Automatica 129 (2021) 109595

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Time-varying IBLFs-based adaptive control of uncertain nonlinear systems with full state constraints☆

Lei Liu [a], Tingting Gao [a], Yan-Jun Liu [a,*], Shaocheng Tong [a], C.L. Philip Chen [b], Lei Ma [a]

[a] College of Science, Liaoning University of Technology, Jinzhou, Liaoning, 121001, China
[b] School of Computer Science & Engineering, South China University of Technology, Guangzhou, Guangdong, 510006, China

ARTICLE INFO

Article history:
Received 13 May 2019
Received in revised form 20 November 2020
Accepted 5 February 2021
Available online 5 April 2021

Keywords:
Adaptive control
Nonlinear systems
Integral barrier Lyapunov functions
Time-varying full state constraints

ABSTRACT

This paper presents an adaptive control design for nonlinear systems with time-varying full state constraints. It is the first time to introduce the novel time-varying Integral Barrier Lyapunov functions (TVIBLFs) into the adaptive control design, which not only overcomes the limitation of conservatism existing in the traditional BLFs, but also guarantees that the full state time-varying constraint bounds are not violated. The TVIBLFs are combined with the backstepping design procedure to construct the controllers and adaptation laws, and the integral mean value theorem is used to differentiate TVIBLFs. It can be proven that all the states are forced in the time-varying regions and the stability of the closed-loop system is achieved. The effectiveness of the proposed adaptive control strategy can be illustrated through a simulation example.

1. Introduction

In recent decades, the adaptive control technology has developed rapidly, which reflects the general trend that modern control system is developing towards intelligent and accurate. Many adaptive backstepping control algorithms were developed for nonlinear systems with uncertain parameters in (Chen, Feng, & Su, 2016; Krstic & Kokotovic, 1996; Krstic & Smyshlyaev, 2008; Smyshlyaev & Krstic, 2007; Sui, Chen, & Tong, 2018; Tong, Min, & Li, 2020; Wang & Lin, 2012, 2015; Wang, Wen and Guo, 2016; Wang, Wen, & Lin, 2017; Wu, Xie, & Zhang, 2007; Xie & Tian, 2009; Yu, Shi, & Zhao, 2018). Driven by practical requirements, some significant backstepping algorithms were proposed and applied to real systems (He, Ge, & Zhang, 2011; Li, Zhao, Chen, Fang, & Liu, 2018). However, some system nonlinearities cannot be linearly parameterized and normally unknown. With the development of neural networks (NNs) and fuzzy logic systems, approximation-based adaptive control designs in (Chen & Pao, 1993; Song, Guo and Huang, 2017; Wang, Liu, Zhang and Chen, 2016; Zhou, Chen, Chen and Li, 2014) were constructed for the nonlinear systems with unknown functions. It is noteworthy that the above results are limited to the unknown nonlinear systems without constraints.

The actual systems are unavoidable to encounter various constraints, and the performance and stability of systems will be affected. In recent years, the application of BLF to solve the control problems of nonlinear systems with constraints has become a hot topic, and it has gradually attracted the attention of more and more scholars. The problems of constant (Tee, Ge, & Tay, 2009) and time-varying (Tee, Ren, & Ge, 2011) output constraints were overcome by establishing adaptive tracking control based on BLF. For nonlinear output constrained systems with unknown functions, some novel adaptive tracking controllers (Li, Li, Liu, Tong, & Chen, 2017; Liu, Liu, Chen, Tong, & Chen, 2020; Zhao, Song and Shen, 2018) were structured based on BLFs and NNs. Subsequently, BLFs-based adaptive control strategies were presented for real systems, such as the Euler–Bernoulli beam system (He & Ge, 2015). Compared with output constraints, full state constraints are more general in actual systems (Liu, Li, Liu, & Tong, 2021). The BLFs-based adaptive control methods were presented to achieve full state constraint satisfactions for strict-feedback systems (Song, Shen, He and Huang, 2017; Zhang, Xia, & Yi, 2017; Zhao & Song, 2019), pure-feedback systems (Kim & Yoo, 2014), stochastic systems and Euler–Lagrange systems (Zhao, Song, Ma and He, 2018). However, the limitation of conservatism in the above results is that the state constraints must be transformed into the error constraints.

To cope with conservative limitation in traditional BLFs, IBLFs were further designed to directly enforce full states within the

☆ This work is supported in part by the National Natural Science Foundation of China under Grants 62025303, 61973147, 61803190, 61773188, in part by the LiaoNing Revitalization Talents Program XLYC1907050. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Shuzhi Sam Ge under the direction of Editor Miroslav Krstic.

∗ Corresponding author.
E-mail addresses: liuleill@live.cn (L. Liu), gaotthh@163.com (T. Gao), liuyanjun@live.com (Y.-J. Liu), jztongsc@163.com (S. Tong), philip.chen@ieee.org (C.L.P. Chen), ml920601@hotmail.com (L. Ma).

https://doi.org/10.1016/j.automatica.2021.109595
0005-1098/

predefined compact sets. With the help of backstepping technique and IBLFs, some novel adaptive control schemes (Tee & Ge, 2012) for nonlinear strict-feedback systems with state constraints have been proposed. Based on the IBLFs, an adaptive NN control approach (Tang, Ge, Tee, & He, 2016) was established for SISO nonlinear state constrained pure-feedback systems with time-varying disturbances to ensure the state constraints were never violated, in which NNs are employed to estimate the uncertain nonlinearities. To solve the explosion of complexity and state constraints, the dynamic surface design based on IBLFs (Kim & Yoo, 2015) was proposed for pure-feedback constrained systems. The authors (Liu, Tong, Chen, & Li, 2017) have further extended these adaptive constraint control strategies to MIMO nonlinear systems with full state constraints. It is worth mentioning that the above results of IBLFs have solved the problem of conservative limitation existing in the traditional approaches. However, these results can only deal with the nonlinear systems with the full state constant constraint problems and it is an urgent problem to control nonlinear systems with full state time-varying constraints based on IBLFs.

Inspired by the above observations, this paper investigates the adaptive tracking control problem for a class of nonlinear systems with uncertain nonlinear functions and time-varying full state constraints. In the framework of backstepping design technique, TVIBLFs and NNs are employed to design adaption laws and controller. Compared with the existing literatures, the main contributions of this paper are as follows: although some adaptive controllers based on BLFs were proposed for nonlinear state constraint systems, they are required to transform the state constraints into error constraints and thus, this results in conservatism for known error bounds. To prevent state constraint violation and overcome the conservatism, the novel TVIBLFs are introduced in this paper; the previous results on IBLFs (Kim & Yoo, 2015; Liu et al., 2017; Tang et al., 2016; Tee & Ge, 2012) only solve constant state constraints and the time-varying IBLFs are first proposed in this paper, which can achieve the full state time-varying constraint satisfactions. Furthermore, the established TVIBLFs-based adaptive neural tracking controller can ensure that all the closed-loop system signals are bounded, the full state constraint satisfactions are achieved. Meanwhile, the tracking error can remain in a small neighborhood of the origin.

2. System descriptions

Consider the nonlinear system in the strict-feedback form

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)\, x_{i+1}, i = 1, \ldots, n-1 \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)\, u \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^T, i = 1, \ldots, n$, u and y are states, input and output of systems, respectively; $f_i(\bar{x}_i)$ and $g_i(\bar{x}_i), i = 1, \ldots, n$ are smooth functions. In this paper, the time-varying full state constraints are considered, i.e. $|x_i| < k_{c_i}(t)$ where $k_{c_i}(t) : R_+ \to R$, $i = 1, \ldots, n$, $\forall t \geq 0$. For this class of nonlinear system, we assume that the existence and uniqueness of solution are satisfied (Zhou, Wen and Yang, 2014).

The control objective is to design an adaptive feedback controller u such that y tracks the desired trajectory y_d while ensuring that all the closed-loop signals are bounded and the time-varying full state constraints are not violated.

Assumption 1 (*Tee & Ge, 2012*)**.** For $k_{c_1}(t) : R_+ \to R$, there exist a function $Y_0(t) : R_+ \to R_+$ and the constants $Y_i > 0, i = 1, \ldots, n$ so that the desired trajectory y_d and its time derivatives satisfy $|y_d(t)| \leq Y_0(t) < k_{c_1}(t)$, $\left|y_d^{(i)}(t)\right| < Y_i, \forall t \geq 0$.

Assumption 2 (*Tee & Ge, 2012*)**.** The smooth functions $g_i(\bar{x}_i)$ $i = 1, \ldots, n$ are known, and there exists a positive constant G such that $0 < G \leq |g_i(\bar{x}_i)|$. For generality, it is assumed that $g_i(\bar{x}_i) > 0$.

Assumption 3 (*Tee et al., 2011*)**.** There exist the constants $K_{c_i}^0$, $i = 1, \ldots, n$, and $K_{c_i}^j, j = 1, \ldots, n$ such that $k_{c_i}(t) \leq K_{c_i}^0$ and $k_{c_i}^{(j)}(t) \leq K_{c_i}^j$, $\forall t \geq 0$.

3. The controller design and stability analysis

In this section, an adaptive controller u will be developed for (1) based on the backstepping design with the IBLF.

