# Investigating Distributed Generation Systems Performance Using Monte Carlo Simulation

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***Abstract*—A novel algorithm to evaluate the performance of electric distribution systems, including distributed generation (DG) is proposed. This algorithm addresses the deterministic and the stochastic natures of these electrical systems. Monte Carlo simulation is employed to solve the system operation randomness problem, taking into consideration the system operation constraints. The uncertainties in the locations, exported penetration level, and the states (on or off) of the DG units constitute the random parameters of the studied systems. The introduced algorithm incorporates these parameters with the traditional Newton–Raphson solution of the power flow equations. Monte Carlo simulation is implemented to perform the analysis of all the possible operation scenarios of the system under study and thus ensure the validity of the results. The proposed algorithm is employed to obtain the hourly power flow solution for a typical DG connected system. The system loading follows several typical load curves based on load bus types. Furthermore, new hourly steady-state operating system parameters are evaluated to describe the system behavior under the DG random operation. The results obtained are presented and discussed.**

***Index Terms*—Distributed generation (DG), Monte Carlo simulation, power flow.**

## NOMENCLATURE

| | |
|---|---|
| $B_{il}$ | Feeder's susceptance connecting buses $i$ and $l$ (Mho). |
| $fl$ | Feeder congestion counter. |
| $G_{il}$ | Feeder's conductance connecting buses $i$ and $l$ (Mho). |
| $H$ | Set of daily hours (hr). |
| $I_i(j)$ | $i$th feeder current at $j$th experiment (Amp.). |
| $\overline{I}$ | Convergence value of the feeder current (Amp.). |
| $i, l$ | Bus indexes. |
| $j$ | Experiment number. |
| $k$ | Random-number generator representing the number of DG units on their on state. |
| $M$ | Total number of primary distributed generator buses. |
| $N$ | Total number of DG units connected to primary distribution system buses. |
| $NE$ | Total number of experiments. |
| $P_g$ | Total hourly centralized power generation (MW). |
| $\overline{P}$ | Convergence value of the bus power (MW). |
| $P_i$ | Active power injected at bus $i$ (MW). |
| $P_i(j)$ | Active power injected at bus $i$ at $j$th experiment (MW). |
| $P_{Gi}(j)$ | $i$th DG random generated power at $j$th experiment (MW). |
| $P_{Gi}$ | $i$th DG generated power (MW). |
| $P_{Ri}$ | $i$th DG rated power (MW). |
| $\overline{P}_G$ | Convergence value of the DG generated power (MW). |
| $P_{lossil}(j)$ | Feeder connecting buses $i$ and $l$ power loss at $j$th experiment (MW). |
| $\overline{P}_{loss}$ | Convergence value of the system power loss (MW). |
| $pe$ | Violated DG power penetration counter. |
| $Pfl$ | Violated combined DG power penetration and feeder congestion counter. |
| $Q_i$ | Reactive power injected at bus $i$ (MVAR). |
| $U$ | DG random power multiplier. |
| $V_i$ | Bus $i$ voltage (volt). |
| $V_i(j)$ | $i$th bus voltage at $j$th experiment (volt). |
| $\overline{V}$ | Convergence value of the bus voltage (volt). |
| $TN$ | Total number of system buses. |
| w | Success Monte Carlo experiment counter. |
| $Z_{il}$ | Feeder's impedance connecting buses $i$ and $l$ (Ohm). |
| $\theta_i$ | $i$th bus voltage angle (degree). |

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## I. INTRODUCTION

DUE to the wide spread use of distributed generation (DG), which is defined as small-scale power generation located at or near to the load being served, several system operating issues have come into sight. These concerns involve both the benefits of using DGs and the problem associated with the wide implementation of DG units in a well-established system for several decades. The concept of implementing small generation in the distribution system has been in practice for several years as backup, standby, and stand-alone generation. However, as a result of the new modern technology achieved in the generation industry and the benefits expected from this technology implementation, a widespread use of these DG units operated in the system has become a fact. The premise of DG is to provide electricity to customers at a reduced cost and a higher efficiency, especially if the proper technology is implemented for its application, such as use of combined heat and power technologies. Other benefits that DG could potentially provide are: reduction of the system power loss; increase of system reliability; reduced emissions; improved power quality; and deferral of transmission or distribution upgrades.

