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Consecutive Synchronization of a Delayed Complex Dynamical Network via Distributed Adaptive Control Approach

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Abstract: In this paper, a consecutive synchronization scheme is investigated to synchronize the nodes of a delayed complex dynamical network with an isolated node via an adaptive control approach. The specific feature of this scheme consists in the structure of the communication links: a communication connection is required between the isolated node and one selected node in the network, and further communication links exist between any node and one neighbor node. In this way, all nodes are connected together like a chain. Based on Lyapunov-Krasovskii theory, some conditions are obtained in the form of linear matrix inequalities to guarantee the consecutive synchronization by the designed distributed adaptive control. To make this synchronization scheme more practical, no constraints have been considered for coupling connection matrix such as being symmetric or zero row sum. Finally, a numerical example is presented to demonstrate the effectiveness of the proposed method.

Keywords: Complex dynamical network, distributed adaptive control, Lyapunov-Krasovskii theory, synchronization, time-delay.

1. INTRODUCTION

Complex dynamical networks (CDNs) have become an attractive research area due to their potential applications to various disciplines such as neural networks [1], social networks [2], ecological networks, communication networks [3], information sciences, and power distribution networks [4]. These complex networks, which consist of a large set of interconnected nodes, have been widely utilized to describe many natural and man-made systems [5, 6]. Among various collective behaviors of complex networks [7], synchronization between the nodes is probably the most important and significant [8–11]. This phenomenon has been discovered in nature such as fireflies in the forest, applause, and description of hearts [12], and also developed in man-made systems such as chaos-based secure communication and distributed computing systems [13]. Due to importance of this issue, various synchronization concepts have been introduced in the literature including complete synchronization [14–16], lag synchronization [17–19], local synchronization [20], cluster synchronization [21, 22], and projective synchronization [23, 24], etc.

In order to synchronize the CDN's nodes with an individual node, conventional methods propose a one-to-one connection between every nodes of the network with the individual node [25–30], as shown in Fig. 1(a). This synchronization scheme creates many practical limitations and difficulties. A one-to-one connection needs a direct communication between every nodes of the network and the isolated node where it may be impossible for many practical applications. Furthermore, this will increase the cost of implementation and make it inefficient in practice. Lee *et al*. [31] proposed an improved scheme by introducing an arbitrary virtual target node in the network, which has been shown in Fig. 1(b). In this way, the virtual node synchronizes itself with the isolated node, and all other nodes in the network synchronize themselves with this virtual node. As a matter of fact, this method has tried to eliminate the problems arising from direct connection of all nodes to an isolated node, located outside of the network, such as communication disturbances and uncertainties. But, the need for direct connection between all nodes and the virtual node still remains. To overcome the drawbacks of this method, a consecutive synchronization scheme is proposed in this paper where each node synchronizes itself with another neighbor node (Fig. 1(c)). In this way, a communication link is just needed to make a route from the isolated node and the other nodes. Therefore, all nodes have been connected together like a chain

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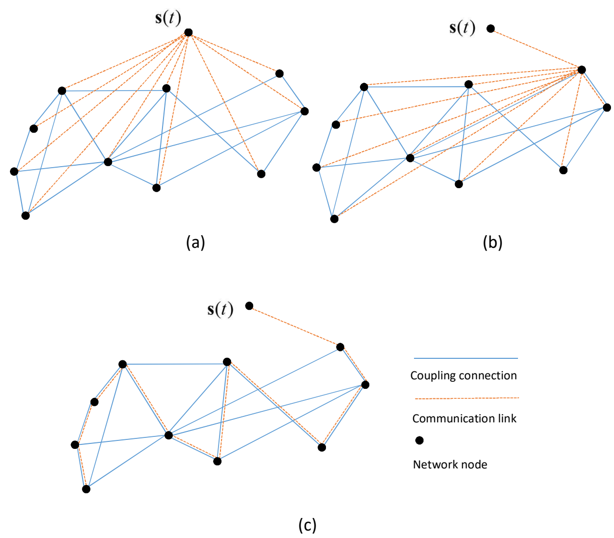

Fig. 1. The framework for synchronization of complex dynamical network in (a) the most previous papers, (b) the method proposed in [31], and (c) this paper.

whereas direct connections between nodes and the isolated node or a virtual node have been eliminated. To the best of the authors' knowledge, this synchronization scheme is presented for the first time, which is the main contribution of this paper. In addition to the other advantages of this synchronization scheme, the following benefits can also be noted:

- Previous synchronization schemes need to broadcast information of the individual node, which yields downgrade the network security. For increasing the level of security, it is better to send the information through some private or secured routes. Therefore, the network designer can opt a route between the individual node and the other nodes based on the network topology. Then, this route can be upgraded by hardware and/or software considerations. Since our proposed scheme has the capability to implement this method, so it can be better in terms of security than other schemes.
- In several cases, however, there is a "natural" ordering of the nodes such that the communication between two subsequent nodes is more advantageous than between every node and a fixed individual node. Applications such as sensor networks (especially for long pipelines), cellular networks, and also some social networks are examples of these networks.