Step 1: The time derivative of tracking error $z_1 = x_1 - y_d$ is defined as follows

$$\dot{z}_1 = f_1(x_1) + g_1(x_1)\, x_2 - \dot{y}_d. \tag{2}$$

Choose the Integral-type Lyapunov function candidate

$$V_1^z = \int_0^{z_1} \frac{\delta k_{c_1}^2(t)}{k_{c_1}^2(t) - (\delta + y_d)^2} d\delta. \tag{3}$$

Remark 1. IBLF is a kind of BLF selected in this paper to deal with the full state constraints problem. At the same time, there are two types of typical traditional BLFs, log-BLF and tan-BLF. The log-BLF is a common BLF, and the tan-BLF is a kind of BLF that can integrate constraint analysis into a general method to deal with both constrained and unconstrained systems. The above two kinds of traditional BLFs have a common disadvantage compared with the IBLF. That is, they need to convert the state constraints into error constraints in the process of dealing with the constraint problems for further derivation, while the IBLF does not need such operation. In addition, this paper studies a kind of TVIBLF that reduces the conservative limitation of constant IBLF and is more general.

Because $|y_d(t)| \leq Y_0(t) < k_{c_1}(t)$, it can be seen that V_1^z is positive definite, continuously differentiable in set $|x_1| < k_{c_1}(t)$, while satisfying the decreasing condition. Then, the following inequality holds

$$\frac{1}{2} z_1^2 \leq V_1^z \leq z_1^2 \int_0^1 \frac{\upsilon k_{c_1}^2(t)}{k_{c_1}^2(t) - (\upsilon z_1 + \operatorname{sgn}(z_1)\, Y_0(t))^2} d\upsilon \tag{4}$$

where the substitution $\delta = \upsilon z_1$ will be used later.

Define

$$\chi_1(t) = \frac{\delta k_{c_1}^2(t)}{k_{c_1}^2(t) - (\delta + y_d)^2} \tag{5}$$

and $\chi_1(t)$ is obviously continuously differentiable in $|x_1| < k_{c_1}(t)$.

The time derivative of V_1^z is described as

$$\begin{aligned} \dot{V}_1^z &= \lim_{\Delta t \to 0} \frac{V_1^z(t + \Delta t) - V_1^z(t)}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{z_1(t)}^{z_1(t+\Delta t)} \chi_1(t + \Delta t)\, d\delta \\ &+ \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_0^{z_1(t)} (\chi_1(t + \Delta t) - \chi_1(t))\, d\delta. \end{aligned} \tag{6}$$

Then, according to the integral mean value theorem and uniformly continuous function $\chi_1(t)$, we have

$$\dot{V}_1^z = \lim_{\Delta t\to 0}\chi_1(\varsigma_1)\frac{z_1(t+\Delta t)-z_1(t)}{\Delta t} + \int_0^{z_1(t)}\lim_{\Delta t\to 0}\frac{\chi_1(t+\Delta t)-\chi_1(t)}{\Delta t}d\delta = \dot{z}_1(t)\chi_1(z_1(t)) + \int_0^{z_1(t)}\frac{d\chi_1(t)}{dt}d\delta \tag{7}$$

where $\varsigma_1 \in (z_1(t), z_1(t+\Delta t))$ and $\chi_1(t)$ is a function of $y_d(t)$ and $k_{c_1}(t)$.

Further, the time derivative of V_1^z satisfies

$$\dot{V}_1^z = \frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2}\dot{z}_1 + \dot{y}_d\int_0^{z_1}\frac{\partial}{\partial y_d}\frac{\delta k_{c_1}^2(t)}{k_{c_1}^2(t)-(\delta+y_d)^2}d\delta + \dot{k}_{c_1}(t)\int_0^{z_1}\frac{\partial}{\partial k_{c_1}(t)}\frac{\delta k_{c_1}^2(t)}{k_{c_1}^2(t)-(\delta+y_d)^2}d\delta \tag{8}$$

where

$$\int_0^{z_1}\frac{\partial}{\partial y_d}\frac{\delta k_{c_1}^2(t)}{k_{c_1}^2(t)-(\delta+y_d)^2}d\delta = z_1\left(\frac{k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2} - \Psi_1(z_1, y_d, k_{c_1})\right) \tag{9}$$

with

$$\Psi_1(z_1, y_d, k_{c_1}) = \int_0^1\frac{k_{c_1}^2(t)}{k_{c_1}^2(t)-(\upsilon z_1+y_d)^2}d\upsilon = \frac{k_{c_1}(t)}{z_1}\left(\tanh^{-1}\left(\frac{z_1+y_d}{k_{c_1}(t)}\right)-\tanh^{-1}\left(\frac{y_d}{k_{c_1}(t)}\right)\right) = \frac{k_{c_1}(t)}{2z_1}\ln\frac{(k_{c_1}(t)+x_1)(k_{c_1}(t)-y_d)}{(k_{c_1}(t)-x_1)(k_{c_1}(t)+y_d)}.$$

The third term in (8) satisfies

$$\int_0^{z_1}\frac{\partial}{\partial k_{c_1}(t)}\frac{\delta k_{c_1}^2(t)}{k_{c_1}^2(t)-(\delta+y_d)^2}d\delta = \int_0^{z_1}-\delta(\delta+y_d)\,d\frac{k_{c_1}(t)}{k_{c_1}^2(t)-(\delta+y_d)^2} = \frac{-z_1(z_1+y_d)k_{c_1}(t)}{k_{c_1}^2(t)-(z_1+y_d)^2} + \int_0^{z_1}\frac{(2\delta+y_d)k_{c_1}(t)}{k_{c_1}^2(t)-(\delta+y_d)^2}d\delta = z_1\left(\frac{-(z_1+y_d)k_{c_1}(t)}{k_{c_1}^2(t)-(z_1+y_d)^2} + \int_0^1\frac{(2\upsilon z_1+y_d)k_{c_1}(t)}{k_{c_1}^2(t)-(\upsilon z_1+y_d)^2}d\upsilon\right) = z_1\left(\frac{-z_1 k_{c_1}(t)}{k_{c_1}^2(t)-(z_1+y_d)^2} + \mathrm{I}_1(z_1, y_d, k_{c_1})\right) \tag{10}$$

where

$$\mathrm{I}_1(z_1, y_d, k_{c_1}) = \frac{-y_d k_{c_1}(t)}{k_{c_1}^2(t)-(z_1+y_d)^2} + \int_0^1\frac{(2\upsilon z_1+y_d)k_{c_1}(t)\,d\upsilon}{k_{c_1}^2(t)-(\upsilon z_1+y_d)^2} = \frac{-y_d k_{c_1}(t)}{k_{c_1}^2(t)-x_1^2} + \frac{k_{c_1}(t)}{z_1}\ln\left(\frac{k_{c_1}^2(t)-x_1^2}{k_{c_1}^2(t)-{y_d}^2}\right) + \frac{y_d}{2z_1}\ln\left(\frac{k_{c_1}^2(t)-y_d^2}{k_{c_1}^2(t)-x_1^2}\right).$$

Remark 2. Using L'Hôpital's rule, that

$$\lim_{z_1\to 0}\Psi_1(z_1, y_d, k_{c_1}) = \frac{k_{c_1}^2(t)}{k_{c_1}^2(t)-{y_d}^2}$$

$$\lim_{z_1\to 0}\mathrm{I}_1(z_1, y_d, k_{c_1}) = \frac{{y_d}^2-3y_d k_{c_1}}{k_{c_1}^2(t)-{y_d}^2}.$$

From Assumption 1, it can be known that $|y_d(t)| \le Y_0 < k_{c_1}(t)$. Thus, $\Psi_1(z_1, y_d, k_{c_1})$ and $\mathrm{I}_1(z_1, y_d, k_{c_1})$ are well defined to be bounded in a neighborhood of $z_1 = 0$.

Introducing $z_2 = x_2 - \alpha_1$, and substituting (2), (9) and (10) into (8) results in

$$\dot{V}_1^z = \frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2}[f_1(x_1)+g_1(x_1)z_2 + g_1(x_1)\alpha_1] - z_1\Psi_1(z_1, y_d, k_{c_1})\dot{y}_d + z_1\mathrm{I}_1(z_1, y_d, k_{c_1})\dot{k}_{c_1}(t) - \frac{z_1^2 k_{c_1}(t)\dot{k}_{c_1}(t)}{k_{c_1}^2(t)-x_1^2}. \tag{11}$$

Define the unknown nonlinear function as

$$\mathrm{P}_1(\mathrm{Z}_1) = f_1(x_1) - \frac{k_{c_1}^2(t)-x_1^2}{k_{c_1}^2(t)}\Psi_1(z_1, y_d, k_{c_1})\dot{y}_d + \frac{k_{c_1}^2(t)-x_1^2}{k_{c_1}^2(t)}\mathrm{I}_1(z_1, y_d, k_{c_1})\dot{k}_{c_1}(t) \tag{12}$$

where $\mathrm{Z}_1 = [x_1, y_d, \dot{y}_d, k_{c_1}, \dot{k}_{c_1}]^T$.

Based on the neural approximation, $\mathrm{P}_1(\mathrm{Z}_1)$ can be described by

$$\mathrm{P}_1(\mathrm{Z}_1) = \xi_1^{*T}S_1(\mathrm{Z}_1) + \varepsilon_1(\mathrm{Z}_1) \tag{13}$$

where ξ_1^* denotes the ideal weight vector, $S_1(\mathrm{Z}_1) = [s_1(\mathrm{Z}_1), \ldots, s_p(\mathrm{Z}_1)]^T$ is the radial basis vector with Gaussian function $s_j(\mathrm{Z}_j)$, $j = 1, \ldots, p$ and the node number $p > 1$, ε_1 is the approximation error satisfying $|\varepsilon_1| \le \bar{\varepsilon}_1$ with the constant $\bar{\varepsilon}_1 > 0$, $\hat{\xi}_1$ is the estimation of ξ_1^* and $\tilde{\xi}_1 = \hat{\xi}_1 - \xi_1^*$.

