Boundary Control of a Flexible Riser With the Application to Marine Installation

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***Abstract*—This paper investigates the control problem of a marine riser installation system. The riser installation system consisting of a vessel, a flexible riser, and a subsea payload is modeled as a distributed parameter system with one partial differential equation and four ordinary differential equations. Based on Lyapunov's direct method, adaptive boundary control is proposed at the top and bottom boundaries of the riser to position the subsea payload to the desired set point and suppress the riser's vibration. With the proposed control, uniform boundedness of the steady-state error between the boundary payload and the desired position is achieved by suitably choosing the design parameters. Numerical simulations are presented for demonstrating the effectiveness of the proposed control.**

***Index Terms*—Adaptive control, boundary control, distributed parameter system, flexible riser, partial differential equation.**

I. Introduction

THE TREND in the offshore industry is toward the increased use of installation systems and floating platforms such as anchored floating production, storage, and offloading (FPSO) vessels in deep water. The riser installation system consisting of an FPSO, a riser, and the subsea payload has been accepted as one of the solutions for the exploitation of oil and gas in the offshore engineering. The marine riser is used for transporting the crude oil or natural gas from the oil well in the ocean floor to the production vessel or platform in the ocean surface [1]. The whole marine installation system is subjected to environmental disturbances including the ocean current, wave, and wind. A typical marine riser installation system is shown in Fig. 1. The surface FPSO, to which the top boundary of the riser is connected, is equipped with a dynamic positioning system with an active thruster. The bottom boundary of the riser is a payload with an end-point thruster attached. This thruster is used for dynamic positioning of the payload.

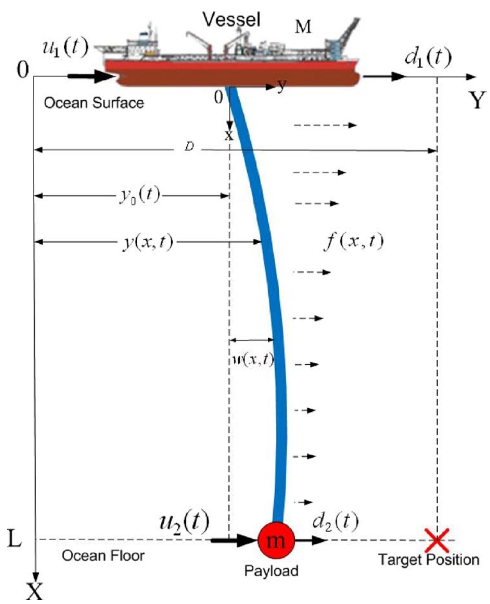

Fig. 1. Typical beam-type marine riser installation system.

Development of a general frame for control of the flexible marine riser system in the presence of the unknown ocean disturbances is quite a challenging research topic. In earlier works of marine flexible risers [2], the modeling of the riser systems is investigated, and the simulations with different numerical methods are provided to verify the effectiveness of the models. In [3], boundary control for the flexible marine riser with actuator dynamics is designed based on Lyapunov's direct method and the backstepping technique. In [4], the boundary control problem of a 3-D nonlinear inextensible riser system is considered via a similar method as that in [3]. In [5], a torque actuator is introduced at the top boundary of the riser to reduce the angle and transverse vibration of the riser with guaranteed closed-loop stability. In [6], boundary control for a coupled nonlinear flexible marine riser with two actuators in transverse and longitudinal directions has been designed to suppress the riser's vibration.

Lyapunov's direct method is widely used for the control of the distributed parameter systems [6]–[21]. Recently, by combining the backstepping method with adaptive control design, a novel boundary controller and observer is designed to stabilize the string and beam model and track the target system. Many remarkable results in this area have been obtained in [22]–[25]. The riser installation system can also be considered as an

Manuscript received June 11, 2012; revised November 27, 2012; accepted December 24, 2012. Date of publication January 9, 2013; date of current version June 21, 2013. This work was supported in part by the National Science Foundation of China under Grant 61203057 and in part by the Fundamental Research Funds for the China Central Universities of the University of Electronic Science and Technology of China (ZYGX2012J087).

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Digital Object Identifier 10.1109/TIE.2013.2238873

overhead crane with the flexible riser. Control design and stability analysis of the overhead crane have been studied in [26]–[30]. In [27], exponential stabilization of an overhead crane with the flexible cable is investigated by using a backstepping approach. In [29], boundary control, well-posedness, and stability are discussed for an overhead crane.

Earlier research [31], [32] on the control of the marine installation systems focuses on the dynamics of the payload, where the dynamics of the riser is ignored for the convenience of the control design. The dynamics of the riser is considered as an external force term to the payload. One drawback of the model is that it can influence the dynamic response of the whole installation system due to the neglect of the coupling between the vessel, the riser, and the payload. To overcome this shortcoming, in this paper, the flexible riser installation system with riser, vessel, and payload dynamics is represented by a set of infinite-dimensional equations coupled with a set of finite-dimensional equations.