To deal with the synchronization problem, several control methods have been introduced in the literature, including sliding mode control [32–35], adaptive control [36–39], impulsive control [40–43], guaranteed cost control [44], and intermittent control [36, 45]. Among these methods, adaptive control methods have been widely used for the synchronization of complex dynamical networks, and the main reason for this can be seen in the simplicity of controller design for networks with a large number of nodes [38, 46, 47]. Most of the published methods have considered some constraints on the coupling matrix such as being symmetric and/or diffusive conditions [38, 48, 49], which limits the application of these methods. In this paper, a distributed adaptive control method has been used that does not consider any restrictive condition on the coupling matrix. In addition, controller calculations are performed at each node separately, only with the adjacent node information. Since there is a delay in many engineering applications and has been considered by many researchers, we have considered the network with delays. Since the time-delay in many engineering applications is unavoidable and investigated by many researchers [50–52], we consider the network with delayed couplings. Based on Lyapunov-Krasovskii theory, some criteria have been obtained in the form of linear matrix inequalities to guarantee the consecutive synchronization with the proposed distributed adaptive control.

This paper is organized as follows: The problem is stated in section 2. Some useful lemmas are also provided in this section. Based on Lyapunov-Krasovskii theory, some distributed adaptive control laws are designed in Section 3. Some criteria are also presented to guarantee the synchronization between all nodes. A numerical example is presented to demonstrate the effectiveness of the proposed method in Section 4. Finally, Section 5 summarizes the paper.

Notations: The notation in this paper is standard. The sign $\otimes$ stands for the Kronecker product and $\|\,.\,\|$ represent the Euclidean vector norm.

2. PROBLEM STATEMENT

A CDN, comprises of N identical nodes coupled together with a constant delay, is considered as

$$\dot{\mathbf{x}}_i(t) = \mathbf{f}(\mathbf{x}_i(t)) + \sum_{j=1}^{N} c_{ij}\mathbf{x}_j(t-\tau) + \mathbf{u}_i(t), \tag{1}$$

for $i = 1,\ldots,N$, where $\mathbf{x}_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T \in \mathbb{R}^n$ represents the state vector of the ith node, $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear vector-valued function, and $\mathbf{u}_i(t)$ denotes the control signal. $\mathbf{C} = [c_{ij}]_{N\times N}$ is an arbitrary coupling connection matrix and τ represents a known constant time-delay.

Remark 1: It is worth to mention that the most papers have studied the synchronization of CDNs under some constraints on the coupling connection matrix $\mathbf{C}$, such as being symmetric or zero row sum [53, 54], i.e. $c_{ii} = -\sum_{j=1, j\neq i}^{N} c_{ij}$, $i = 1, 2, \ldots, N$. These assumptions restrict the applicability of their methods to real-world problems.

Assumption 1: The vector function $\mathbf{f}:\mathbb{R}^n\to\mathbb{R}^n$ satisfies the Lipschitz condition, i.e., for any $\mathbf{x},\mathbf{y}\in\mathbb{R}^n$:

$$\|\mathbf{f}(\mathbf{x})-\mathbf{f}(\mathbf{y})\|\le l\|\mathbf{x}-\mathbf{y}\|,$$

where l is a known positive constant.

Definition 1: The CDN (1) is said to be consecutively synchronized for any initial conditions if $\lim_{t\to\infty}\|\mathbf{x}_1(t)-\mathbf{s}(t)\| = \lim_{t\to\infty}\|\mathbf{x}_i(t)-\mathbf{x}_{i-1}(t)\| = 0$, $i = 2,3,\ldots,N$, where $\mathbf{s}(t)\in\mathbb{R}^n$ is an isolated node's state vector and satisfies

$$\dot{\mathbf{s}}(t)=\mathbf{f}(\mathbf{s}(t)). \tag{2}$$

Error vectors are introduced as

$$\begin{cases}\mathbf{e}_1(t)=\mathbf{x}_1(t)-\mathbf{s}(t),\\ \mathbf{e}_i(t)=\mathbf{x}_i(t)-\mathbf{x}_{i-1}(t), \quad i=2,3,\ldots,N.\end{cases} \tag{3}$$

The aim is to achieve consecutive synchronization of the CDN for any initial condition under the assumption of the following information structure: for $i=1,\ldots,N$, vectors $e_i(t)$, $x_i(t-\tau)$ and, for $i=2,\ldots,N$, additionally the control $u_{i-1}(t)$ are available for the controller of the ith node at each time instant $t\in[0,\infty)$.