Based on (12) and (13), (11) can be further rewritten as

$$\dot{V}_1^z = \frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2}\left(\xi_1^{*T}S_1(\mathrm{Z}_1)+\varepsilon_1(\mathrm{Z}_1)\right) - \frac{z_1^2 k_{c_1}(t)\dot{k}_{c_1}(t)}{k_{c_1}^2(t)-x_1^2} + \frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2}(g_1(x_1)z_2 + g_1(x_1)\alpha_1). \tag{14}$$

Design the virtual controller α_1 as

$$\alpha_1 = \frac{1}{g_1(x_1)}\left(-(\kappa_1+\bar{\kappa}_1(t))z_1 - \hat{\xi}_1^T S_1(\mathrm{Z}_1) - \frac{1}{2}\frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2}\right) \tag{15}$$

where κ_1 stands for a positive design parameter, $\bar{\kappa}_1(t) = \sqrt{(\dot{k}_{c_1}(t)/k_{c_1}(t))^2+\beta_1}$ and $\beta_1 > 0$ is the design parameter.

According to the Young's inequality, we have

$$\frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2}\varepsilon_1(\mathrm{Z}_1) \le \frac{1}{2}\left(\frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2}\right)^2 + \frac{1}{2}\bar{\varepsilon}_1^2.$$

It is worth noting that $\bar{\kappa}_1(t) + \dot{k}_{c_1}(t)/k_{c_1}(t) \ge 0$ and it has

$$\dot{V}_1^z \le -\frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2}\tilde{\xi}_1^T S_1(\mathrm{Z}_1) - \frac{\kappa_1 z_1^2 k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2} + \frac{1}{2}\bar{\varepsilon}_1^2 + \frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2}g_1(x_1)z_2. \tag{16}$$

Design the following Lyapunov function

$$V_1 = V_1^{z} + V_1^{\xi}. \quad (17)$$

where $V_1^{\xi} = \frac{1}{2}\widetilde{\xi}_1^T \Gamma_1^{-1}\widetilde{\xi}_1$ and $\Gamma_1^T = \Gamma_1 > 0$ is the design parameter.

Then, the time derivative of V_1 yields

$$\dot{V}_1 \le \widetilde{\xi}_1^T \left(-\frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t) - x_1^2} S_1(Z_1) + \Gamma_1^{-1}\dot{\hat{\xi}}_1 \right) + \frac{1}{2}\bar{\varepsilon}_1^2 - \frac{\kappa_1 z_1^2 k_{c_1}^2(t)}{k_{c_1}^2(t) - x_1^2} + \frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t) - x_1^2} g_1(x_1) z_2. \quad (18)$$

Step i, $2 \le i \le n-1$**:** Define the error variable $z_i = x_i - \alpha_{i-1}$, and its time derivative is given as

$$\dot{z}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i) x_{i+1} - \dot{\alpha}_{i-1} \quad (19)$$

where α_{i-1} can be seen as a function of $\bar{x}_{i-1}$, $y_d, \ldots, y_d^{(i-1)}$, $\hat{\xi}_1, \ldots, \hat{\xi}_{i-1}$, $k_{c_1}, \ldots, k_{c_1}^{(i-1)}$, $k_{c_2}, \ldots, k_{c_2}^{(i-2)}, \ldots, k_{c_{i-1}}, \dot{k}_{c_{i-1}}$, and it has

$$\dot{\alpha}_{i-1} = \sum_{m=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_m} \dot{x}_m + \sum_{m=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\xi}_m} \dot{\hat{\xi}}_m + \sum_{m=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(m)}} y_d^{(m+1)} + \sum_{m=1}^{i-1} \sum_{l=0}^{i-m} \frac{\partial \alpha_{i-1}}{\partial k_{c_m}^{(l)}(t)} k_{c_m}^{(l+1)}(t). \quad (20)$$

It needs to note that α_1 is a function of x_1, y_d, $\dot{y}_d$ $\hat{\xi}_1$, k_{c_1} and $\dot{k}_{c_1}$.

Choose the Integral-type Lyapunov function candidate

$$V_i^{z} = \int_0^{z_i} \frac{\delta k_{c_i}^2(t)}{k_{c_i}^2(t) - (\delta + \alpha_{i-1})^2} d\delta \quad (21)$$

with $\delta = \upsilon z_i$ and the following inequality holds

$$\frac{1}{2} z_i^2 \le V_i^{z} \le z_i^2 \int_0^1 \frac{\upsilon k_{c_i}^2(t)}{k_{c_i}^2(t) - (\upsilon z_i + \operatorname{sgn}(z_i) A_{i-1})^2} d\upsilon. \quad (22)$$

The virtual controllers $\alpha_1, \ldots, \alpha_{n-1}$ are continuously differentiable functions and satisfy $|\alpha_{i-1}| \le A_{i-1} < k_{c_i}(t)$ where A_{i-1}, $i = 2, \ldots, n$ are positive constants (Tee & Ge, 2012).

Similar to (6) and (7) in Step 1, the time derivative of V_i^{z} can be given as

$$\dot{V}_i^{z} = \frac{z_i k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2} \dot{z}_i + \dot{\alpha}_{i-1} \int_0^{z_i} \frac{\partial}{\partial \alpha_{i-1}} \frac{\delta k_{c_i}^2(t)}{k_{c_i}^2(t) - (\delta + \alpha_{i-1})^2} d\delta + \dot{k}_{c_i}(t) \int_0^{z_i} \frac{\partial}{\partial k_{c_i}(t)} \frac{\delta k_{c_i}^2(t)}{k_{c_i}^2(t) - (\delta + \alpha_{i-1})^2} d\delta$$

where

$$\int_0^{z_i} \frac{\partial}{\partial \alpha_{i-1}} \frac{\delta k_{c_i}^2(t)}{k_{c_i}^2(t) - (\delta + \alpha_{i-1})^2} d\delta = z_i \left(\frac{k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2} - \Psi_i(z_i, \alpha_{i-1}, k_{c_i}) \right) \quad (23)$$

with

$$\Psi_i(z_i, \alpha_{i-1}, k_{c_i}) = \int_0^1 \frac{k_{c_i}^2(t)}{k_{c_i}^2(t) - (\upsilon z_i + \alpha_{i-1})^2} d\upsilon = \frac{k_{c_i}(t)}{z_i} \left(\tanh^{-1}\left(\frac{z_i + \alpha_{i-1}}{k_{c_i}(t)} \right) - \tanh^{-1}\left(\frac{\alpha_{i-1}}{k_{c_i}(t)} \right) \right) = \frac{k_{c_i}(t)}{2 z_i} \ln \frac{(k_{c_i}(t) + x_i)(k_{c_i}(t) - \alpha_{i-1})}{(k_{c_i}(t) - x_i)(k_{c_i}(t) + \alpha_{i-1})}$$

and

$$\int_0^{z_i} \frac{\partial}{\partial k_{c_i}(t)} \frac{\delta k_{c_i}^2(t)}{k_{c_i}^2(t) - (\delta + \alpha_{i-1})^2} d\delta = \int_0^{z_i} -\delta(\delta + \alpha_{i-1})\, d\frac{k_{c_i}(t)}{k_{c_i}^2(t) - (\delta + \alpha_{i-1})^2} = z_i \left(\frac{-(z_i + \alpha_{i-1}) k_{c_i}(t)}{k_{c_i}^2(t) - (z_i + \alpha_{i-1})^2} + \int_0^1 \frac{(2\upsilon z_i + \alpha_{i-1}) k_{c_i}(t)}{k_{c_i}^2(t) - (\upsilon z_i + \alpha_{i-1})^2} d\upsilon \right) = z_i \left(\frac{-z_i k_{c_i}(t)}{k_{c_i}^2(t) - (z_i + \alpha_{i-1})^2} + \mathrm{I}_i(z_i, \alpha_{i-1}, k_{c_i}) \right) \quad (24)$$

where

$$\mathrm{I}_i(z_i, \alpha_{i-1}, k_{c_i}) = \frac{-\alpha_{i-1} k_{c_i}(t)}{k_{c_i}^2(t) - (z_i + \alpha_{i-1})^2} + \int_0^1 \frac{(2\upsilon z_i + \alpha_{i-1}) k_{c_i}(t)}{k_{c_i}^2(t) - (\upsilon z_i + \alpha_{i-1})^2} d\upsilon = \frac{-\alpha_{i-1} k_{c_i}(t)}{k_{c_i}^2(t) - x_i^2} + \frac{k_{c_i}(t)}{z_i} \ln\left(\frac{k_{c_i}^2(t) - x_i^2}{k_{c_i}^2(t) - \alpha_{i-1}^2} \right) + \frac{\alpha_{i-1}}{2 z_i} \ln\left(\frac{k_{c_i}^2(t) - \alpha_{i-1}^2}{k_{c_i}^2(t) - x_i^2} \right).$$

Similar to Remark 2, $\Psi_i(z_i, \alpha_{i-1}, k_{c_i})$ and $\mathrm{I}_i(z_i, \alpha_{i-1}, k_{c_i})$ are well defined to be bounded in a neighborhood of $z_i = 0$.