II. Problem Formulation and Preliminaries

Fig. 1 shows a typical marine riser installation system. The top boundary of the marine riser is connected to the vessel, and the bottom boundary of the marine riser is at the underwater payload. D is the desired target position of the payload. The control inputs are from thrusters on vessel and payload, respectively. Frame $X{-}Y$ is the fixed inertia frame, and frame $x{-}y$ is the local reference frame fixed along the vertical direction of the surface vessel. From Fig. 1, we have

$$y(x,t) := y_0(t) + w(x,t) \tag{1}$$

where $y(x,t)$ denotes the position of the marine riser with respect to frame $X{-}Y$ at the position x for time t, $y_0(t)$ represents position of the vessel, and $w(x,t)$ is elastic transverse deflection with respect to frame $x{-}y$ at the position x for time t. Due to the connection between the vessel and the top boundary of the marine riser, we have $w(0,t) = 0$; then, $y(0,t) := y_0(t)$, $\dot{y}(0,t) =: \dot{y}_0(t)$, and $\ddot{y}(0,t) =: \ddot{y}_0(t)$ are the position, velocity, and acceleration of the vessel, respectively. In this paper, we consider the transverse degree of freedom only. We assume that the original position of the vessel is directly above the subsea payload with no horizontal offset and the payload is filled with seawater.

Remark 1: For clarity, notions $(\cdot)' = \partial(\cdot)/\partial x$ and $(\dot{\cdot}) = \partial(\cdot)/\partial t$ are used throughout this paper.

A. Dynamics of a Marine Riser Installation System

The kinetic energy of the installation system E_k can be represented as

$$E_k = \frac{1}{2}M\left[\dot{y}(0,t)\right]^2 + \frac{1}{2}\rho\int_0^L \left[\dot{y}(x,t)\right]^2 dx + \frac{1}{2}m\left[\dot{y}(L,t)\right]^2 \tag{2}$$

where x and t represent the independent spatial and time variables, respectively, L is length of the marine riser, M denotes mass of the surface vessel, m is mass of the bottom payload, and ρ denotes the uniform mass per unit length of the marine riser.

The potential energy E_p due to the strain energy of the marine riser can be obtained from

$$E_p = \frac{1}{2}T\int_0^L \left[w'(x,t)\right]^2 dx + \frac{1}{2}EI\int_0^L \left[w''(x,t)\right]^2 dx \tag{3}$$

where EI denotes bending stiffness of the marine riser and T is tension of the marine riser. The definition of $y(x,t)$ yields $y'(x,t) = w'(x,t)$. Then, we have

$$E_p = \frac{1}{2}T\int_0^L \left[y'(x,t)\right]^2 dx + \frac{1}{2}EI\int_0^L \left[y''(x,t)\right]^2 dx. \tag{4}$$

The virtual work done by ocean current disturbances on the vessel, the marine riser, and the payload is given by

$$\delta W_f = \int_0^L f(x,t)\delta y(x,t)dx + d_1(t)\delta y(0,t) + d_2(t)\delta y(L,t) \tag{5}$$

where $d_1(t)$ denotes unknown boundary disturbance on the vessel, $d_2(t)$ is unknown boundary disturbance on the payload, and $f(x,t)$ is unknown spatiotemporally varying load. The virtual work done by damping on the vessel, the riser, and the payload is represented by

$$\delta W_d = -\int_0^L c\dot{y}(x,t)\delta y(x,t)dx - c_1\dot{y}(0,t)\delta y(0,t) - c_2\dot{y}(L,t)\delta y(L,t) \tag{6}$$

where c denotes the damping coefficient of the marine riser, c_1 is the damping coefficient of the vessel, and c_2 represents the damping coefficient of the payload.

The virtual work done by the boundary controls$u_1(t)$ and $u_2(t)$ is written as

$$\delta W_m = u_1(t)\delta w(0,t) + u_2(t)\delta w(L,t) \tag{7}$$

where $u_1(t)$ and $u_2(t)$ are boundary control forces applied to the top and bottom boundaries of the marine riser, respectively. The total virtual work done by ocean current disturbances, damping, and boundary control on the system is

$$\delta W = \delta W_f + \delta W_d + \delta W_m. \tag{8}$$

Using Hamilton's principle and applying the variation operator and integrating (2), (4), and (8) by parts, respectively, we obtain the governing equation of the system as

$$\rho\ddot{y}(x,t) + EIy''''(x,t) - Ty''(x,t) + c\dot{y}(x,t) = f(x,t) \tag{9}$$

$\forall(x,t) \in (0,L) \times [0,\infty)$, and the boundary conditions of the system as

$$y'(0,t) = 0 \tag{10}$$

$$y''(L,t) = 0 \tag{11}$$

$$EIy'''(0,t) + c_1\dot{y}(0,t) + M\ddot{y}(0,t) = u_1(t) + d_1(t) \tag{12}$$

$$-EIy'''(L,t) + Ty'(L,t) + c_2\dot{y}(L,t) = u_2(t) + d_2(t) - m\ddot{y}(L,t) \tag{13}$$

$\forall t \in [0,\infty)$.

Property 1 [33, p. 131]: If the kinetic energy of the system (9)–(13), given by (2), is bounded $\forall t \in [0,\infty)$, then $\dot{y}(x,t)$, $\dot{y}'(x,t)$, and $\dot{y}''(x,t)$ are bounded $\forall(x,t) \in [0,L] \times [0,\infty)$.

Property 2 [33, p. 132]: If the potential energy of the system (9)–(13), given by (4), is bounded $\forall t \in [0,\infty)$, then $y'(x,t)$ and $y''(x,t)$ are bounded $\forall(x,t) \in [0,L] \times [0,\infty)$.

In our paper, the boundedness properties 1 and 2 hold since the control inputs and external disturbances are not included in the kinetic energy and the potential energy. That is to say, the control inputs and external disturbances will not affect the boundedness properties of the system, if the kinetic energy and potential energy are bounded.