By differentiating the error vectors (3) and substituting equations (1) and (2) into them, the synchronization error dynamic can be written as

$$\left.\begin{aligned}\dot{\mathbf{e}}_1(t)=&\mathbf{f}(\mathbf{x}_1(t))-\mathbf{f}(\mathbf{s}(t))+\sum_{j=1}^{N}c_{1j}\mathbf{x}_j(t-\tau)+\mathbf{u}_1(t),\\ \dot{\mathbf{e}}_i(t)=&\mathbf{f}(\mathbf{x}_i(t))-\mathbf{f}(\mathbf{x}_{i-1}(t))+\sum_{j=1}^{N}(c_{ij}-c_{(i-1)j})\\ &\mathbf{x}_j(t-\tau)+\mathbf{u}_i(t)-\mathbf{u}_{i-1}(t), \quad i=2,3,\ldots,N.\end{aligned}\right\} \tag{4}$$

Reformulating (4) yields

$$\left.\begin{aligned}\dot{\mathbf{e}}_1(t)&=\mathbf{f}(\mathbf{x}_1(t))-\mathbf{f}(\mathbf{s}(t))+\bar{c}_1\mathbf{x}_1(t-\tau)\\ &\quad+\sum_{j=2}^{N}\left(\sum_{k=j}^{N}c_{1k}\right)\mathbf{e}_j(t-\tau)+\mathbf{u}_1(t),\\ \dot{\mathbf{e}}_i(t)&=\mathbf{f}(\mathbf{x}_i(t))-\mathbf{f}(\mathbf{x}_{i-1}(t))\\ &\quad+\sum_{j=2}^{i}\left(\sum_{k=1}^{j-1}(c_{(i-1)k}-c_{ik})\right)\mathbf{e}_j(t-\tau)\\ &\quad+\sum_{j=i+1}^{N}\left(\sum_{k=j}^{N}(c_{ik}-c_{(i-1)k})\right)\mathbf{e}_j(t-\tau)\\ &\quad+(\bar{c}_i-\bar{c}_{i-1})\mathbf{x}_i(t-\tau)+\mathbf{u}_i(t)-\mathbf{u}_{i-1}(t),\\ &\quad i=2,3,\ldots,N-1,\\ \dot{\mathbf{e}}_N(t)&=\mathbf{f}(\mathbf{x}_N(t))-\mathbf{f}(\mathbf{x}_{N-1}(t))\\ &\quad+(\bar{c}_N-\bar{c}_{N-1})\mathbf{x}_N(t-\tau)\\ &\quad+\sum_{j=2}^{N}\left(\sum_{k=1}^{j-1}(c_{(N-1)k}-c_{Nk})\right)\mathbf{e}_j(t-\tau)\\ &\quad+\mathbf{u}_N(t)-\mathbf{u}_{N-1}(t),\end{aligned}\right\} \tag{5}$$

where $\bar{c}_i=\sum_{j=1}^{N}c_{ij}$, $i=1,2,\ldots,N$.

We are now in a position to ensure the stability of the synchronization error dynamic (5) by providing a suitable adaptive controller. This will be done in the next section.

3. MAIN RESULTS

In this section, we propose a distributed adaptive controller to achieve the consecutive synchronization. The following theorem provides this adaptive controller structure and corresponding adaptive laws.

Theorem 1: For any $\tau>0$, the CDN (1) will achieve the consecutive synchronization asymptotically with the isolated node $\mathbf{s}(t)$, based on Definition 1, by utilizing adaptive controllers

$$\mathbf{u}_1(t)=-k_1(t)\mathbf{e}_1(t)-a_1(t)\mathbf{x}_1(t-\tau), \tag{6}$$

$$\mathbf{u}_i(t)=-k_i(t)\mathbf{e}_i(t)-a_i(t)\mathbf{x}_i(t-\tau)+\mathbf{u}_{i-1}(t),\quad i=2,3,\ldots,N, \tag{7}$$

with update laws

$$\dot{k}_i(t)=\|\mathbf{e}_i(t)\|^2, \quad i=1,2,\ldots,N, \tag{8}$$

$$\dot{a}_i(t)=\mathbf{e}_i^T(t)\mathbf{x}_i(t-\tau), \quad i=1,2,\ldots,N, \tag{9}$$

if there exist some positive constants p_i and g_i, $i=1,\ldots,N$, satisfying the following LMI:

$$\Xi=\begin{bmatrix} l\mathbf{I}_N-\mathbf{P}+\mathbf{G} & \Phi\\ * & -\mathbf{G}\end{bmatrix}<0, \tag{10}$$

where $\mathbf{P}=\mathrm{diag}\{p_1,p_2,\ldots,p_N\}$, $\mathbf{G}=\mathrm{diag}\{g_1,g_2,\ldots,g_N\}$, and $\Phi=[\Phi_{ij}]_{N\times N}$ is a matrix defined as follows

$$\Phi_{ij}=\begin{cases}\sum_{k=j}^{N}(c_{ik}-c_{(i-1)k}), & i<j, i\neq 1, j\neq 1,\\ \sum_{k=1}^{j-1}(c_{(i-1)k}-c_{ik}), & i\geq j, i\neq 1, j\neq 1,\\ \sum_{k=j}^{N}c_{1k}, & i=1, j\neq 1,\\ 0, & j=1.\end{cases}$$

Proof: See Appendix A.1. □

As we mentioned before, consecutive synchronization implies that every node in the network synchronize itself with one another node, probably its neighbor, which we call it "parent node". The control laws given in (6) and (7) represent that every node in the network just needs the information of its parent node to synchronize itself to the isolated node without any direct connection. Therefore, this synchronization scheme need local information for each node to reach collective behavior of the whole network, which makes it more practical than the other published methods.

Remark 2: As it is clear, the adaptive controllers (6) and (7), and update laws (8) and (9), do not need any parameters to be obtained from solving the LMI condition (10). Therefore, the LMI (10) can be solved easily with a few number of decision variables even for very large networks. This advantage does not exist in methods such as state feedback and sliding mode controllers, which makes designing of them difficult for large networks. However, the only drawback of this method is the need to control all network nodes. This problem can be fixed with applying pinning adaptive control strategy.

In some particular cases, we can suppose that the conditions $c_{ii} = -\sum_{j=1, j\neq i}^{N} c_{ij}$, $i = 1, 2, \ldots, N$, hold in the network. For this situation, the Theorem 1 can be reduced to the following corollary.

Corollary 1: For any $\tau > 0$, the CDN (1), with zero row sum condition of coupling connection matrix, will achieve the consecutive synchronization asymptotically with the isolated node $\mathbf{s}(t)$, based on Definition 1, by utilizing adaptive controllers

$$\mathbf{u}_1(t) = -k_1(t)\mathbf{e}_1(t), \tag{11}$$

$$\mathbf{u}_i(t) = -k_i(t)\mathbf{e}_i(t) + \mathbf{u}_{i-1}(t), \quad i = 2, 3, \ldots, N, \tag{12}$$

with update laws

$$\dot{k}_i(t) = \|\mathbf{e}_i(t)\|^2, \quad i = 1, 2, \ldots, N, \tag{13}$$

if there exist some positive constants p_i and g_i, $i = 1, \ldots, N$, satisfying the LMI (10).

Proof: See Appendix A.2. □

4. ILLUSTRATIVE EXAMPLE

Consider the well-known Lorenz chaotic system given by

$$\begin{cases} \dot{x}_1(t) = a(x_2(t) - x_1(t)), \\ \dot{x}_2(t) = cx_1(t) - x_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) = x_1(t)x_2(t) - bx_3(t), \end{cases} \tag{14}$$

where $a = 10$, $b = 8/3$, and $c = 28$, [55]. The chaotic behavior of this system for initial state vector as $\mathbf{x}(0) = [-0.2, -0.3, 0.2]^T$ is shown in Fig. 2. It can be calculated that this dynamic system satisfies the Lipschitz condition, defined in Assumption 1, with $l = 50$. Suppose a complex dynamical network introduced in (1) with five nodes where each node is described by (14). We consider $\tau = 0.5$ and the coupling connection matrix as

$$\mathbf{C} = \begin{bmatrix} -1 & 1.5 & -0.5 & 1 & 0.3 \\ 1 & -2 & 1 & 0.2 & -0.8 \\ 0 & 0.5 & -1 & -2 & 1 \\ -1 & -1 & -2 & -1 & 0.5 \\ 2 & 3 & 4 & 0.5 & -2 \end{bmatrix}.$$

Regard to the considered coupling connection matrix, one can obtain

$$\Phi = \begin{bmatrix} 0 & 2.3 & 0.8 & 1.3 & 0.3 \\ 0 & -2 & -0.4 & -1.9 & -1.1 \\ 0 & 1 & -1.5 & -0.4 & 1.8 \\ 0 & 1 & 2.5 & 3.5 & -0.5 \\ 0 & -3 & -7 & -13 & -14.5 \end{bmatrix}.$$