From (23) and (24), the following equation can be obtained

$$\dot{V}_i^{z} = \frac{z_i k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2} [f_i(\bar{x}_i) + g_i(\bar{x}_i) z_i + g_i(\bar{x}_i) \alpha_{i-1}] - z_i \Psi_i(z_i, \alpha_{i-1}, k_{c_i}) \dot{\alpha}_{i-1} + z_i \mathrm{I}_i(z_i, \alpha_{i-1}, k_{c_i}) \dot{k}_{c_i}(t) - \frac{z_i^2 k_{c_i}(t) \dot{k}_{c_i}(t)}{k_{c_i}^2(t) - x_i^2}. \quad (25)$$

Define the unknown nonlinear function as

$$\mathrm{P}_i(Z_i) = f_i(\bar{x}_i) - \frac{k_{c_i}^2(t) - x_i^2}{k_{c_i}^2(t)} \Psi_i(z_i, \alpha_{i-1}, k_{c_i}) \dot{\alpha}_{i-1} + \frac{k_{c_i}^2(t) - x_i^2}{k_{c_i}^2(t)} \mathrm{I}_i(z_i, \alpha_{i-1}, k_{c_i}) \dot{k}_{c_i}(t) \quad (26)$$

where $Z_i = \left[\bar{x}_i, y_d, \ldots, y_d^{(i)}, k_{c_1}, \ldots, k_{c_1}^{(i)}, k_{c_2}, \ldots, k_{c_2}^{(i-1)}, \ldots, k_{c_i}, \dot{k}_{c_i}, \hat{\xi}_1, \ldots, \hat{\xi}_{i-1}\right]^T$. Based on the neural approximation, $\mathrm{P}_i(Z_i)$ can be described by

$$\mathrm{P}_i(Z_i) = \xi_i^{*T} S_i(Z_i) + \varepsilon_i(Z_i) \quad (27)$$

where $|\varepsilon_i| \le \bar{\varepsilon}_i$ with the constant $\bar{\varepsilon}_i > 0$. (25) becomes

$$\dot{V}_i^{z} = \frac{z_i k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2} \left(\xi_i^{*T} S_i(Z_i) + \varepsilon_i(Z_i) \right) - \frac{z_i^2 k_{c_i}(t) \dot{k}_{c_i}(t)}{k_{c_i}^2(t) - x_i^2} + \frac{z_i k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2} \left(g_i(\bar{x}_i) z_{i+1} + g_i(\bar{x}_i) \alpha_i \right). \quad (28)$$

The virtual controller α_i is designed as follows

$$\alpha_i = \frac{1}{g_i(\bar{x}_i)} \left[-(\kappa_i + \bar{\kappa}_i(t)) z_i - \frac{1}{2} \frac{z_i k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2} - \hat{\xi}_i^T S_i(Z_i) - \frac{k_{c_{i-1}}^2(t) \left(k_{c_i}^2(t) - x_i^2 \right)}{k_{c_i}^2(t) \left(k_{c_{i-1}}^2(t) - x_{i-1}^2 \right)} g_{i-1}(\bar{x}_{i-1}) z_{i-1} \right] \quad (29)$$

where κ_i represents a positive design parameter, $\overline{\kappa}_i(t) = \sqrt{\left(\dot{k}_{c_i}(t)/k_{c_i}(t)\right)^2 + \beta_i}$ with $\beta_i > 0$, $\hat{\xi}_i$ is the estimation of ξ_i^* and define $\widetilde{\xi}_i = \hat{\xi}_i - \xi_i^*$.

Using the Young's inequality, it is easy to get

$$\frac{z_i k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2}\varepsilon_i(Z_i) \le \frac{1}{2}\left(\frac{z_i k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2}\right)^2 + \frac{1}{2}\overline{\varepsilon}_i^2. \tag{30}$$

Based on (29), (30) and the fact $\overline{\kappa}_i(t) + \dot{k}_{c_i}(t)/k_{c_i}(t) \ge 0$, (28) can be rewritten as

$$\begin{aligned}\dot{V}_i^z \le &-\frac{z_i k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2}\widetilde{\xi}_i^T S_i(Z_i) + \frac{1}{2}\overline{\varepsilon}_i^2 + \frac{z_i k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2}g_i(\overline{x}_i) z_{i+1} \\ &- \frac{\kappa_i z_i^2 k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2} - \frac{z_i k_{c_{i-1}}^2(t)}{k_{c_{i-1}}^2(t) - x_{i-1}^2}g_{i-1}(\overline{x}_{i-1}) z_{i-1}. \end{aligned}\tag{31}$$

Define the following function as

$$V_i = V_{i-1} + V_i^z + V_i^\xi. \tag{32}$$

where $V_i^\xi = \frac{1}{2}\widetilde{\xi}_i^T \Gamma_i^{-1}\widetilde{\xi}_i$ and $\Gamma_i^T = \Gamma_i > 0$ is the design parameter.

From the step $i-1$, it can be obtained that

$$\begin{aligned}\dot{V}_{i-1} \le &\sum_{m=1}^{i-1}\widetilde{\xi}_m^T\left(-\frac{z_m k_{c_m}^2(t)}{k_{c_m}^2(t) - x_m^2}S_m(Z_m) + \Gamma_m^{-1}\dot{\hat{\xi}}_m\right) + \frac{1}{2}\sum_{m=1}^{i-1}\overline{\varepsilon}_m^2 \\ &- \sum_{m=1}^{i-1}\frac{\kappa_m z_m^2 k_{c_m}^2(t)}{k_{c_m}^2(t) - x_m^2} + \frac{z_{i-1}k_{c_{i-1}}^2(t)}{k_{c_{i-1}}^2(t) - x_{i-1}^2}g_{i-1}(\overline{x}_{i-1}) z_i. \end{aligned}\tag{33}$$

Therefore, the time derivative of V_i can be expressed as

$$\begin{aligned}\dot{V}_i \le &\sum_{m=1}^{i}\widetilde{\xi}_m^T\left(-\frac{z_m k_{c_m}^2(t)}{k_{c_m}^2(t) - x_m^2}S_m(Z_m) + \Gamma_m^{-1}\dot{\hat{\xi}}_m\right) + \frac{1}{2}\sum_{m=1}^{i}\overline{\varepsilon}_m^2 \\ &- \sum_{m=1}^{i}\frac{\kappa_m z_m^2 k_{c_m}^2(t)}{k_{c_m}^2(t) - x_m^2} + \frac{z_i k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2}g_i(\overline{x}_i) z_{i+1}. \end{aligned}\tag{34}$$

Step n: The time derivative of error $z_n = x_n - \alpha_{n-1}$ is

$$\dot{z}_n = f_n(\overline{x}_n) + g_n(\overline{x}_n) u - \dot{\alpha}_{n-1}. \tag{35}$$

Based on Step i with $i = n-1$, $\dot{\alpha}_{n-1}$ is given by

$$\begin{aligned}\dot{\alpha}_{n-1} = &\sum_{m=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_m}\dot{x}_m + \sum_{m=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\xi}_m}\dot{\hat{\xi}}_m \\ &+ \sum_{m=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(m)}}y_d^{(m+1)} + \sum_{m=1}^{n-1}\sum_{l=0}^{n-m}\frac{\partial\alpha_{n-1}}{\partial k_{c_m}^{(l)}(t)}k_{c_m}^{(l+1)}(t). \end{aligned}\tag{36}$$

Consider the Integral-type Lyapunov function candidate

$$V_n^z = \int_0^{z_n}\frac{\delta k_{c_n}^2(t)}{k_{c_n}^2(t) - (\delta + \alpha_{n-1})^2}d\delta \tag{37}$$

where $\delta = \upsilon z_n$ and the following inequality holds

$$\frac{1}{2}z_n^2 \le V_n^z \le z_n^2\int_0^1\frac{\upsilon k_{c_n}^2(t)}{k_{c_n}^2(t) - (\upsilon z_n + \operatorname{sgn}(z_n) A_{n-1})^2}d\upsilon. \tag{38}$$

Similar to Step 1, the time derivative of V_n^z is

$$\begin{aligned}\dot{V}_n^z = &\frac{z_n k_{c_n}^2(t)}{k_{c_n}^2(t) - x_n^2}\dot{z}_n + \dot{\alpha}_{n-1}\int_0^{z_n}\frac{\partial}{\partial\alpha_{n-1}}\frac{\delta k_{c_n}^2(t)}{k_{c_n}^2(t) - (\delta + \alpha_{n-1})^2}d\delta \\ &+ \dot{k}_{c_n}(t)\int_0^{z_n}\frac{\partial}{\partial k_{c_n}(t)}\frac{\delta k_{c_n}^2(t)}{k_{c_n}^2(t) - (\delta + \alpha_{n-1})^2}d\delta \end{aligned}\tag{39}$$

where

$$\begin{aligned}&\int_0^{z_n}\frac{\partial}{\partial\alpha_{n-1}}\frac{\delta k_{c_n}^2(t)}{k_{c_n}^2(t) - (\delta + \alpha_{n-1})^2}d\delta \\ &= z_n\left(\frac{k_{c_n}^2(t)}{k_{c_n}^2(t) - x_n^2} - \Psi_n(z_n, \alpha_{n-1}, k_{c_n})\right) \end{aligned}\tag{40}$$

with

$$\begin{aligned}\Psi_n(z_n, \alpha_{n-1}, k_{c_n}) &= \int_0^1\frac{k_{c_n}^2(t)}{k_{c_n}^2(t) - (\upsilon z_n + \alpha_{n-1})^2}d\upsilon \\ &= \frac{k_{c_n}(t)}{z_n}\left(\tanh^{-1}\left(\frac{z_n + \alpha_{n-1}}{k_{c_n}(t)}\right) - \tanh^{-1}\left(\frac{\alpha_{n-1}}{k_{c_n}(t)}\right)\right) \\ &= \frac{k_{c_n}(t)}{2z_n}\ln\frac{(k_{c_n}(t) + x_n)(k_{c_n}(t) - \alpha_{n-1})}{(k_{c_n}(t) - x_n)(k_{c_n}(t) + \alpha_{n-1})}\end{aligned}$$