On the other hand, a lot of problems have been created due to DG integration into the system. These problems are complex and have many related aspects that have to be studied thoroughly. Among these aspects are the evaluation of the impact of

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DG on distribution system performance [1]–[3], the investigation of system reliability and availability under the new structure [4]–[6], the assessment of interfacing problems and the difficulties associated with the parallel operation of DG with the existing system [7], the examination of different nonconventional protective schemes to avoid islanding [8], [9], and the analysis of the economical facets related to the installation and operation of DGs [10]. An attempt to address the adequacy assessment of DG systems is done using a Monte Carlo-based method, which has been investigated by Hegazy *et al.* using DG random state (on or off) [11]. A determination of the allowable DG penetration level is carried out based on harmonic limit consideration in [12], which is restricted to radial distribution feeders with uniform, linearly increasing or decreasing load pattern. Furthermore, [12]'s method only provides the amount of DG penetration level, which cannot be implemented in load flow analysis in order to calculate other system parameters and values.

The research in all these directions was based on the assumption that the DG units have known locations (except [12] that does not consider DG units location) and are running all the time with their full capacity. In the real-life systems, the operation of these DG units undergoes different scenarios according to the strategies of the electricity producers, the needs of the consumers, and the zonal load time variation characteristics. Therefore, some uncertainties are introduced in the operation of such units, and thus, stochastic modeling of systems involving DG units becomes of great interest.

The sources of uncertainty in the operation of DG connected systems at a certain hour of the day include: the number of the running DG units at this hour; the locations of these units; and the power exported to the system by these units. These uncertainties affect the modeling and evaluation of the system capacity, generation scheduling, power losses, buses' voltages, and feeders' power flow. Knowing these steady-state system parameters helps greatly in predicting the electric system's behavior and its impact on system planning, design, operation, and the electricity market at all levels: generation companies (Gencos), independent market/system operator (IMO/ISO), local distribution companies (LDCs), and even the customer. Previous publications attempted to address the stochastic nature of the traditional power delivery systems and presented analytical formulation of the stochastic power flow of the system [18]–[20].

In this paper, the proposed solution incorporates Monte Carlo simulation with the traditional Newton–Raphson method to ensure the coverage of all the possible operating scenarios of the system based on the operating system boundaries and thus the accuracy of the solution. Different system parameters are calculated to show the importance of the proposed algorithm. The details of the proposed algorithm are given in the next section.

## II. Stochastic Power Flow

### A. Modeling of System Components

The main components of the DG connected electric systems are feeders, synchronous condensers, shunt susceptance, loads, transformers, and the DG units. Each feeder section is represented by its series impedance and shunt reactance. The synchronous condensers are modeled with their limits. The loads are represented by constant power models (constant $\mathrm{P} + \mathrm{jQ}$), which are simulated hourly according to typical load curve variations [13]. Distribution transformers are modeled with their series parallel equivalent circuits taking into consideration their turn's ratio [13]. All DG units are considered as PV units, which will cause a load reduction at their connected buses. The following traditional power flow equations are then formulated and solved iteratively according to the method described in the next subsection:

$$P_i = \sum_{l=1}^{TN} |V_i||V_l| \left[G_{il}\cos(\theta_i - \theta_l) + B_{il}\sin(\theta_i - \theta_r)\right] \quad i \in 1, 2, \ldots, TN \tag{1}$$

$$Q_i = \sum_{l=1}^{TN} |V_i||V_l| \left[G_{il}\sin(\theta_i - \theta_l) - B_{il}\cos(\theta_i - \theta_r)\right] \quad i \in 1, 2, \ldots, TN. \tag{2}$$