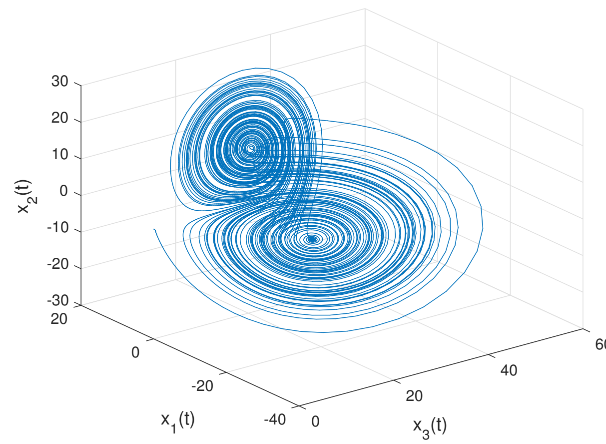

Fig. 2. Chaotic behavior of the Lorenz system,

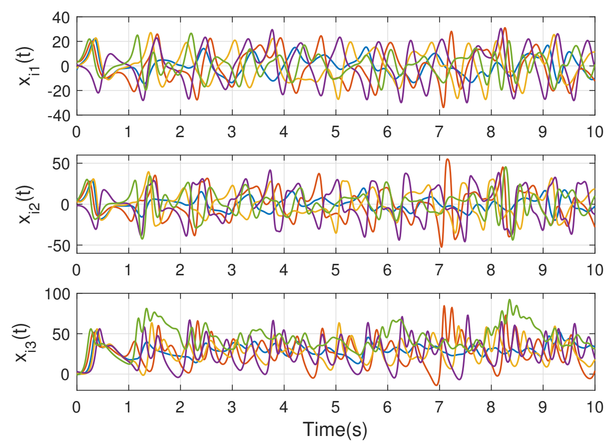

Fig. 3. The state trajectories of all the nodes without control effort,

Note that sum of the elements in each row in the coupling matrix is not necessarily to be zero. The state trajectories of all nodes without control are shown in Fig. 3. It is obvious that the states are not synchronized together without applying the control effort. For this system, the LMI (10) has feasible solution with the following matrices

$$\mathbf{P} = 10^4 \times \text{diag}\{2.0011, 2.0011, 2.0010, 2.0009, 2.0002\},$$

$$\mathbf{G} = 10^4 \times \text{diag}\{1.0532, 1.0532, 1.0532, 1.0532, 1.0532\}.$$

Therefore, Theorem 1 guarantees that control signals (6) and (7) with update laws (8) and (9) synchronize the complex dynamical network. By applying these controllers, the state trajectories of the nodes have been synchronized to the isolated node's states (Fig. 4). Fig. 5 shows the synchronization errors converged to zero after about 3 seconds. Figs. 6 and 7 also show the evolution of $k_i(t)$ and $a_i(t)$, $i = 1, 2, \ldots, 5$, respectively. These figures show that these parameters converge to their final value

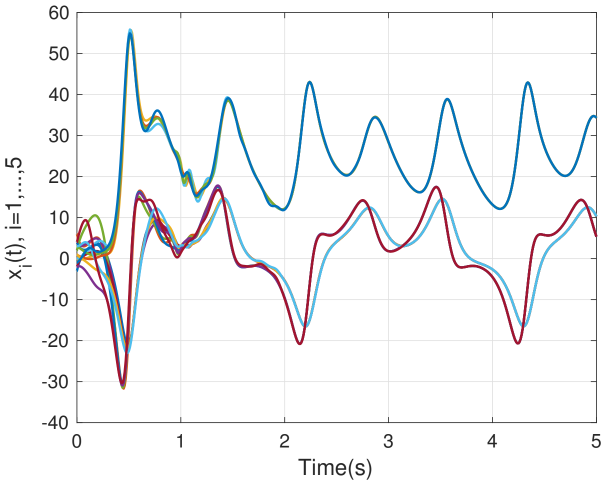

Fig. 4. The state trajectories of all the nodes with control effort.

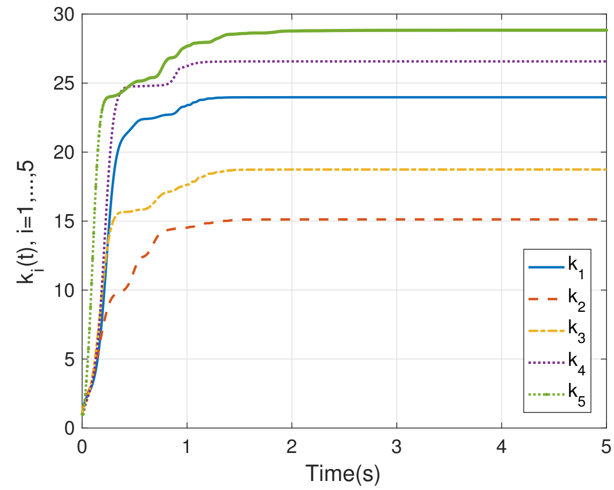

Fig. 6. The evolution of $k_i(t)$, $i = 1, 2, \ldots, 5$.

