fly path angle of the interception missile is $\mathbf{50^o}$ and the speed is 1200 m/s. The flight path angle and speed of the target missile are constant and they are $\mathbf{185^o}$ and 800 m/s, respectively. The designated impact angle is $\mathbf{150^o}$, which accounts for the flight path angle of $\mathbf{35^o}$ under the mid-course guidance.

In the first simulation, we verify the effectiveness of the proposed mid-course guidance law by comparing the errors due to different update periods. **Figure** 4 shows the missile trajectories when the mid-course guidance law is updated every 2 Hz (blue-dash line), 30 times (pink-dot line), and once (green-dash line). According to the simulation results, when updates once, the designated target position is (40571, 28465) m while the missile reaches (41207, 26837) m under the mid-course guidance. The miss-distance is calculated to be 1748 meters. The final flight path angle of the missile is $\mathbf{22.06^o}$, and the error of the flight path angle is $\mathbf{12.94^o}$. When updates 30 times, the designated target position is (40540, 28440) m while the missile reaches (40982, 27466) m. The miss-distance is calculated to be 1069 meters. The final flight path angle of the missile is $\mathbf{25.34^o}$, and the error of that is $\mathbf{9.66^o}$. When updates every 2 Hz, the designated target position is (40296, 29370) m while the missile reaches (39712, 29904) m. The miss-distance is calculated to be 791 meters. The final flight path angle of the missile is $\mathbf{34.44^o}$, and the error of that is $\mathbf{0.56^o}$. According to those results, the errors are getting smaller when the number of update increases.

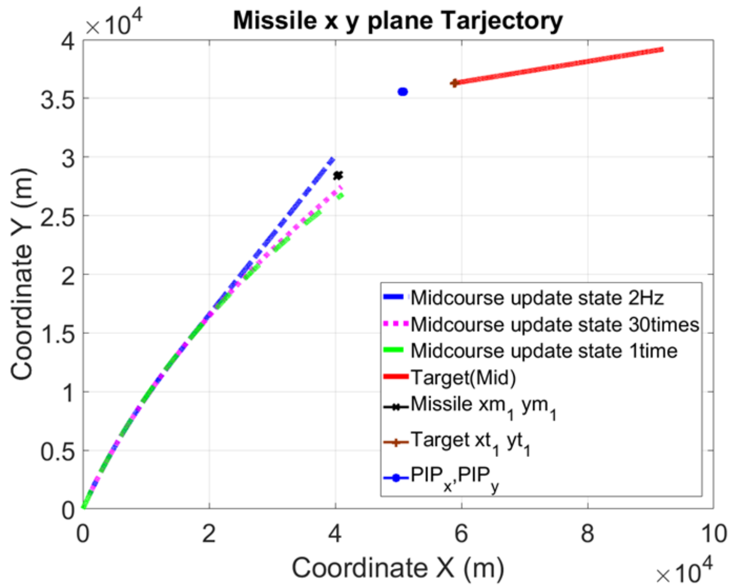

Figure 4. Missile trajectory under various update frequencies. The updates of the missile dynamics and target position are performed for once, 30 times, and every 2 Hz during the entire guidance period.

Another simulation shows the missile trajectory when adapting different linearization processes of the guidance law. According to the simulation results shown in Figure 5, when the guidance command is obtained from a linearized model without the updates of the operation point (black-dash line in the plot), the missile reaches (37881, 34618) m. The error with the target position is 6439 meters, which is much larger than the frequently updated case of 791 meters. This simulation result shows the importance of updating the operation point in the proposed guidance law.

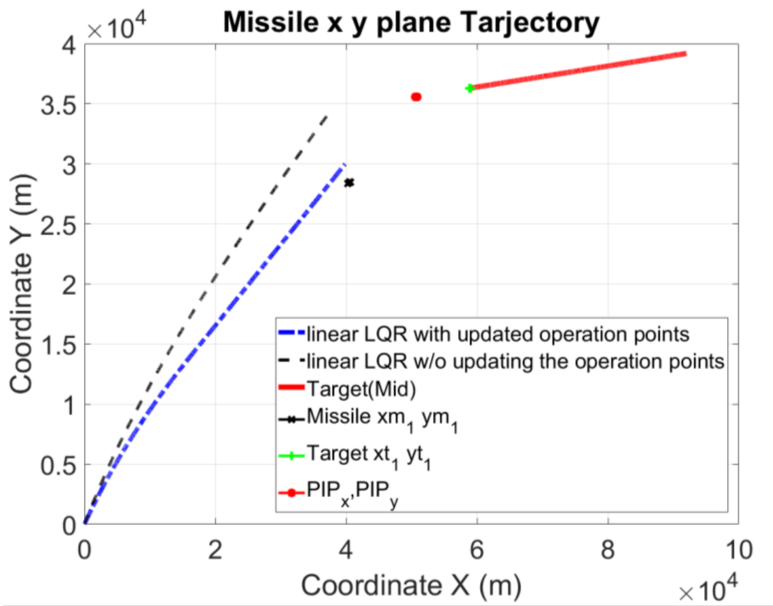

Figure 5. Missile trajectory when adapting different linearization processes of the guidance law.

Figure 6 and Figure 7 show the simulation results of the missile engagement under both mid-course and terminal guidance. According to the missile trajectory shown in Figure 6, the impact angle is

149.99^o and the miss distance is 0.002m at the end of the terminal guidance. Figure 7 indicates that there exists a large normal acceleration (control input) at the instance of the guidance switch. Consequently, the flight path angle of the interception missile experiences a large variation during the terminal guidance. It starts from 34.44^o to 23^o and finally reaches 35.01^o at the end.