### B. Applications of Monte Carlo Method

Assume a primary distribution system with $M$ buses and several meshed feeder sections. Each bus may have a load, a synchronous condenser, a shunt susceptance, and/or a DG unit connected to it. The maximum number of the DG units is the same as the number of buses $N(N \in M)$ that have DG connection feasibility. However, the number of DG units in their "on state" at any given hour is random. Therefore, the bus identity whether it is a load bus or a generator bus, and its loading value will also be a random one. To formulate the power flow equations, the bus loading value and its identity must be identified first. To carry out this step, a random-number generator is employed to generate an integer $k$, where $k \in [1, \ldots, N]$ to represent the expected number of DGs in their "on state." The locations of these DGs are then identified, by generating a random sequence of $k$ integers, where $k \in [1, \ldots, N]$; this sequence represents the buses' number where the DGs are connected. The power flow equations are then formulated, including the DG buses, to be the randomly selected ones.

In the proposed algorithm, the power exported from each DG unit to the primary distribution system is assumed to be varying randomly. To simulate this feature, the magnitude of the generator power is calculated each time the simulation is recalled. The DG nominal rating is multiplied by a random number $U$, where $U$ is a uniformly distributed random number sequence generally between [0, 1]. However, the minimum value of $U$ can be adjusted according to the historical primary distribution system loading characteristic and behavior. Therefore, the $U$ value can vary from overnight to day loading hours

$$P_{Gi} = P_{Ri} \times U. \tag{3}$$

The solution of the power flow equations is performed employing Newton–Raphson iteration with the three random features ($k$, locations, $\mathrm{P_G}$) included. Then, the random process of assigning DGs in their on state to certain buses in the system is repeated, and the formulated system equations are updated. The updated system load flow equations are then solved with the DG powers calculated using (3). The process of updating the system

equations and solving them leads to the calculations of total DG contribution power, total system centralized power generation, each bus voltage and injected power, each feeder power flow and total system power losses. The final values of these parameters are achieved by running Monte Carlo simulation. Monte Carlo simulation entails the update and solution of the system power flow equations and the evaluations of (4)–(8) until the convergence for all variables have been reached. The updating process here refers to the inclusion of the new number, new locations and the percentage of exported power of each DG unit in the solution. Several constraints can be introduced to the Monte Carlo model to replicate real-life problems. Power constraint can be introduced to the model to represent the actual upper limit of the DG power exported to the system. Thermal feeder's constraint can be brought into the model. These two proposed constraints are used to eliminate these situations.

It is to be noted that, unlike the deterministic approach, running Monte Carlo simulation at this stage does not require any additional calculations; it needs only the updating of the already developed system equations

$$\bar{P}_{Gi} = \frac{1}{NE}\sum_{j=1}^{NE} P_{Gi}(j) \tag{4}$$

$$\bar{I}_i^2 = \frac{1}{NE}\sum_{j=1}^{NE} I_i^2(j) \tag{5}$$

$$\bar{V}_i = \frac{1}{NE}\sum_{j=1}^{NE} V_i(j) \tag{6}$$

$$\bar{P}_i = \frac{1}{NE}\sum_{j=1}^{NE} P_i(j) \tag{7}$$

$$\bar{P}_{lossil} = \frac{1}{NE}\sum_{j=1}^{NE} \left(P_{lossil}(j) - P_{lossli}(j)\right). \tag{8}$$

The flow chart of the Monte Carlo-based algorithm is shown in Fig. 1, and the step-by-step procedure is discussed as follows.