B. Problem Formulation

The control objective is to design the boundary control to position the subsea payload to the desired set point D as well as suppress the vibrations of the marine riser in the presence of the unknown ocean disturbances. The control forces $u_1(t)$ and $u_2(t)$ are from the thruster in the vessel and the thruster attached in the subsea payload, respectively. The adaptive control is designed to compensate for the system parametric uncertainties and the unknown disturbances.

Assumption 1: For the distributed load $f(x,t)$ on the marine riser, the disturbance $d_1(t)$ on the vessel, and the disturbance $d_2(t)$ on the payload, we assume that there exist constants $\bar{f} \in \mathbb{R}^+$, $\bar{d}_1 \in \mathbb{R}^+$ and $\bar{d}_2 \in \mathbb{R}^+$, such that $|f(x,t)| \le \bar{f}$, $\forall(x,t) \in [0,L] \times [0,\infty)$, $|d_1(t)| \le \bar{d}_1$, $\forall t \in [0,\infty)$, and $|d_2(t)| \le \bar{d}_2$, $\forall t \in [0,\infty)$.

Remark 2: This is a reasonable assumption as the time-varying disturbances $f(x,t)$, $d_1(t)$, and $d_2(t)$ have finite energy and hence are bounded, i.e., $f(x,t) \in \mathcal{L}_\infty$, $d_1(t) \in \mathcal{L}_\infty$, and $d_2(t) \in \mathcal{L}_\infty$. The knowledge of the exact values for $f(x,t)$, $d_1(t)$, and $d_2(t)$ is not required.

C. Preliminaries

For the convenience of stability analysis, we present the following lemmas and properties for the subsequent development.

Proposition 1: Let $\phi_1(x,t), \phi_2(x,t) \in \mathbb{R}$ with $x \in [0,L]$ and $t \in [0,\infty)$; then, the following inequality holds:

$$|\phi_1\phi_2| = \left|\left(\frac{1}{\sqrt{\delta}}\phi_1\right)(\sqrt{\delta}\phi_2)\right| \le \frac{1}{\delta}\phi_1^2 + \delta\phi_2^2 \tag{14}$$

$\forall\phi_1, \phi_2 \in \mathbb{R}$, *and* $\delta > 0$.

Lemma 1 [6], [34]: Let $\phi(x,t) \in \mathbb{R}$ be a function defined on $x \in [0,L]$ and $t \in [0,\infty)$ that satisfies the boundary condition

$$\phi(0,t) = 0, \qquad \forall t \in [0,\infty) \tag{15}$$

then, the following inequality holds:

$$\phi^2 \le L\int_0^L [\phi']^2 dx, \qquad \forall x \in [0,L]. \tag{16}$$

If, in addition to (15), the function $\phi(x,t)$ satisfies the boundary condition

$$\phi'(0,t) = 0, \qquad \forall t \in [0,\infty) \tag{17}$$

then, the following inequality also holds:

$$[\phi']^2 \le L\int_0^L [\phi'']^2 dx, \qquad \forall x \in [0,L]. \tag{18}$$

III. Control Design

In this section, two boundary control laws at the top and bottom boundaries of the marine riser have been designed to position the subsea payload to the desired target as well as to reduce the vibrations of the riser and the vessel. Lyapunov's direct method is used to analyze the stability of the closed-loop system. We present the control design for both known and uncertain versions of the plant and stability analysis of the closed-loop system.

A. Control Design for the Known System

When the system parameters are known, to stabilize the system given by governing (9) and boundary condition (10)–(13), we propose the following model-based boundary control:

$$u_1(t) = -k_v\dot{y}(0,t) - \text{sgn}\,[y(0,t)]\,\bar{d}_1 \tag{19}$$

$$u_2(t) = -P(t)\Phi - k_s u_a(t) - k_p\,(y(L,t) - D) - \text{sgn}\,[u_a(t)]\,\bar{d}_2 \tag{20}$$

where k_s, k_p, and k_v are the positive control gains and vector $P(t)$ and the parameter vector Φ are defined as

$$P(t) = \begin{bmatrix} \dot{y}'(L,t) - \dot{y}'''(L,t) & -y'(L,t) \\ y'''(L,t) & -\dot{y}(L,t) \end{bmatrix} \tag{21}$$

$$\Phi = [\,m \quad T \quad EI \quad c_2\,]^{\mathrm{T}}. \tag{22}$$

The auxiliary signal $u_a(t)$ is defined as

$$u_a(t) = \dot{y}(L,t) + y'(L,t) - y'''(L,t). \tag{23}$$

Remark 3: The proposed boundary control does not require distributed sensing, and all the signals in the boundary control can be measured by sensors or obtained by a backward difference algorithm. $y(0,t)$ and $y(L,t)$ can be sensed by two

Global Positioning Systems located in the vessel and the endpoint thruster, respectively. $y'(0,t)$ can be sensed by a laser displacement sensor in the top boundary of the riser, and $y'(L,t)$ can be measured by an inclinometer at the bottom boundary of the riser. $y'''(L,t)$ can be obtained by a shear force sensor. After using a backward difference algorithm to $y'(L,t)$ and $y'''(L,t)$, respectively, we can obtain $\dot{y}'(L,t)$ and $\dot{y}'''(L,t)$.