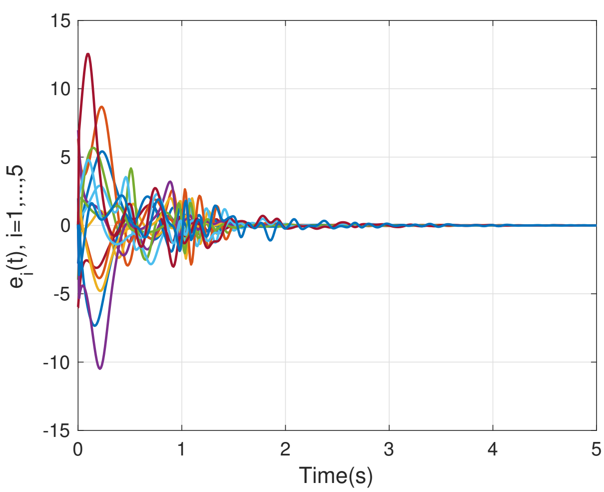

Fig. 5. Synchronization errors for all the nodes.

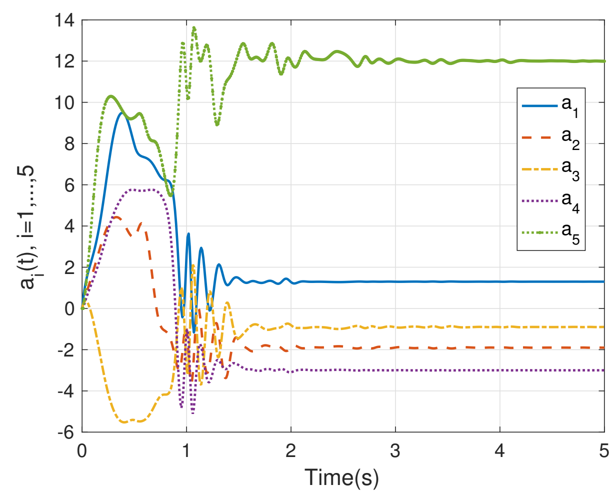

Fig. 7. The evolution of $a_i(t)$, $i = 1, 2, \ldots, 5$.

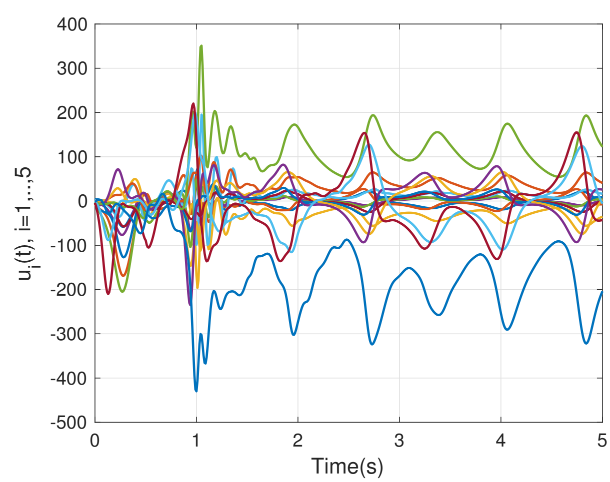

Fig. 8. Control signals.

after forsaking the transient time. The control signals are shown in Fig. 8.

Remark 3: Maybe, it can be thought that this synchronization scheme will stretch the convergence time of synchronization. This conception comes from the hierarchy structure of the proposed synchronization scheme. It is worth to mention that the synchronization convergence time for this scheme is not so different from the conventional synchronization scheme, because every node synchronize itself to its parent node and that node also to its parent node until the first node simultaneously. For proof of this claim, we consider conventional scheme with defining $\mathbf{e}_i(t) = \mathbf{x}_i(t) - \mathbf{s}(t)$, $i = 1, 2, \ldots, 5$. Fig. 9 shows the synchronization errors with conventional method which implies that the convergence time of synchronization is about 3 seconds.