while

$$\begin{aligned}&\int_0^{z_n}\frac{\partial}{\partial k_{c_n}(t)}\frac{\delta k_{c_n}^2(t)}{k_{c_n}^2(t) - (\delta + \alpha_{n-1})^2}d\delta \\ &= \int_0^{z_n} -\delta(\delta + \alpha_{n-1})\, d\frac{k_{c_n}(t)}{k_{c_n}^2(t) - (\delta + \alpha_{n-1})^2} \\ &= z_n\left(\frac{-(z_n + \alpha_{n-1})k_{c_n}(t)}{k_{c_n}^2(t) - (z_n + \alpha_{n-1})^2} + \int_0^1\frac{(2\upsilon z_n + \alpha_{n-1})k_{c_n}(t)}{k_{c_n}^2(t) - (\upsilon z_n + \alpha_{n-1})^2}d\upsilon\right) \\ &= z_n\left(\frac{-z_n k_{c_n}(t)}{k_{c_n}^2(t) - (z_n + \alpha_{n-1})^2} + \mathrm{I}_n(z_n, \alpha_{n-1}, k_{c_n})\right)\end{aligned}\tag{41}$$

with

$$\begin{aligned}\mathrm{I}_n(z_n, \alpha_{n-1}, k_{c_n}) = &\frac{-\alpha_{n-1}k_{c_n}(t)}{k_{c_n}^2(t) - (z_n + \alpha_{n-1})^2} \\ &+ \int_0^1\frac{(2\upsilon z_n + \alpha_{n-1})k_{c_n}(t)}{k_{c_n}^2(t) - (\upsilon z_n + \alpha_{n-1})^2}d\upsilon \\ = &\frac{-\alpha_{n-1}k_{c_n}(t)}{k_{c_n}^2(t) - x_n^2} + \frac{k_{c_n}(t)}{z_n}\ln\left(\frac{k_{c_n}^2(t) - x_n^2}{k_{c_n}^2(t) - \alpha_{n-1}^2}\right) \\ &+ \frac{\alpha_{n-1}}{2z_n}\ln\left(\frac{k_{c_n}^2(t) - \alpha_{n-1}^2}{k_{c_n}^2(t) - x_n^2}\right).\end{aligned}$$

Same as Remark 2, $\Psi_n(z_n, \alpha_{n-1}, k_{c_n})$ and $\mathrm{I}_n(z_n, \alpha_{n-1}, k_{c_n})$ are well defined to be bounded in a neighborhood of $z_n = 0$.

Substituting (40) and (41) into (39) leads to

$$\begin{aligned}\dot{V}_n^z = &\frac{z_n k_{c_n}^2(t)}{k_{c_n}^2(t) - x_n^2}(f_n(\overline{x}_n) + g_n(\overline{x}_n) u) - \frac{z_n^2 k_{c_n}(t)\dot{k}_{c_n}(t)}{k_{c_n}^2(t) - x_n^2} \\ &- z_n\Psi_n(z_n, \alpha_{n-1}, k_{c_n})\dot{\alpha}_{n-1} + z_n\mathrm{I}_n(z_n, \alpha_{n-1}, k_{c_n})\dot{k}_{c_n}(t)\end{aligned}\tag{42}$$

Define the unknown nonlinear function as

$$\begin{aligned}\mathrm{P}_n(Z_n) = &f_n(\overline{x}_n) - \frac{k_{c_n}^2(t) - x_n^2}{k_{c_n}^2(t)}\Psi_n(z_n, \alpha_{n-1}, k_{c_n})\dot{\alpha}_{n-1} \\ &+ \frac{k_{c_n}^2(t) - x_n^2}{k_{c_n}^2(t)}\mathrm{I}_n(z_n, \alpha_{n-1}, k_{c_n})\dot{k}_{c_n}(t)\end{aligned}\tag{43}$$

where $Z_n = \left[\overline{x}_n, y_d, \ldots, y_d^{(n)}, k_{c_1}, \ldots, k_{c_1}^{(n)}, k_{c_2}, \ldots, k_{c_2}^{(n-1)}, \ldots, k_{c_n}, \dot{k}_{c_n}, \hat{\xi}_1, \ldots, \hat{\xi}_n\right]^T$. Based on the neural approximation, $\mathrm{P}_n(Z_n)$ can be described by

$$\mathrm{P}_n(Z_n) = \xi_n^{*T}S_n(Z_n) + \varepsilon_n(Z_n) \tag{44}$$

where $|\varepsilon_n| \le \overline{\varepsilon}_n$ with the constant $\overline{\varepsilon}_n > 0$.

Further, (42) becomes

$$\dot{V}_n^z = \frac{z_n k_{c_n}^2(t)}{k_{c_n}^2(t) - x_n^2}\left(\xi_n^{*T} S_n(Z_n) + \varepsilon_n(Z_n)\right) - \frac{z_n^2 k_{c_n}(t)\dot{k}_{c_n}(t)}{k_{c_n}^2(t) - x_n^2} + \frac{z_n k_{c_n}^2(t)}{k_{c_n}^2(t) - x_n^2} g_n(\bar{x}_n) u. \tag{45}$$

The controller u as following

$$u = \frac{1}{g_n(\bar{x}_n)}\left[-(\kappa_n + \bar{\kappa}_n(t)) z_n - \frac{1}{2}\frac{z_n k_{c_n}^2(t)}{k_{c_n}^2(t) - x_n^2} - \hat{\xi}_n^T S_n(Z_n) - \frac{k_{c_{n-1}}^2(t)\left(k_{c_n}^2(t) - x_n^2\right)}{k_{c_n}^2(t)\left(k_{c_{n-1}}^2(t) - x_{n-1}^2\right)} g_{n-1}(\bar{x}_{n-1}) z_{n-1}\right] \tag{46}$$

where κ_n represents a positive design parameter, $\bar{\kappa}_n(t) = \sqrt{\left(\dot{k}_{c_n}(t)/k_{c_n}(t)\right)^2 + \beta_n}$ with $\beta_n > 0$ and $\tilde{\xi}_n = \hat{\xi}_n - \xi_n^*$.

According to the Young's inequality, we have

$$\frac{z_n k_{c_n}^2(t)}{k_{c_n}^2(t) - x_n^2}\varepsilon_n(Z_n) \le \frac{1}{2}\left(\frac{z_n k_{c_n}^2(t)}{k_{c_n}^2(t) - x_n^2}\right)^2 + \frac{1}{2}\bar{\varepsilon}_n^2. \tag{47}$$

Noting that $\bar{\kappa}_n(t) + \dot{k}_{c_n}(t)/k_{c_n}(t) \ge 0$, (45) becomes

$$\dot{V}_n^z \le -\frac{z_n k_{c_n}^2(t)}{k_{c_n}^2(t) - x_n^2}\tilde{\xi}_n^T S_n(Z_n) - \frac{\kappa_n z_n^2 k_{c_n}^2(t)}{k_{c_n}^2(t) - x_n^2} + \frac{1}{2}\bar{\varepsilon}_n^2 - \frac{z_n k_{c_{n-1}}^2(t)}{k_{c_{n-1}}^2(t) - x_{n-1}^2} g_{n-1}(\bar{x}_{n-1}) z_{n-1}. \tag{48}$$

Construct the following function

$$V_n^{\xi} = \frac{1}{2}\tilde{\xi}_n^T \Gamma_n^{-1}\tilde{\xi}_n \tag{49}$$

where $\Gamma_n^T = \Gamma_n > 0$ is the design parameter.

Based on (17) and (32), it has

$$V_n = V_{n-1} + V_n^z + V_n^{\xi}. \tag{50}$$

The time derivative of V_{n-1} along with step $n-1$ is

$$\dot{V}_{n-1} \le \sum_{m=1}^{n-1}\tilde{\xi}_m^T\left(-\frac{z_m k_{c_m}^2(t)}{k_{c_m}^2(t) - x_m^2} S_m(Z_m) + \Gamma_m^{-1}\dot{\hat{\xi}}_m\right) + \frac{1}{2}\sum_{m=1}^{n-1}\bar{\varepsilon}_m^2 - \sum_{m=1}^{n-1}\frac{\kappa_m z_m^2 k_{c_m}^2(t)}{k_{c_m}^2(t) - x_m^2} + \frac{z_{n-1} k_{c_{n-1}}^2(t)}{k_{c_{n-1}}^2(t) - x_{n-1}^2} g_{n-1}(\bar{x}_{n-1}) z_n. \tag{51}$$

Then, the time derivative of V_n becomes

$$\dot{V}_n \le \sum_{m=1}^{n}\tilde{\xi}_m^T\left(-\frac{z_m k_{c_m}^2(t)}{k_{c_m}^2(t) - x_m^2} S_m(Z_m) + \Gamma_m^{-1}\dot{\hat{\xi}}_m\right) - \sum_{m=1}^{n}\frac{\kappa_m z_m^2 k_{c_m}^2(t)}{k_{c_m}^2(t) - x_m^2} + \frac{1}{2}\sum_{m=1}^{n}\bar{\varepsilon}_m^2. \tag{52}$$

Adaptive laws are established as

$$\dot{\hat{\xi}}_m = \Gamma_m\left(\frac{z_m k_{c_m}^2(t)}{k_{c_m}^2(t) - x_m^2} S_m(Z_m) - \tau_m\hat{\xi}_m\right), m = 1, \ldots, n \tag{53}$$

where τ_m is a positive constant.