The existence of the large command input (accelerations) is because there exist errors in the target position and the flight path angle from the mid-course guidance. These two errors would contribute to both the line-of-sight error $(\lambda - \lambda_d)$ and the change rate of the line-of-sight $(\dot{\lambda})$ in the terminal phase. Consequently, the terminal guidance (see equation (14), (15)) responds to these two values and requests a large command input. The acceleration command varies from -10g to 7g during the terminal guidance. It suggests that the position error from the mid-course guidance may contribute more than the angle error. How to adjust the design parameters shown in equation (9) and (14) to minimize the control inputs at the terminal guidance are still under investigation.

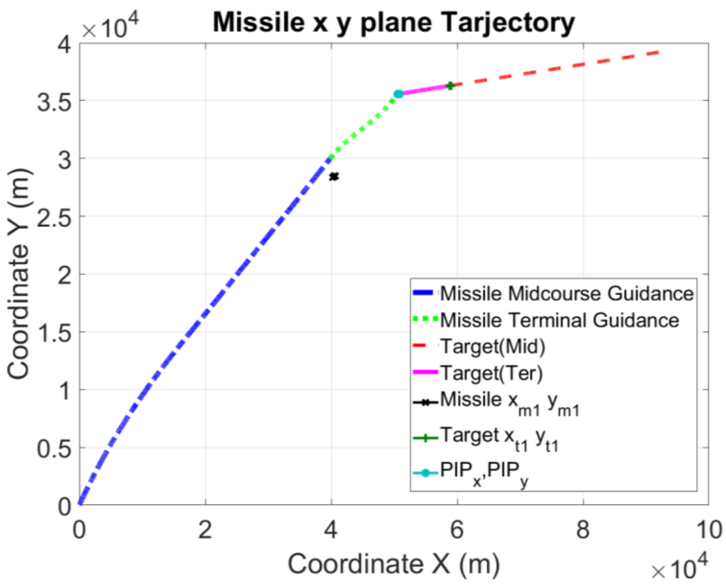

Figure 6. Flight trajectory of interception missile and target missile under proposed mid-course and terminal guidance

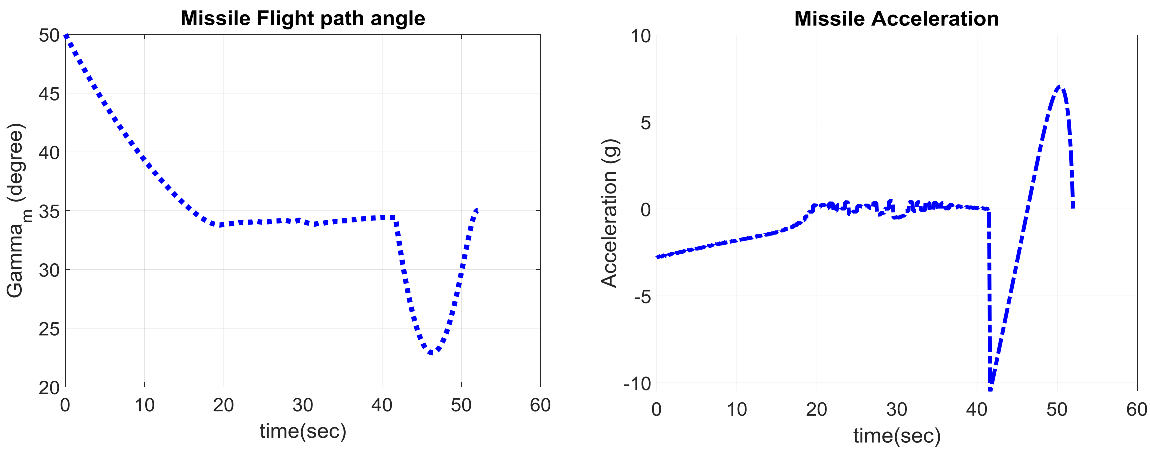

Figure 7. Flight path angle and the acceleration command of the interception missile.

6. Conclusion

This paper proposed novel mid-course guidance for achieving a constrained impact angle. This mid-course guidance is developed to regulate the missile dynamics such that the missile can hit the target at a specified impact angle without sharply changing its flight trajectory during the terminal guidance. The proposed guidance law achieves this goal by recursively updating the target dynamics for the mid-course guidance, and applying linear LQR controls with frequently updated control parameters.

Simulations results verify the importance and effectiveness of the proposed guidance law and its associated update procedures. In a simulation case, the interception missile is 100 kilometers away from the target missile, the flight speed of the two missiles are 1200 m/s and 800 m/s, respectively. The requested impact angle is 150^o. The proposed method achieves the positioning error and flight-path-angle error of 791 meters and 0.56^o, at the end of the mid-course guidance. When working with the previously developed terminal guidance, the miss distance and impact-angle error are 0.002m and 0.001^o at the end of the engagement. The missile guidance still presents large acceleration commands (~10g) when adapting to the terminal guidance law. This is likely due to the positioning error from the mid-course guidance. And, this error could be adjusted by choosing different cost functions of the optimal controls. More study is ongoing to improve the performance.

Acknowledgements
This research work was supported by the National Chung-Shan Institute of Science & Technology under XB07154P-CS, also appreciate personnel of missile institute and researcher Z.D. Zhang.

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