1) Determine the system buses that feasibly could have DG (maximum number of DG in the system, $N$).
2) Estimate the hourly operating DG number, location, and power and run the model.
   a) Generate random integer $k$ each hour in the day, to represent the expected number of DGs operating at that hour (operating DG number).
   b) Generate random sequence of the selected random integer $k$ each hour in the day, to represent the system's buses, out of $N$ buses, that have DGs operating at that hour (bus identity either load or PV bus).
   c) Generate random power multiplier $U$, to calculate each DG exported power (DG penetration power). The minimum value of $U$ is hourly dependent and can be estimated from the system historical data.
   d) Perform the traditional Newton–Raphson load flow.
   e) Calculate $pe$ (represents the number of experiments where DG operation exceeds the allowable penetration level), $fl$ (represents the number of experiments where overloading any feeder in the system occurs), and $pfl$ (represents the number of experiments where either exceeding the DG penetration level and/or occurrence of feeders' thermal capacity violation during the process of creating the random operation of DG units).
   f) Perform the Monte Carlo convergence equations until all system parameters reach saturation.
   g) Repeat Step 2 for all hours of the day.
3) Calculate the hourly DG converged power contribution and system steady-state converged electric parameters.

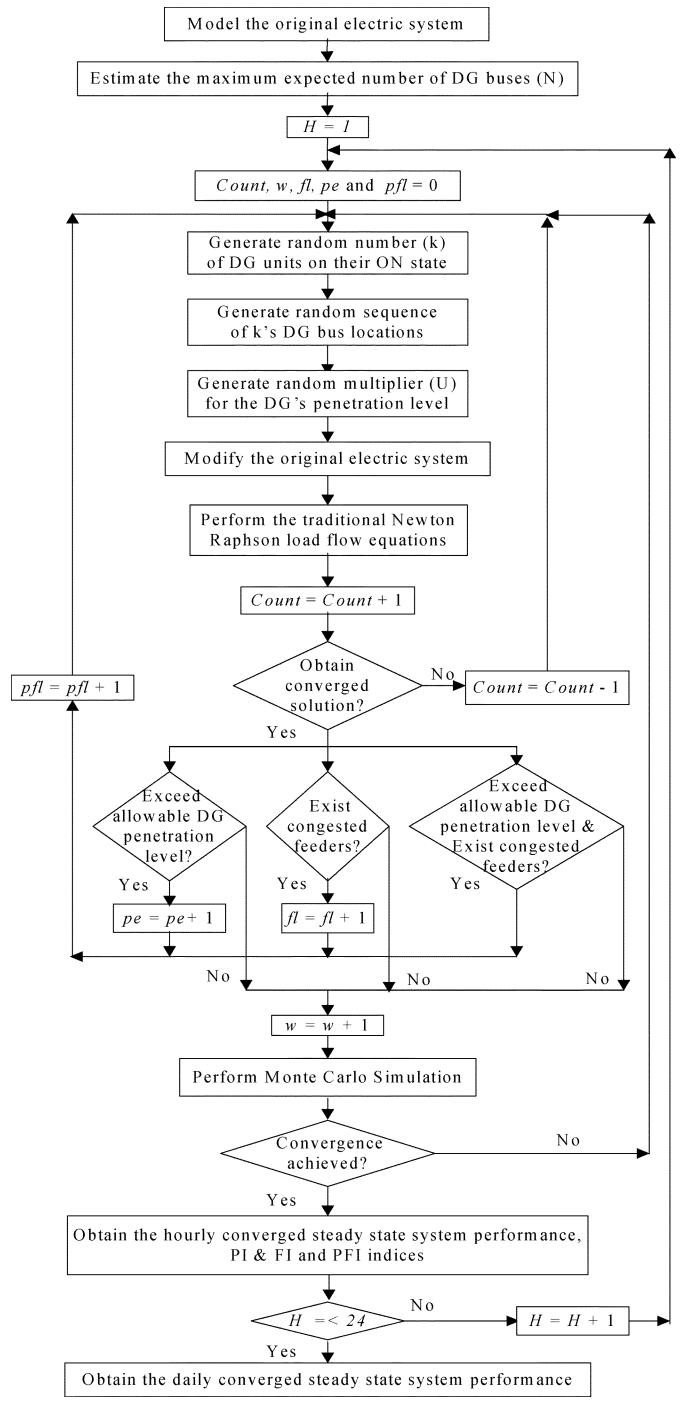


Fig. 1. Monte Carlo-based load flow algorithm.