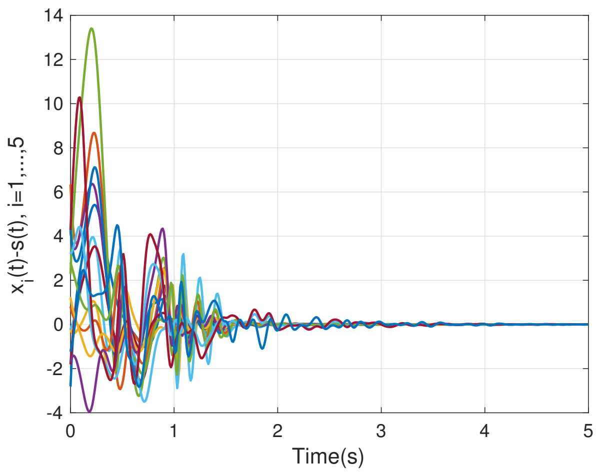

Fig. 9. Synchronization errors with conventional method.

5. CONCLUSION

For a delayed CDN, a new synchronization scheme has been introduced in this paper to overcome some practical limitations of conventional synchronization schemes. To achieve the synchronization, a distributed adaptive control was presented. With the help of Lyapunov-Krasovskii theory, some conditions were obtained in the form of LMIs that ensure the synchronization with respect to the proposed controller. This method was finally applied and simulated on the Lorenz chaotic system, which showed the effectiveness of the proposed method. It has been found that the proposed method has some limitations such as the need to control all the network nodes. Removing these restrictions, by utilizing adaptive pinning control, outlines the road map of our future research.

APPENDIX A

A.1. Proof of Theorem 1

Consider the following Lyapunov functional:

$$V(t) = \sum_{i=1}^{N} V_i(t), \tag{A.1}$$

where

$$\begin{aligned} V_i(t) =& \frac{1}{2}\left(\mathbf{e}_i^T(t)\mathbf{e}_i(t) + (k_i(t)-p_i)^2 + (\bar{a}_i - a_i(t))^2 \right. \\ & \left. + 2\int_{t-\tau}^{t} g_i \mathbf{e}_i^T(s)\mathbf{e}_i(s)ds\right), \quad i = 1,2,\ldots,N, \end{aligned}$$

and $\bar{a}_1 = \bar{c}_1$, $\bar{a}_i = \bar{c}_i - \bar{c}_{i-1}$, $i = 2, 3, \ldots, N$.

Taking the derivative of $V_i(t)$, $i = 1, 2, \ldots, N$, along the solutions of (5) yields:

$$\begin{aligned} \dot{V}_i(t) =& \mathbf{e}_i^T(t)\dot{\mathbf{e}}_i(t) + (k_i(t)-p_i)\dot{k}_i(t) \\ & - (\bar{a}_i - a_i(t))\dot{a}_i(t) + g_i \mathbf{e}_i^T(t)\mathbf{e}_i(t) \\ & - g_i \mathbf{e}_i^T(t-\tau)\mathbf{e}_i(t-\tau), \quad i = 1,2,\ldots,N. \end{aligned} \tag{A.2}$$

By substituting $\dot{k}_i(t)$ and $\dot{a}_i(t)$ from (8) and (9) into (A.2), one yields

$$\begin{aligned} \dot{V}_i(t) =& \mathbf{e}_i^T(t)\dot{\mathbf{e}}_i(t) + (k_i(t)-p_i)\mathbf{e}_i^T(t)\mathbf{e}_i(t) \\ & - (\bar{a}_i - a_i(t))\mathbf{e}_i^T(t)\mathbf{x}_i(t-\tau) + g_i \mathbf{e}_i^T(t)\mathbf{e}_i(t) \\ & - g_i \mathbf{e}_i^T(t-\tau)\mathbf{e}_i(t-\tau), \end{aligned} \tag{A.3}$$

for $i = 1,2,\ldots,N$. Taking $\dot{\mathbf{e}}_1(t)$ from (5), and $\mathbf{u}_1(t)$ from (6) into $\dot{V}_1(t)$, we have

$$\begin{aligned} \dot{V}_1(t) =& \mathbf{e}_1^T(t)(\mathbf{f}(\mathbf{x}_1(t)) - \mathbf{f}(\mathbf{s}(t))) + \bar{c}_1 \mathbf{e}_1^T(t)\mathbf{x}_1(t-\tau) \\ & + \sum_{j=2}^{N}\left(\sum_{k=j}^{N} c_{1k}\right)\mathbf{e}_1^T(t)\mathbf{e}_j(t-\tau) \\ & - k_1(t)\mathbf{e}_1^T(t)\mathbf{e}_1(t) - a_1(t)\mathbf{e}_1^T(t)\mathbf{x}_1(t-\tau) \\ & + (k_1(t)-p_1)\mathbf{e}_1^T(t)\mathbf{e}_1(t) \\ & - (\bar{a}_1 - a_1(t))\mathbf{e}_1^T(t)\mathbf{x}_1(t-\tau) + g_1\mathbf{e}_1^T(t)\mathbf{e}_1(t) \\ & - g_1 \mathbf{e}_1^T(t-\tau)\mathbf{e}_1(t-\tau). \end{aligned} \tag{A.4}$$