Remark 4: The control design is based on the assumption on the boundedness of the disturbances $d_1(t)$ and $d_2(t)$. However, assumption on disturbances may result in the conservative control design. Disturbance observers [19] can be used to estimate the disturbances $d_1(t)$ and $d_2(t)$ for a Timoshenko beam, where the assumption on the boundedness of the time-varying disturbances is not required.

Consider Lyapunov function candidate

$$V(t) = V_1(t) + V_2(t) + \Delta(t) \tag{24}$$

where the energy term $V_1(t)$, an auxiliary term $V_2(t)$, and a small crossing term $\Delta(t)$ are defined as

$$\begin{aligned} V_1(t) = &\frac{\beta}{2}\rho\int_0^L [\dot{y}(x,t)]^2\,dx + \frac{\beta}{2}T\int_0^L [y'(x,t)]^2\,dx \\ &+ \frac{\beta}{2}EI\int_0^L [y''(x,t)]^2\,dx + \frac{\beta}{2}M\,[\dot{y}(0,t)]^2 \\ &+ \frac{\beta k_p}{2}\,[y(L,t)-D]^2 \end{aligned} \tag{25}$$

$$V_2(t) = \frac{1}{2}mu_a^2(t) \tag{26}$$

$$\Delta(t) = \alpha\rho\int_0^L x\dot{y}(x,t)y'(x,t)dx \tag{27}$$

where α is the positive weighting constant.

Proposition 2: Lyapunov function candidate given by (24) is upper and lower bounded as

$$0 \le \lambda_1(V_1+V_2) \le V(t) \le \lambda_2(V_1+V_2)$$

where λ_1 and λ_2 are two positive constants.

Proof: Equation (27) yields

$$|\Delta(t)| \le \alpha\rho L\int_0^L \left([y'(x,t)]^2 + [\dot{y}(x,t)]^2\right)dx \le \alpha_1 V_1(t) \tag{28}$$

where

$$\alpha_1 = \frac{2\alpha\rho L}{\min(\beta\rho,\beta T)}. \tag{29}$$

Then, we obtain

$$-\alpha_1 V_1(t) \le \Delta(t) \le \alpha_1 V_1(t). \tag{30}$$

Considering α is a small positive weighting constant satisfying $0 < \alpha < (\min(\beta\rho,\beta T)/2\rho L)$, we obtain

$$\alpha_2 = 1-\alpha_1 = 1-\frac{2\alpha\rho L}{\min(\beta\rho,\beta T)} > 0 \tag{31}$$

$$\alpha_3 = 1+\alpha_1 = 1+\frac{2\alpha\rho L}{\min(\beta\rho,\beta T)} > 1. \tag{32}$$

Given Lyapunov function candidate in (24), we obtain

$$0 \le \lambda_1\left(V_1(t)+V_2(t)\right) \le V(t) \le \lambda_2\left(V_1(t)+V_2(t)\right) \tag{33}$$

where $\lambda_1 = \min(\alpha_2,1) = \alpha_2$ and $\lambda_2 = \max(\alpha_3,1) = \alpha_3$ are positive constants. ■

Lemma 2: The time derivative of Lyapunov function in (24) can be upper bounded with

$$\dot{V}(t) \quad \le \quad -\lambda V(t) + \varepsilon \tag{34}$$

where λ and ε are two positive constants.

Proof: Differentiating (24) with respect to time leads to

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{\Delta}(t). \tag{35}$$

Substituting the governing equations, the boundary conditions, and the proposed control laws and applying Proposition 1 and Lemma 1, we have

$$\begin{aligned} &\dot{V}(t) \\ &\le -\left(\beta c + \frac{\alpha\rho}{2} - \beta\delta_3 - \frac{\alpha cL}{\delta_7}\right)\int_0^L [\dot{y}(x,t)]^2\,dx \\ &\quad - \left(\frac{\alpha T}{2} - 4k_pL - \alpha L\delta_6 - \alpha cL\delta_7\right)\int_0^L [y'(x,t)]^2\,dx \\ &\quad - \Bigg(\frac{3\alpha EI}{2} + \frac{\beta EIL}{2} - \beta\delta_1 L|T-EI| \\ &\qquad - \beta\delta_2 EIL - \alpha\delta_5 EIL^2\Bigg) \\ &\quad \times\int_0^L [y''(x,t)]^2\,dx - \beta(k_v+c_1)\,[\dot{y}(0,t)]^2 \\ &\quad - k_p\left(1-\frac{\beta}{2\delta_4}\right)[y(L,t)-D]^2 \\ &\quad - \left(k_s - k_p - \frac{\beta EI}{2}\right)u_a^2 \\ &\quad - \left(\frac{\beta EI}{2} - \frac{\beta EI}{\delta_2} - \frac{\alpha EIL}{\delta_5}\right)[y'''(L,t)]^2 \\ &\quad - \left(\frac{\beta EI}{2} - \frac{\alpha\rho L}{2} - \frac{\beta k_p\delta_4}{2} - \frac{\beta|T-EI|}{\delta_1}\right)[\dot{y}(L,t)]^2 \\ &\quad + \left(\frac{\beta}{\delta_3} + \frac{\alpha L}{\delta_6}\right)\int_0^L \bar{f}^2(x,t)dx + 4k_pD^2 \\ &\le -\lambda_3\left(V_1(t)+V_2(t)\right) + \varepsilon \end{aligned} \tag{36}$$

where $\delta_1 - \delta_7$ are positive constants and ε is a positive constant defined as (37) and (38), shown at the bottom of the page. Other constants k_s, k_v, k_p, α, β, and $\delta_1 - \delta_7$ are chosen to satisfy the following conditions:

$$\frac{\beta EI}{2} - \frac{\alpha\rho L}{2} - \frac{\beta k_p \delta_4}{2} - \frac{\beta|T-EI|}{\delta_1} \geq 0 \tag{39}$$

$$\frac{\beta EI}{2} - \frac{\beta EI}{\delta_2} - \frac{\alpha EIL}{\delta_5} \geq 0. \tag{40}$$