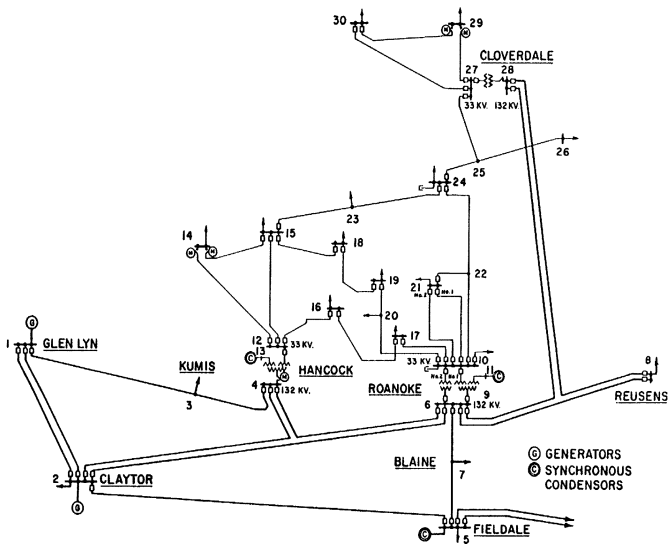


Fig. 2. Thirty-bus IEEE system under study.

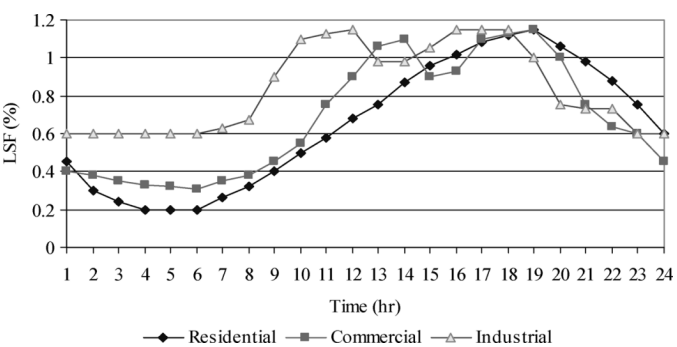


Fig. 3. Typical buses load curve characteristics for the system under study.

## III. Applications

The proposed system under study is the typical IEEE 30-bus system shown in Fig. 2 [14]. The system is modeled with all of its detailed parameters using MATLAB. The system is comprised of two centralized generating stations at buses 1 and 2 and three primary distribution substations (132/33 kV) at buses 10, 12, and 27. The primary distribution feeders working at 33 kV are connecting buses 10 to 30 (except bus 28 at transmission level, synchronous condenser buses 11 and 13) and junction nodes 20, 22, and 25. The primary distribution feeder's thermal capacity is assumed to be 20 MVA for all primary distribution feeders based on the system's steady-state load flow analysis under normal loading conditions. The loads supplied by these primary distribution feeders are aggregated and pointed out directly from the main feeder buses and junction nodes. The load characteristics are shown in Fig. 3 [15] with a load scaling factor (LSF). LSF varies with time as a percentage of its nominal load according to three typical daily load curves (residential, commercial, and industrial load curves). Each primary distribution bus is assigned to certain load curve types, as given in Table I.