By applying the Young's inequality, it has

$$\tilde{\xi}_m^T\left(\frac{-z_m k_{c_m}^2(t)}{k_{c_m}^2(t) - x_m^2} S_m(Z_m) + \Gamma_m^{-1}\dot{\hat{\xi}}_m\right) = -\tau_m\tilde{\xi}_m^T\hat{\xi}_m \le \frac{-\tau_m\left\|\tilde{\xi}_m\right\|^2}{2} + \frac{\tau_m\left\|\xi_m^*\right\|^2}{2}. \tag{54}$$

Based on the above steps, it has the following inequality

$$\dot{V}_n^z \le -\frac{1}{2}\sum_{m=1}^{n}\tau_m\left\|\tilde{\xi}_m\right\|^2 + \frac{1}{2}\sum_{m=1}^{n}\tau_m\left\|\xi_m^*\right\|^2 - \sum_{m=1}^{n}\frac{\kappa_m z_m^2 k_{c_m}^2(t)}{k_{c_m}^2(t) - x_m^2} + \frac{1}{2}\sum_{m=1}^{n}\bar{\varepsilon}_m^2. \tag{55}$$

Lemma 1 (Tee & Ge, 2012). *For $|x_i| < k_{c_i}(t), i = 1, \ldots, n, \forall t \ge 0$, V_i^z satisfies the inequality $V_i^z \le \frac{z_i^2 k_{c_i}^2(t)}{k_{c_i}^2(t) - x_i^2}$.*

Theorem 1. *Consider the nonlinear system (1) under Assumptions 1–3, with the actual controller (46) and the adaptive law (53), and if the initial conditions satisfy $|x_i(0)| < k_{c_i}(0)$, then the following properties can be guaranteed that*

(1) The error signals $z_i(t), i = 1, \ldots, n$ remain in the compact sets;
(2) The states are never violated their constraint bounds, i.e., $|x_i| < k_{c_i}(t)$, $\forall t > 0$;
(3) All the closed-loop signals are bounded.

Proof. According to $V_i = V_{i-1} + V_i^z + V_i^{\xi}, i = 1, \ldots, n$, one has

$$V_n = \sum_{m=1}^{n}\int_0^{z_m}\frac{\delta k_{c_m}^2(t)}{k_{c_m}^2(t) - (\delta + \alpha_{m-1})^2}d\delta + \frac{1}{2}\sum_{m=1}^{n}\tilde{\xi}_m^T\Gamma_m^{-1}\tilde{\xi}_m$$

where $V_0 = 0$ and $\alpha_0 = y_d$. With Lemma 1, we have

$$V_n \le \sum_{m=1}^{n}\frac{k_{c_m}^2(t) z_m^2}{k_{c_m}^2(t) - x_m^2} + \frac{1}{2}\sum_{m=1}^{n}\tilde{\xi}_m^T\Gamma_m^{-1}\tilde{\xi}_m.$$

Define $\mathrm{A}_n = \min\{\kappa_m, \tau_m\lambda_{\min}(\Gamma_m), m = 1, \ldots, n\}$ and $\mathrm{B}_n = \frac{1}{2}\sum_{m=1}^{n}\tau_m\left\|\xi_m^*\right\|^2 + \frac{1}{2}\sum_{m=1}^{n}\bar{\varepsilon}_m^2$. Therefore, the following inequality holds

$$\dot{V}_n \le -\mathrm{A}_n V_n + \mathrm{B}_n. \tag{56}$$

Multiplying both sides of (56) by $e^{\mathrm{A}_n t}$, one has $d\left(e^{\mathrm{A}_n t}V_n\right)/dt \le e^{\mathrm{A}_n t}\mathrm{B}_n$. Then, integrating it over $[0, t]$, one gets

$$e^{\mathrm{A}_n t}V_n\Big|_0^t \le \frac{\mathrm{B}_n}{\mathrm{A}_n}e^{\mathrm{A}_n t}\Big|_0^t$$

Further, it yields

$$V_n(t) \le e^{-\mathrm{A}_n t}\left(V_n(0) - \mathrm{B}_n/\mathrm{A}_n\right) + \mathrm{B}_n/\mathrm{A}_n \le V_n(0)e^{-\mathrm{A}_n t} + \mathrm{B}_n/\mathrm{A}_n. \tag{57}$$

Then, according to (4), (22) and (38), we can obtain $z_m^2 \le 2V_n(t) \le 2e^{-\mathrm{A}_n t}\left(V_n(0) - \mathrm{B}_n/\mathrm{A}_n\right) + 2\mathrm{B}_n/\mathrm{A}_n \le 2\mathrm{B}_n/\mathrm{A}_n + 2V_n(0)e^{-\mathrm{A}_n t}$ and $\left\|\tilde{\xi}_m\right\|^2 \le 2\lambda_{\max}(\Gamma_m)\left(\mathrm{B}_n/\mathrm{A}_n + V_n(0)e^{-\mathrm{A}_n t}\right)$. Further, it has

$$\left\|\tilde{\xi}_m\right\| \le \sqrt{2\lambda_{\max}(\Gamma_m)\left(V_n(0)e^{-\mathrm{A}_n t} + \mathrm{B}_n/\mathrm{A}_n\right)}$$

$$|z_m| \le \sqrt{2V_n(0)e^{-\mathrm{A}_n t} + 2\mathrm{B}_n/\mathrm{A}_n}$$

Therefore, z_m and $\tilde{\xi}_m$ are bounded, and $\hat{\xi}_m$ is also bounded. At the same time, we have $|z_1| \le \Theta_1$ where $\Theta_1 = \sqrt{2e^{-\mathrm{A}_n t}\left(V_n(0) - \mathrm{B}_n/\mathrm{A}_n\right) + 2\mathrm{B}_n/\mathrm{A}_n}$.

If $V_n(0) = B_n/A_n$, it has $|z_1| \leq \Theta_1 = \sqrt{2B_n/A_n}$. If $V_n(0) \neq B_n/A_n$, it can follow that given any $\Theta_1 > \sqrt{2B_n/A_n}$, there exists T such that for any $t > T$, it has $|z_1| \leq \Theta_1$. Thus, z_1 can be arbitrarily small by choosing the appropriate design parameters.

Using the contradiction to prove that the full state constraints are not violated. First, assume that there exist some $t = T$ and $m \in \{1, \dots, n\}$ such that $|x_m(T)| = k_{c_m}(T)$. Then, according to (57), we know that $V_n|_{t=T}$ is bounded, which implies that $\sum_{m=1}^{n} \int_0^{z_m(T)} \frac{\delta k_{c_m}^2(T)}{k_{c_m}^2(T)-(\delta+\alpha_{m-1})^2} d\delta$ and are $\int_0^{z_m(T)} \frac{\delta k_{c_m}^2(T)}{k_{c_m}^2(T)-(\delta+\alpha_{m-1})^2} d\delta$, $m = 1, \dots, n$ also bounded.

In addition, integrating $\int_0^{z_m(T)} \frac{\delta k_{c_m}^2(T)}{k_{c_m}^2(T)-(\delta+\alpha_{m-1})^2} d\delta$ by parts, we obtain

$$\begin{aligned}
&\int_0^{z_m(T)} \frac{\delta k_{c_m}^2(T)}{k_{c_m}^2(T)-(\delta+\alpha_{m-1})^2} d\delta \\
&= k_{c_m}(T)\left(\delta \tanh^{-1}\frac{\delta+\alpha_{m-1}}{k_{c_m}(T)}\right)\Bigg|_0^{z_m} \\
&\quad - k_{c_m}(T)\int_0^{z_m} \tanh^{-1}\frac{\delta+\alpha_{m-1}}{k_{c_m}(T)} d\delta \\
&= k_{c_m}(T)\,\alpha_{m-1}(T) \ln\frac{(1+\alpha_{m-1}(T))(1-x_m(T))}{(1-\alpha_{m-1}(T))(1+x_m(T))} \\
&\quad + \frac{k_{c_m}^2(T)}{2}\ln\frac{\left(k_{c_m}^2(T)-\alpha_{m-1}^2(T)\right)}{\left(k_{c_m}^2(T)-x_m^2(T)\right)}
\end{aligned}$$

when $|x_m(T)| = k_{c_m}(T)$, $\int_0^{z_m(T)} \frac{\delta k_{c_m}^2(T)}{k_{c_m}^2(T)-(\delta+\alpha_{m-1})^2} d\delta$, $m = 1, \dots, n$ become unbounded and contradict with their boundedness. So $|x_m(T)| \neq k_{c_m}(T)$ and for the given initial state $x_m(0) \in \{x_m \in R \mid |x_m| < k_{c_m}(T)\}$, it is clear that $|x_m(T)| < k_{c_m}(T)$.

By Assumptions 1 and 3, $|y_d(t)| \leq Y_0(t) \leq K_{c_1}^0$, $|\dot{y}_d(t)| < Y_1$ and $\dot{k}_{c_1}(t) \leq K_{c_1}^1, \forall t \geq 0$ can be obtained, so the virtual controller $\alpha_1\left(x_1, y_d, \dot{y}_d, k_{c_1}, \dot{k}_{c_1}, \hat{\xi}_1\right)$ must be bounded. Similarly, the boundedness of the virtual controllers $\alpha_{m-1}, m = 3, \dots, n$ and actual controller u can be proved. In summary, all the closed-loop signals z_m, $\hat{\xi}_m$, x_m and u are bounded. This completes the proof of Theorem 1.