From (33) and (36), we have

$$\dot{V}(t) \leq -\lambda V(t) + \varepsilon \tag{41}$$

where $\lambda = \lambda_3/\lambda_2$. ■

Theorem 1: For the system dynamics described by (9) and boundary conditions (10)–(13), under Assumption 1, and the boundary control (19) and (20), given that the initial conditions are bounded, the transverse deflection $w(x,t)$ of the closed-loop system and the position error $e(t) = y(L,t) - D$ are uniformly bounded and converge to some bounded sets defined by

$$\Omega_w := \{w(x,t) \in \mathbb{R}| \quad |w(x,t)| \leq D_w\} \tag{42}$$

$$\Omega_e := \{e(t) \in \mathbb{R}| \quad |e| \leq D_e\} \tag{43}$$

where $D_w = \sqrt{(2L/\beta T\lambda_1)(V(0) + (\varepsilon/\lambda))}$ and $D_e = \sqrt{(2/\beta k_p\lambda_1)(V(0) + (\varepsilon/\lambda))}$.

Proof: Multiplying (34) by $e^{\lambda t}$ yields

$$\frac{\partial}{\partial t}(Ve^{\lambda t}) \leq \varepsilon e^{\lambda t}. \tag{44}$$

Integrating the aforementioned inequality, we obtain

$$V \leq \left(V(0) - \frac{\varepsilon}{\lambda}\right)e^{-\lambda t} + \frac{\varepsilon}{\lambda} \leq V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda} \in \mathcal{L}_\infty \tag{45}$$

which implies V is bounded. Utilizing (16) and (25), we have

$$\frac{\beta}{2L}Tw^2(x,t) \leq \frac{\beta}{2}T\int_0^L [w'(x,t)]^2\,dx = \frac{\beta}{2}T\int_0^L [y'(x,t)]^2\,dx$$
$$\leq V_1(t) \leq V_1(t) + V_2(t) \leq \frac{V(t)}{\lambda_1} \in \mathcal{L}_\infty. \tag{46}$$

Appropriately rearranging the terms of the aforementioned equation (46), we obtain that $w(x,t)$ is uniformly bounded as follows:

$$|w(x,t)| \leq \sqrt{\frac{2L}{\beta T\lambda_1}\left(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda}\right)} \tag{47}$$

$\forall(x,t) \in [0,L] \times [0,\infty)$. Combining (25) and (46) yields

$$\frac{\beta k_p}{2}[y(L,t) - D]^2 \leq V_1(t) \leq \frac{V(t)}{\lambda_1} \in \mathcal{L}_\infty.$$

Furthermore, we have

$$|y(L,t) - D| \leq \sqrt{\frac{2}{\beta k_p\lambda_1}\left(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda}\right)} \tag{48}$$

$\forall t \in [0,\infty)$. ■

Remark 5: Even though the $y(L,t)$ may be far from the desired position D, it is guaranteed that the steady bottom boundary state error $y(L,\infty) - D$ can be made arbitrarily small provided that the design parameters are appropriately selected. It is easily seen that the increase in the control gains k_v, k_s, and k_p will result in a better tracking performance.

Remark 6: From aforementioned analysis, we can obtain that V_1 and V_2 are bounded $\forall t \in [0,\infty)$. The use of boundedness of V_1 and V_2 produces $\dot{y}(x,t)$; $y'(x,t)$ is bounded $\forall(x,t) \in [0,L] \times [0,\infty)$, and u_a is bounded $\forall t \in [0,\infty)$. Then, we can obtain that potential energy (4) is bounded. Using Property 2, we can further obtain that $y''(x,t)$ is bounded $\forall(x,t) \in [0,L] \times [0,\infty)$. From the boundedness of $\dot{y}(x,t)$, we can state that $\dot{y}(0,t)$ and $\dot{y}(L,t)$ are bounded $\forall t \in [0,\infty)$. Therefore, we can conclude that the kinetic energy of the system (2) is also bounded. Using Property 1, we can obtain that $\dot{y}'(x,t)$ is also bounded $\forall(x,t) \in [0,L] \times [0,\infty)$. Applying Assumption 1, (9), and the aforementioned statements, we can state that $\ddot{y}(x,t)$ is also bounded $\forall(x,t) \in [0,L] \times [0,\infty)$. From the aforementioned information, it is shown that the proposed controls (19) and (20) ensure all internal system signals including $w(x,t)$, $y'(x,t)$, $\dot{y}(x,t)$, $\dot{y}'(x,t)$, and $\ddot{y}(x,t)$ that are uniformly bounded $\forall(x,t) \in [0,L] \times [0,\infty)$. Since $y'(x,t)$, $\dot{y}(x,t)$, and $\dot{y}'(x,t)$ are all bounded $\forall(x,t) \in [0,L] \times [0,\infty)$, we can conclude that the boundary controls (19) and (20) are also bounded $\forall t \in [0,\infty)$.