The primary distribution system total load is 120 MVA at normal operating conditions $(\mathrm{LSF} = 1)$. The total load of the IEEE 30-bus system at normal operating condition is 320 MVA. A maximum number of eleven DG units are assigned to operate at certain primary distribution buses, which have loads only. The capacity of each DG unit is assumed to be 4 MVA, with a total installed DG units' capacity of 44 MVA, working nominally at 0.9 power factor. The DG units are simulated with their required active power and constrained by the maximum and minimum reactive power that can be generated. These units are operating randomly, as discussed in the previous section. The maximum allowable total DG's penetration level is considered as a percentage of the total load to be served in the 33-kV area based on system historical data.

TABLE I
System Buses Types

| Area Load Curve Type | Bus Number without DG | Bus Number with DG |
|---|---|---|
| Residential | 10, 12, 20, 22, 23, 25 | 15, 16, 17, 18,19 |
| Commercial | 4, 29 | 14, 21, 24, 29 |
| Industrial | 2, 3, 5, 7, 8 | 26, 30 |

The proposed Monte Carlo-based power flow algorithm is employed to estimate the steady-state hourly variation values for the system generation scheduled power, the distribution substation power, the system bus voltages and angles, the primary distribution feeder section currents, and the total system power losses under different DG operating conditions under the system operating boundary conditions. Three different case studies are presented here, to demonstrate the efficiency of the proposed algorithm. These cases are represented according to the total DG penetration level exported to the primary distribution system (the 33-kV area in the proposed system under study) with respect to its hourly load level and the feeder's thermal operating capacity as follows.

1) In the first case, the total value of the DG units' penetration level exported to the system is limited only by the DG's installed capacity.
2) In the second case, the cumulative DG units exported power is limited to three assumed penetration levels, 15%, 25% and 35% of the total primary distribution area loads.
3) Finally, in the third case, both the DG units exported power and feeders' thermal capacity limits are imposed to simulate the real system operation states and conditions.

In all of the above cases, the number of DG units in their "on state," the buses to which these DGs are connected and their generated power, which is delivered to the primary distribution system, are the three random parameters of interest that are updated in each experiment. The total power exported by DG units to the system is constrained by the total DG units' capacity and the total allowable penetration level to the primary distribution system. The random DG operating power depends on the random value, $U$. The value of $U$ is set to be between [0.3–1] for overnight operating hours (1–8 a.m. and 11–12 p.m.), while the $U$ value is taken between [0.5–1] in day hours (9 a.m.–10 p.m.) according to a typical historical data of system's load behavior.

The proposed Monte Carlo-based solution is used to provide the IMO/ISO with an hourly converged system steady-state-generated centralized power, feeders' power flow, power losses, and buses' voltages, taking into consideration the power exported to the system by DG units independent of their random

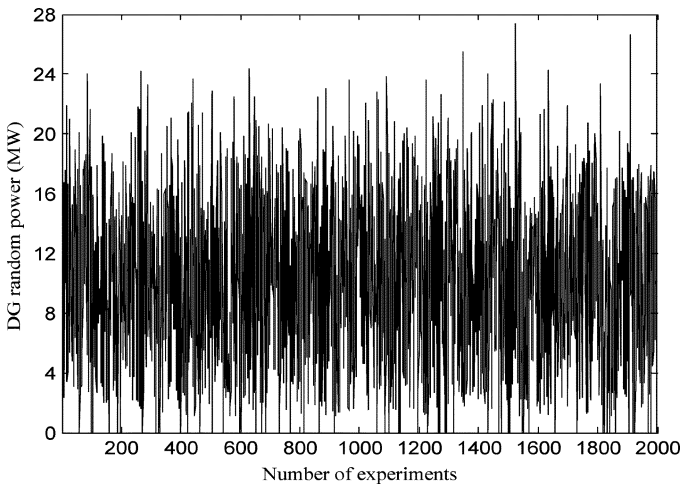


Fig. 4. Total DG random power for each individual at 6 a.m.