4. Simulation example

In order to verify the effectiveness of the proposed scheme, the nonlinear systems are considered as follows

$$\begin{cases}
\dot{x}_1 = 0.5\sin x_1^2 + x_2 \\
\dot{x}_2 = 0.1(x_1+x_2)^2 + (0.2+\cos(2.4x_1x_2))\,u \\
y = x_1
\end{cases}$$

where $z_1 = x_1 - y_d$ and $z_2 = x_2 - \alpha_1$.

The actual controller is given as follows

$$\begin{aligned}
u = \frac{1}{g_2(\bar{x}_2)}\Bigg(&-(\kappa_2+\bar{\kappa}_2(t))z_2 - \hat{\xi}_2^T S_2(Z_2) \\
&-\frac{1}{2}\frac{z_2 k_{c_2}^2(t)}{k_{c_2}^2(t)-x_2^2} - \frac{k_{c_1}^2(t)\left(k_{c_2}^2(t)-x_2^2\right)}{k_{c_2}^2(t)\left(k_{c_1}^2(t)-x_1^2\right)} g_1(x_1)\,z_1\Bigg)
\end{aligned}$$

where $Z_2 = \left[x_1, x_2, y_d, \dot{y}_d, \ddot{y}_d, k_{c_1}, \dot{k}_{c_1}, \ddot{k}_{c_1}, k_{c_2}, \dot{k}_{c_2}, \hat{\xi}_1\right]^T$.

The desired trajectory is described by $y_d(t) = 0.5\sin(2t)$ and the time-varying state constraints are defined as $|x_1| \leq k_{c_1}(t) = 0.63 + 0.1\sin(0.9t)$ and $|x_2| \leq k_{c_2}(t) = 0.1\sin(1.5t) + 1.2$. Since $y = x_1$, for a tracking control, it always expect that x_1 tracks the desired trajectory as soon as possible. This will be verified by Fig. 1, which shows that $|x_1| \leq 0.5$. Thus, one gets $|2.4x_1x_2| \leq 2.4 \times 0.5 \times 1.3 = 1.56 < \pi/2$. It deduces that $\cos 2.4x_1x_2 > 0$. This means that the control gain $0.2 + \cos 2.4x_1x_2$ is positive, satisfying Assumption 2.

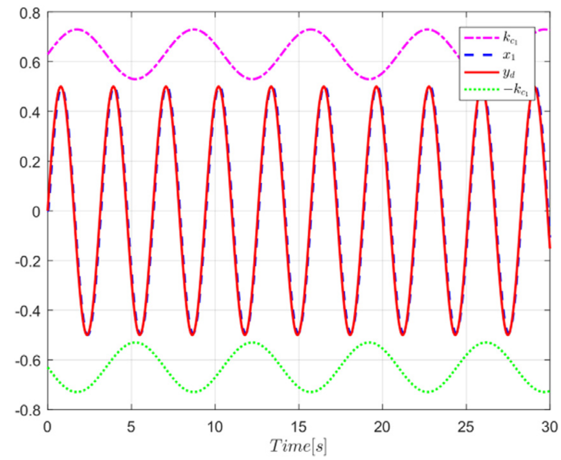

Fig. 1. The trajectories of $x_1 = y$ and y_d.

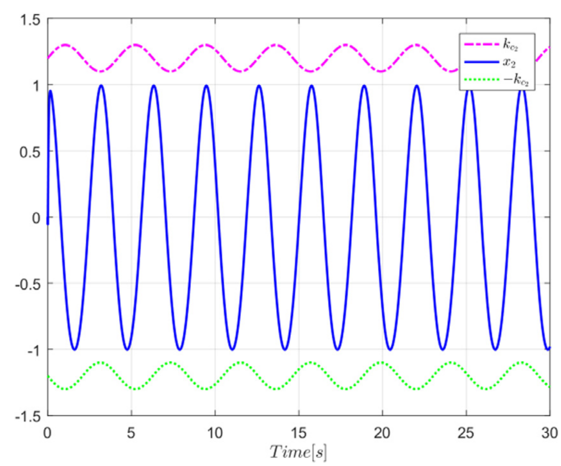

Fig. 2. The trajectory of x_2.

The virtual controller is given as follows

$$\alpha_1 = \frac{1}{g_1(x_1)}\left(-(\kappa_1+\bar{\kappa}_1(t))z_1 - \hat{\xi}_1^T S_1(Z_1) - \frac{1}{2}\frac{z_1 k_{c_1}^2(t)}{k_{c_1}^2(t)-x_1^2}\right)$$

where $Z_1 = \left[x_1, y_d, \dot{y}_d, k_{c_1}, \dot{k}_{c_1}\right]^T$. The adaptive laws are designed as

$$\dot{\hat{\xi}}_m = \Gamma_m\left(\frac{z_m k_{c_m}^2(t)}{k_{c_m}^2(t)-x_m^2} S_m(Z_m) - \tau_m \hat{\xi}_m\right), m = 1, 2.$$

In this paper, the design parameters are chosen as $\kappa_1 = 25$, $\kappa_2 = 20$, $\beta_1 = 0.1$, $\beta_2 = 0.2$, $\Gamma_1 = diag([0.4, \dots, 0.4])_{20\times 20}$, $\Gamma_2 = diag([0.5, \dots, 0.5])_{20\times 20}$, $\tau_1 = 0.8$ and $\tau_2 = 0.3$.

The simulation results are shown in Figs. 1–4. Fig. 1 shows the trajectories of the output and tracking signal. It can be seen from Fig. 1 that the proposed scheme has a good tracking performance under the time-varying state constraints. Figs. 1 and 2 illustrate that the time-varying constraint bounds of the state variables are not violated. Fig. 3 illustrates the error trajectory. The adaptation laws and the actual controller are diagrammed in Fig. 4. At the same time, the boundedness of all the closed-loop signals can be shown in Figs. 1–4.

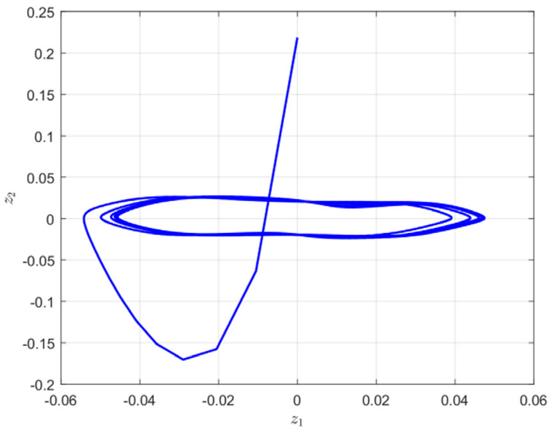

Fig. 3. The phase portrait of z_1 and z_2.

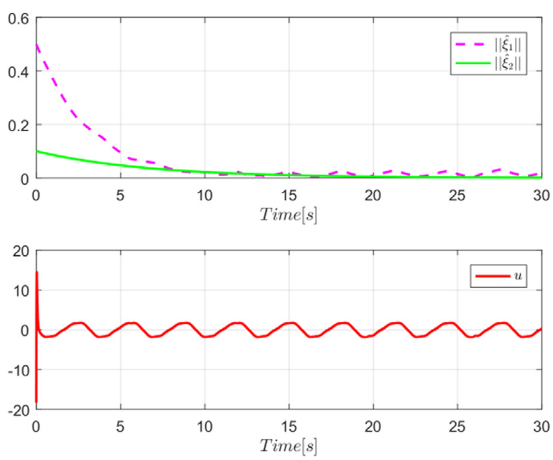

Fig. 4. The trajectories of $\left\|\hat{\xi}_1\right\|$, $\left\|\hat{\xi}_2\right\|$ and u.

5. Conclusion

In this paper, an adaptive NN control scheme for a class of strict feedback systems with time-varying full state constraints has been proposed based on IBLFs. By utilizing the advantages of the IBLFs solving the constraints problems, the controller has been designed with the help of the backstepping design algorithm. Then the stability of the closed-loop system can be verified by using the Lyapunov theorem. Finally, the simulation results have showed that the time-varying full state constraints are not violated, all the closed-loop signals are bounded, and a good tracking performance is achieved. In addition, IBLFs can be applied to deal with uncertain stochastic systems or switched systems with time-varying constraints as a further research direction.

Acknowledgments

This work is supported in part by the National Natural Science Foundation of China under Grants 62025303, 61973147, 61803190, 61773188, in part by the LiaoNing Revitalization Talents Program XLYC1907050.

References

Chen, X., Feng, Y., & Su, C. Y. (2016). Adaptive control for continuous-time systems with actuator and sensor hysteresis. *Automatica*, *64*, 196–207.

Chen, C. P., & Pao, Y. H. (1993). An integration of neural network and rule-based systems for design and planning of mechanical assemblies. *IEEE Transactions on Systems, Man, and Cybernetics*, *23*(5), 1359–1371.

He, W., & Ge, S. S. (2015). Vibration control of a flexible beam with output constraint. *IEEE Transactions on Industrial Electronics*, *62*(8), 5023–5030.

He, W., Ge, S. S., & Zhang, S. (2011). Adaptive boundary control of a flexible marine installation system. *Automatica*, *47*(12), 2728–2734.

Kim, B. S., & Yoo, S. J. (2014). Approximation-based adaptive control of uncertain non-linear pure-feedback systems with full state constraints. *IET Control Theory & Applications*, *8*(17), 2070–2081.