$$\varepsilon = \left(\frac{\beta}{\delta_3} + \frac{\alpha L}{\delta_4}\right)\int_0^L \bar{f}^2 dx + 4k_p D^2 \tag{37}$$

$$\lambda_3 = \min\left(\frac{2\beta c + \alpha\rho - 2\beta\delta_3 - 2\frac{\alpha c L}{\delta_7}}{\beta\rho}, \frac{2(k_v + c_1)}{M}, \frac{\alpha T - 8k_p L - 2\alpha L\delta_6 - 2\alpha c L\delta_7}{\beta T}, \frac{k_p\left(2 - \frac{\beta}{\delta_4}\right)}{\beta},\right.$$
$$\left.\frac{\left(3\alpha + \beta L - 2\beta\delta_2 L - 2\alpha\delta_5 L^2\right)EI - 2\beta\delta_1 L|T - EI|}{\beta EI}, \frac{2k_s - 2k_p - \beta EI}{m}\right) > 0 \tag{38}$$

B. Adaptive Control Design for the Unknown System

When the system parameters are unknown, we propose the adaptive boundary control as

$$u_1(t) = -k_v \dot{y}(0,t) - \operatorname{sgn}[y(0,t)]\, \bar{d}_1 \tag{49}$$

$$u_2(t) = -P\hat{\Phi}(t) - k_s u_a(t) - k_p(y(L,t) - D) - \operatorname{sgn}[u_a(t)]\, \bar{d}_2 \tag{50}$$

where k_s, k_p, and k_v are the positive control gains and P and $u_a(t)$ are defined as in (21) and (23). The parameter estimate vector $\hat{\Phi}(t)$ is given as

$$\hat{\Phi}(t) = \begin{bmatrix} \widehat{m}(t) & \widehat{T}(t) & \widehat{EI}(t) & \widehat{c_2}(t) \end{bmatrix}^{\mathrm{T}}. \tag{51}$$

The adaptation laws are designed as

$$\dot{\hat{\Phi}}(t) = \Gamma P^{\mathrm{T}} u_a - \zeta_0 \Gamma \hat{\Phi} \tag{52}$$

where $\Gamma \in \mathbb{R}^{4\times 4}$ is a diagonal positive-definite matrix and ζ_0 is a positive constant.

The parameter estimate error vector $\tilde{\Phi} \in \mathbb{R}^3$ is defined as

$$\tilde{\Phi}(t) = \Phi - \hat{\Phi}(t) \tag{53}$$

where Φ is defined in (22).

Consider Lyapunov function candidate

$$V_a(t) = V(t) + \frac{1}{2}\tilde{\Phi}^{\mathrm{T}}(t)\Gamma^{-1}\tilde{\Phi}(t) \tag{54}$$

where $V(t)$ is defined as in (24).

Proposition 3: Lyapunov function candidate given by (54) is upper and lower bounded as

$$0 \le \lambda_{1a}\left(V_1 + V_2 + \left\|\tilde{\Phi}(t)\right\|^2\right) \le V_a(t) \le \lambda_{2a}\left(V_1 + V_2 + \left\|\tilde{\Phi}(t)\right\|^2\right) \tag{55}$$

where λ_{1a} and λ_{2a} are two positive constants.

Proof: We define all the eigenvalues of Γ as real and positive and the maximum and minimum eigenvalues of matrix Γ as $\lambda_{\max}$ and $\lambda_{\min}$, respectively. Utilizing the properties of matrix Γ and Rayleigh–Ritz theorem [20], we have

$$\frac{1}{2\lambda_{\max}}\left\|\tilde{\Phi}(t)\right\|^2 \le \frac{1}{2}\tilde{\Phi}^{\mathrm{T}}(t)\Gamma^{-1}\tilde{\Phi}(t) \le \frac{1}{2\lambda_{\min}}\left\|\tilde{\Phi}(t)\right\|^2. \tag{56}$$

Combining (33) and (56), we have (55), where $\lambda_{1a} = \min(\lambda_1, (1/2\lambda_{\max}))$ and $\lambda_{2a} = \max(\lambda_2, (1/2\lambda_{\min}))$ are two positive constants. ■

Lemma 3: The time derivative of Lyapunov function in (24) can be upper bounded with

$$\dot{V}_a(t) \le -\lambda_a V_a(t) + \psi \tag{57}$$

where λ_a and ψ are two positive constants.

Proof: Differentiating (54) with respect to time leads to

$$\dot{V}_a(t) = \dot{V}(t) + \tilde{\Phi}^{\mathrm{T}}(t)\Gamma^{-1}\dot{\tilde{\Phi}}(t). \tag{58}$$

Substituting (50) and the adaptation law (52) into (58), we have

$$\dot{V}_a(t) \le -\lambda_{3a}\left(V_1(t) + V_2(t) + \left\|\tilde{\Phi}(t)\right\|^2\right) + \frac{\zeta_0}{2}\|\Phi\|^2 + \varepsilon \tag{59}$$

where ε is defined as in (37) and $\lambda_{3a} = \min(\lambda_3, (\zeta_0/2))$ is a positive constant. Then, we have (57), where $\lambda_a = \lambda_{3a}/\lambda_{2a}$ and $\psi = (\zeta_0/2)\|\Phi\|^2 + \varepsilon > 0$. ■

Theorem 2: For the system dynamics described by (9) and boundary conditions (10)–(13), under Assumption 1, and the boundary control (49) and (50), given that the initial conditions are bounded, the transverse deflection $w(x,t)$ of the closed-loop system and the position error $e(t) = y(L,t) - D$ are uniformly bounded and converge to some bounded sets defined by

$$\Omega_{wa} := \{w(x,t) \in \mathbb{R}| \quad |w(x,t)| \le D_{wa}\} \tag{60}$$

$$\Omega_{ea} := \{e(t) \in \mathbb{R}| \quad |e(t)| \le D_{ea}\} \tag{61}$$

where $D_{wa} = \sqrt{(2L/\beta T\lambda_{1a})(V_a(0) + (\psi/\lambda_a))}$ and $D_{ea} = \sqrt{(2/\beta k_p \lambda_{1a})(V_a(0) + (\psi/\lambda_a))}$.