By application of Assumption 1, equation (A.4) can be written as

$$\begin{aligned} \dot{V}_1(t) \le& \mathbf{e}_1^T(t)(l_1 + g_1 - p_1)\mathbf{e}_1(t) \\ & - g_1 \mathbf{e}_1^T(t-\tau)\mathbf{e}_1(t-\tau) \\ & + \sum_{j=2}^{N}\left(\sum_{k=j}^{N} c_{1k}\right)\mathbf{e}_1^T(t)\mathbf{e}_j(t-\tau). \end{aligned} \tag{A.5}$$

Same way as described in (A.3)–(A.5), $\dot{V}_i(t)$, $i = 2,3,\ldots,N$, are obtained as

$$\begin{aligned} \dot{V}_i(t) \le& \mathbf{e}_i^T(t)(l_i + g_i - p_i)\mathbf{e}_i(t) - g_i \mathbf{e}_i^T(t-\tau)\mathbf{e}_i(t-\tau) \\ & + \sum_{j=2}^{i}\left(\sum_{k=1}^{j-1}(c_{(i-1)k} - c_{ik})\right)\mathbf{e}_i^T(t)\mathbf{e}_j(t-\tau) \\ & + \sum_{j=i+1}^{N}\left(\sum_{k=j}^{N}(c_{ik} - c_{(i-1)k})\right)\mathbf{e}_i^T(t)\mathbf{e}_j(t-\tau), \\ & i = 2,3,\ldots,N-1, \end{aligned} \tag{A.6}$$

and

$$\begin{aligned} \dot{V}_N(t) \le& \mathbf{e}_N^T(t)(l_N + g_N - p_N)\mathbf{e}_N(t) \\ & + \sum_{j=2}^{N}\left(\sum_{k=1}^{j-1}(c_{(N-1)k} - c_{Nk})\right)\mathbf{e}_N^T(t)\mathbf{e}_j(t-\tau) \\ & - g_N \mathbf{e}_N^T(t-\tau)\mathbf{e}_N(t-\tau). \end{aligned} \tag{A.7}$$

Considering (A.5)–(A.7), it is straightforward to show that

$$\dot{V}(t) = \sum_{i=1}^{N}\dot{V}_i(t) \le \xi^T(t)\left(\Xi \otimes \mathbf{I}_n\right)\xi(t), \tag{A.8}$$

where $\xi(t) = \left[\mathbf{e}^T(t), \mathbf{e}^T(t-\tau)\right]^T$, $\mathbf{e}(t) = \left[\mathbf{e}_1^T(t), \ldots, \mathbf{e}_N^T(t)\right]^T$, $\mathbf{e}(t-\tau) = [\mathbf{e}_1^T(t-\tau), \ldots, \mathbf{e}_N^T(t-\tau)]^T$, and the other parameters are given in (10). If the LMI given in (10) holds, then $\dot{V}(t) < 0$, which implies that $\mathbf{e}(t) \to 0$ when $t \to \infty$. This completes the proof. □

A.2. Proof of Corollary 1

Zero row sum condition of coupling connection matrix, i.e., $\bar{c}_i = \sum_{j=1}^{N} c_{ij}$, $i = 1, 2, \ldots, N$, implies that $\bar{a}_1 = \bar{c}_1 = 0$, $\bar{a}_i = \bar{c}_i - \bar{c}_{i-1} = 0$, $i = 2, 3, \ldots, N$. Therefore, the terms $\mathbf{x}_i(t-\tau)$, $i = 1, 2, \ldots, N$, will be removed from the error dynamic system (5). Thus, there is no need to estimate and compensate these terms in control laws (6) and (7). From this reason, the Lyapunov-Krasovskii functional candidates are as

$$V(t) = \sum_{i=1}^{N} V_i(t), \tag{A.9}$$

where

$$\begin{aligned} V_i(t) =& \frac{1}{2}\Big(\mathbf{e}_i^T(t)\mathbf{e}_i(t) + (k_i(t) - p_1)^2 \\ &+ 2\int_{t-\tau}^{t} g_i \mathbf{e}_i^T(s)\mathbf{e}_i(s)ds\Big), \quad i = 1, 2, \ldots, N. \end{aligned}$$

Rest of the proof is similar to the proof of Theorem 1. □

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