Kim, B. S., & Yoo, S. J. (2015). Adaptive control of nonlinear pure-feedback systems with output constraints: integral barrier Lyapunov functional approach. *International Journal of Control, Automation and Systems*, *13*(1), 249–256.

Krstic, M., & Kokotovic, P. V. (1996). Adaptive nonlinear output-feedback schemes with marino-tomei controller. *IEEE Transactions on Automatic Control*, *41*(2), 274–280.

Krstic, M., & Smyshlyaev, A. (2008). Adaptive boundary control for unstable parabolic PDEs—Part i: Lyapunov design. *IEEE Transactions on Automatic Control*, *53*(7), 1575–1591.

Li, D. P., Li, D. J., Liu, Y. J., Tong, S., & Chen, C. P. (2017). Approximation-based adaptive neural tracking control of nonlinear MIMO unknown time-varying delay systems with full state constraints. *IEEE Transactions on Cybernetics*, *47*(10), 3100–3109.

Li, T. S., Zhao, R., Chen, C. L. P., Fang, L. Y., & Liu, C. (2018). Finite-time formation control of under-actuated ships using nonlinear sliding mode control. *IEEE Transactions on Cybernetics*, *48*(11), 3243–3253.

Liu, L., Li, X. S., Liu, Y. J., & Tong, S. (2021). Neural network based adaptive event trigger control for a class of electromagnetic suspension systems. *Control Engineering Practice*, *106*, Article 104675.

Liu, L., Liu, Y. J., Chen, A. Q., Tong, S., & Chen, C. L. P. (2020). Integral barrier Lyapunov function based adaptive control for switched nonlinear systems. *Science China. Information Sciences*, *63*(3), 132203:1–132203:14.

Liu, Y. J., Tong, S., Chen, C. L. P., & Li, D. J. (2017). Adaptive NN control using integral barrier Lyapunov functionals for uncertain nonlinear block-triangular constraint systems. *IEEE Transactions on Cybernetics*, *47*(11), 3747–3757.

Smyshlyaev, A., & Krstic, M. (2007). Adaptive boundary control for unstable parabolic PDEs—Part II: Estimation-based designs. *Automatica*, *43*(9), 1543–1556.

Song, Y., Guo, J., & Huang, X. (2017). Smooth neuroadaptive PI tracking control of nonlinear systems with unknown and nonsmooth actuation characteristics. *IEEE Transactions on Neural Networks and Learning Systems*, *28*(9), 2183–2195.

Song, Y., Shen, Z., He, L., & Huang, X. (2017). Neuroadaptive control of strict feedback systems with full-state constraints and unknown actuation characteristics: an inexpensive solution. *IEEE Transactions on Cybernetics*, (99), 1–9.

Sui, S., Chen, C. L. P., & Tong, S. (2018). Neural network filtering control design for nontriangular structure switched nonlinear systems in finite time. *IEEE Transactions on Neural Networks and Learning Systems*, *30*(7), 2153–2162.

Tang, Z. L., Ge, S. S., Tee, K. P., & He, W. (2016). Robust adaptive neural tracking control for a class of perturbed uncertain nonlinear systems with state constraints. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, *46*(12), 1618–1629.

Tee, K. P., & Ge, S. S. (2012). Control of state-constrained nonlinear systems using integral barrier Lyapunov functionals. In *2012 IEEE 51st IEEE conference on decision and control (CDC)* (pp. 3239–3244).

Tee, K. P., Ge, S. S., & Tay, E. H. (2009). Barrier Lyapunov functions for the control of output-constrained nonlinear systems. *Automatica*, *45*(4), 918–927.

Tee, K. P., Ren, B., & Ge, S. S. (2011). Control of nonlinear systems with time-varying output constraints. *Automatica*, *47*(11), 2511–2516.

Tong, S., Min, X., & Li, Y. (2020). Observer-based adaptive fuzzy tracking control for strict-feedback nonlinear systems with unknown control gain functions. *IEEE Transactions on Cybernetics*, *50*(9), 3903–3913.

Wang, C., & Lin, Y. (2012). Multivariable adaptive backstepping control: a norm estimation approach. *IEEE Transactions on Automatic Control*, *57*(4), 989–995.

Wang, C., & Lin, Y. (2015). Decentralized adaptive tracking control for a class of interconnected nonlinear time-varying systems. *Automatica*, *54*, 16–24.

Wang, F., Liu, Z., Zhang, Y., & Chen, C. P. (2016). Adaptive fuzzy control for a class of stochastic pure-feedback nonlinear systems with unknown hysteresis. *IEEE Transactions on Fuzzy Systems*, *24*(1), 140–152.

Wang, C., Wen, C., & Guo, L. (2016). Decentralized output-feedback adaptive control for a class of interconnected nonlinear systems with unknown actuator failures. *Automatica*, *71*, 187–196.

Wang, C., Wen, C., & Lin, Y. (2017). Adaptive actuator failure compensation for a class of nonlinear systems with unknown control direction. *IEEE Transactions on Automatic Control*, *62*(1), 385–392.

Wu, Z. J., Xie, X. J., & Zhang, S. Y. (2007). Adaptive backstepping controller design using stochastic small-gain theorem. *Automatica*, *43*(4), 608–620.

Xie, X. J., & Tian, J. (2009). Adaptive state-feedback stabilization of high-order stochastic systems with nonlinear parameterization. *Automatica*, *45*(1), 126–133.

Yu, J., Shi, P., & Zhao, L. (2018). Finite-time command filtered backstepping control for a class of nonlinear systems. *Automatica*, *92*, 173–180.

Zhang, T., Xia, M., & Yi, Y. (2017). Adaptive neural dynamic surface control of strict-feedback nonlinear systems with full state constraints and unmodeled dynamics. *Automatica*, *81*, 232–239.

Zhao, K., & Song, Y. (2019). Removing the feasibility conditions imposed on tracking control designs for state-constrained strict-feedback systems. *IEEE Transactions on Automatic Control*, *64*(3), 1265–1272.

Zhao, K., Song, Y., Ma, T., & He, L. (2018). Prescribed performance control of uncertain Euler–Lagrange systems subject to full-state constraints. *IEEE Transactions on Neural Networks and Learning Systems*, *29*(8), 3478–3489.

Zhao, K., Song, Y., & Shen, Z. (2018). Neuroadaptive fault-tolerant control of nonlinear systems under output constraints and actuation faults. *IEEE Transactions on Neural Networks and Learning Systems*, *29*(2), 286–298.

Zhou, J., Chen, C. P., Chen, L., & Li, H. X. (2014). A collaborative fuzzy clustering algorithm in distributed network environments. *IEEE Transactions on Fuzzy Systems*, *22*(6), 1443–1456.

Zhou, J., Wen, C., & Yang, G. H. (2014). Adaptive backstepping stabilization of nonlinear uncertain systems with quantized input signal. *IEEE Transactions on Automatic Control*, *59*(2), 460–464.

Lei Liu received the B.S. degree in information and computing science and the M.S. degree in applied mathematics from the Liaoning University of Technology, Jinzhou, China, in 2010 and 2013, respectively. He received the Ph.D. degree in 2017 from the Northeastern University, Shenyang, China. Currently, he is an associate professor at the Liaoning University of Technology, Jinzhou, China.

His current research interests include constraint control, fault-tolerant control, fault detection and diagnosis, optimal control, and their industrial applications.

Tingting Gao received the B.S. degree in information and computing science and the M.S. degree in applied mathematics from the Liaoning University of Technology, Jinzhou, China, in 2016 and 2019, respectively. She is currently pursuing the Ph. D. degree in Navigation College, Dalian Maritime University.

Her research interests include nonlinear control, adaptive control, stochastic control, neural network control, and constraint control.

Yan-Jun Liu received the B.S. degree in Applied Mathematics and the M.S. degree in Control Theory and Control Engineering from Shenyang University of Technology, Shenyang, China, in 2001 and 2004, respectively. He received the Ph.D. degree in Control Theory and Control Engineering from Dalian University of Technology, Dalian, China, in 2007.

He is currently a Professor with the College of Science, Liaoning University of Technology. His research interests include intelligent control theory and its application, and swarm intelligence. He has served on Associate Editor for several journals, including IEEE Transactions on Cybernetics, IEEE Transactions on Neural Networks and Learning Systems and IEEE/CAA Journal of Automatica Sinica.

Shaocheng Tong received the B.S. degree in mathematics from Jinzhou Normal College, Jinzhou, China, in 1982, the M.S. degree in mathematics from Dalian Marine University, Dalian, China, in 1988, and the Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China, in 1997.

He is currently a Professor with the College of Science, Liaoning University of Technology. His current research interests include fuzzy and neural networks control theory and nonlinear control, adaptive control, and system identification.

C.L. Philip Chen received his M.S. degree in electrical engineering from the University of Michigan, Ann Arbor, in 1985 and the Ph.D. degree in electrical engineering from Purdue University, West Lafayette, IN, in 1988.

He is currently the Dean of the School of computer Science & Engineering, South China University of Technology, China. Dr. Chen is a Fellow of the IEEE, AAAS, IAPR, Chinese Association of Automation (CAA), and Hong Kong Institute of Engineers' (HKIE). He is the Editor-in-Chief of IEEE Transactions on Cybernetics.

Lei Ma received the B.S. degree in information and computing science and the M.S. degree in applied mathematics from the Liaoning University of Technology, Jinzhou, China, in 2015 and 2019, respectively. He is currently pursuing the Ph.D degree in control theory and control engineering, Northeastern University, Shenyang, China.

His current research interests include nonlinear control, adaptive control, neural network control and constraint control.