Proof: The proof of Theorem 2 is straightforward using the method of proof in Theorem 1 and hence is omitted for conciseness. ■

IV. Numerical Simulations

Simulations for a marine riser installation system under ocean disturbances are carried out to demonstrate the effectiveness of the proposed boundary control. In this paper, we select the finite-difference method to simulate the system performance with the proposed boundary control. The riser, initially at rest, is excited by a distributed transverse disturbance due to ocean current. The corresponding initial conditions of the marine installation system are given as follows: $y(x,0) = w(x,0) = \dot{y}(x,0) = \dot{w}(x,0) = 0$. The system parameters are given as follows. $L = 1000.00$ m, $D_r = 0.152$ m, $M = 9.60 \times 10^7$ kg, $m = 4 \times 10^5$ kg, $c_1 = 9.00 \times 10^7$ N · S/m, $c_2 = 2.00 \times 10^5$ N · S/m, $T = 8.11 \times 10^7$ N, $EI = 1.5 \times 10^7$ N · m^2, $\rho = 108.02$ kg/m, $\rho_s = 1024.00$ kg/m^3, $c = 1.00$ N · S/m^2, and $D = 50.00$ m. The surface current $U(t)$ is expressed as [13, eq. (62)], where $\bar{U} = 2$ ms^{-1} is the mean flow current and $U' = 0.2$ is the amplitude of the oscillating flow. In the simulation, we assume that the full current load is applied from $x = 1000$ m to $x = 0$ m and thereafter linearly declines to zero at the ocean floor, $x = 0$, to obtain a depth-dependent ocean current profile $U(x,t)$. The distributed load $f(x,t)$ is generated by (16) in [13] with $C_H = 1$, $\theta = 0$, and $S_t = 0.2$. The high-frequency oscillation of $f(L,t)$ is due to the vortex shedding frequency and the natural frequency of the riser. The disturbance $d_1(t)$ on the vessel is generated by $d_1(t) = \rho_s C_H U^2(t) \times 10^3$, where the drag confident $C_H = 1$ and $U(t)$ is the surface current. The disturbance $d_2(t)$ on the payload is given by $d_2(t) = [3 + 0.8\sin(0.7t) + 0.2\sin(0.5t) + 0.2\sin(0.9t)] \times 10^4$.

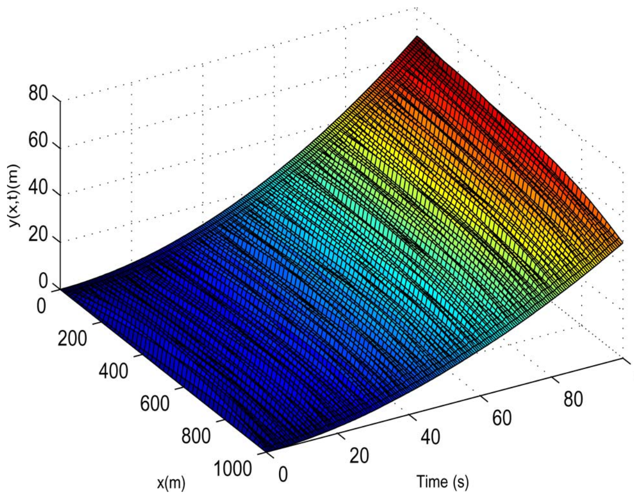

Fig. 2. Position of the riser without control.

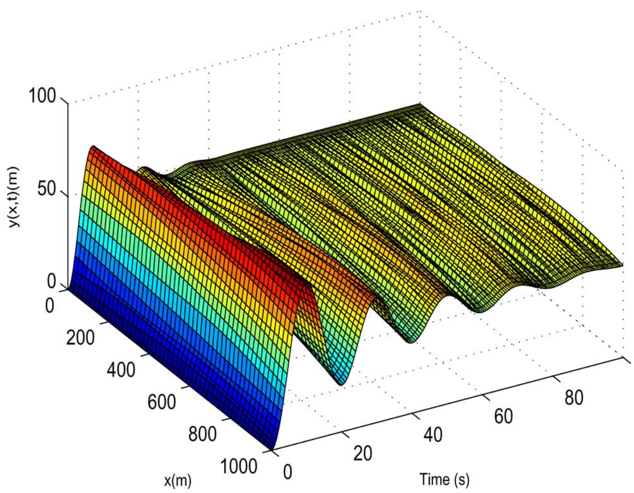

Fig. 3. Position of the riser with model-based boundary control.

In this paper, we carry out simulations for two cases of the flexible riser installation system: 1) exact model-based control, i.e., EI, T, m, and c_2 are all known and 2) adaptive control for the system parametric uncertainty, i.e., EI, T, m, and c_2 are unknown. The position of the riser for free vibration, i.e., $u_1(t) = u_2(t) = 0$, exposed to ocean disturbance is shown in Fig. 2. It is clear that the system is unstable and the vibration of the riser is quite large. The position of the riser with the proposed model-based control (19) and (20), by choosing $k_v = 2 \times 10^7$, $k_p = 1.5 \times 10^5$, and $k_s = 1 \times 10^6$ under ocean disturbance, is shown in Fig. 3. The corresponding boundary control inputs are shown in Fig. 4. It is shown that the model-based control is able to bring the subsea payload to the desired position $D = 50$ m and stabilize the riser at the some bounds of its equilibrium position. The position of the riser with the adaptive boundary control (49) and (50), by choosing $k_v = 2 \times 10^7$, $k_p = 1.5 \times 10^5$, $k_s = 1 \times 10^7$, $\Gamma = \mathrm{diag}\{1, 1, 1, 1\}$, and $\zeta_0 = 0.001$ under ocean disturbance, is shown in Fig. 5. The corresponding boundary control inputs are shown in Fig. 6. Fig. 5 illustrates that the payload will be positioned to the desired set point and the riser's vibration will be suppressed significantly.

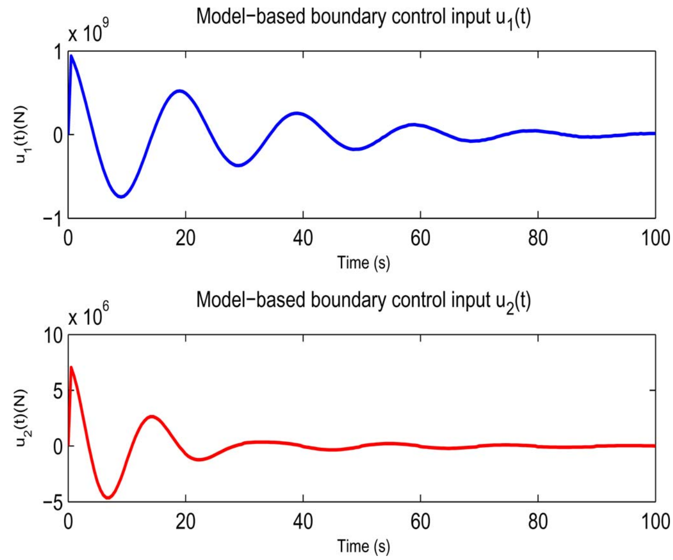

Fig. 4. Model-based boundary control inputs.

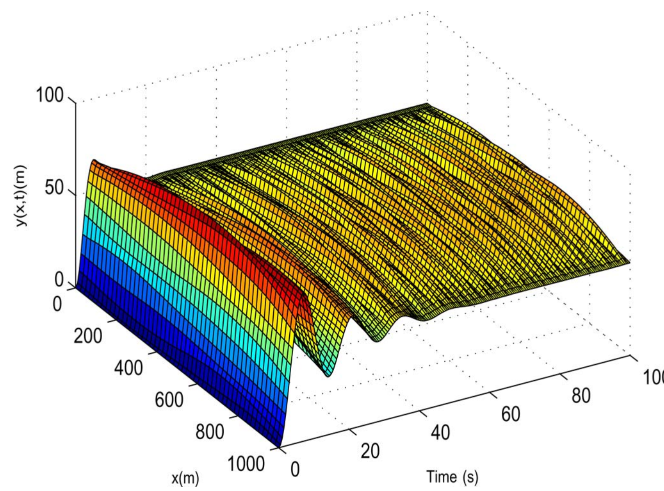

Fig. 5. Position of the riser with adaptive boundary control.

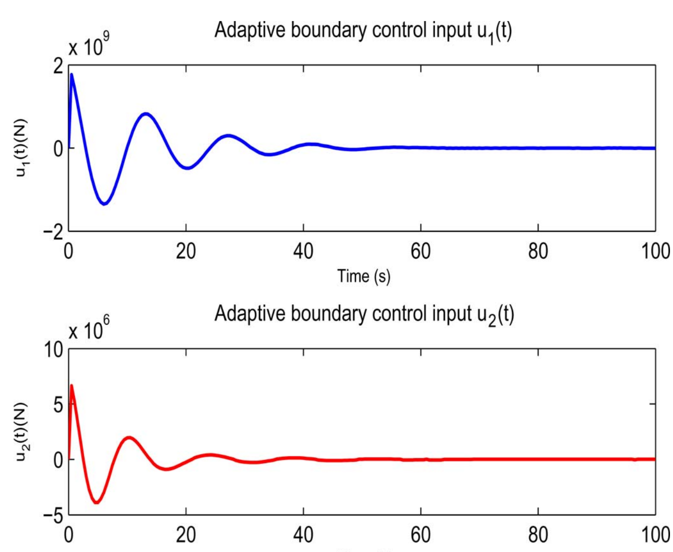

Fig. 6. Adaptive boundary control inputs.

V. Conclusion

In this paper, both position control and vibration suppression have been considered for a flexible marine riser installation system in the presence of the ocean disturbance. The model of the system has been represented by a partial differential equation via Hamilton's principle. Adaptive control has been developed to compensate for the system parametric uncertainties and the unknown disturbances based on Lyapunov's direct method. The states of the system have been proved to converge to some bounded sets by appropriately choosing the design parameters. The simulations have been carried out to illustrate the effectiveness of the proposed control.

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