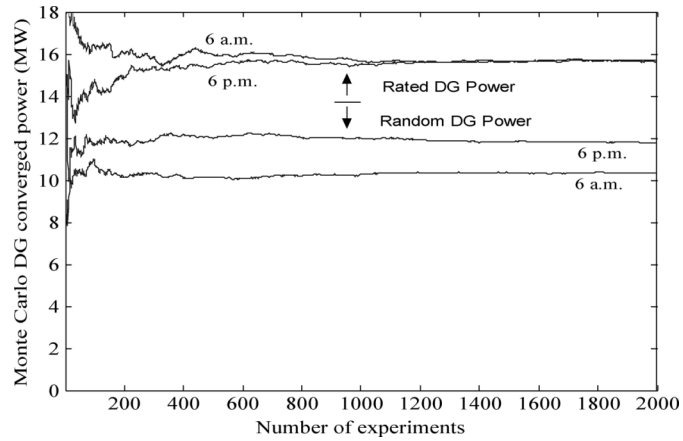


Fig. 5. DG converged penetration level (no operating constraint).

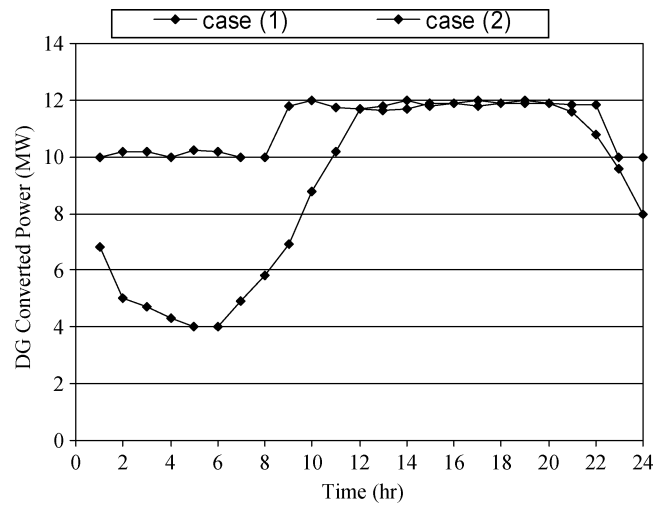


Fig. 6. DG Monte Carlo convergence of hourly penetration level.

operation. These values can be used to estimate the system behavior and conditions for the short time forecasting in the electricity spot market. Therefore, the ISO can reschedule the centralized system generation, redistribute the power flow to overcome feeders' congestion occurrence and avoid the high congestion areas' electricity price, estimate the amount and location of the auxiliary service required, and readjust the spot market price by estimating more accurately the system forecast. These converged values can asset Genco to estimate better generation-cost bidding and provide estimation for their possibility to participate in the ancillary service market to overcome their revenue loss due to their partial loss of an amount of generation required due to the existence of DG units operation. Moreover, the given results can help LDC to estimate the amount of power to be purchased from the main grid. All of the above benefits can be estimated, regardless of the randomness of the DG aspects in any given system. The convergence of the system's operating parameters under all given cases are presented in the following sections.

## IV. Analysis and Results

### A. Total DG Penetration Level

*1) Free Constraint Case:* In this case, the proposed algorithm is executed using (1)–(8) without imposing any system operating constraints for each hour in the day. The random process of DG exported power to the primary distribution system is calculated hourly. Fig. 4 shows a sample of the number of experiments carried out; each experiment represents the summation of all individual random DG generated power in the system at 6 a.m. A sample of the convergence process for the total DG penetration level exported to the system, for all experiments such as those shown in Fig. 4, is depicted in Fig. 5. Fig. 5 also shows the DG converged penetration levels if the DG units are allowed to produce their full ratings whenever they are in their "on" state. The converged value when the DG produces random power reaches 10.3277 MW (6 a.m., the lowest total system loading) for overnight loading condition and 11.7549 MW (6 p.m., the highest total system loading) for day loading condition, while the converged value when the DG units inject their rated capacity are around 15.69 MW all the time. The constant level of the DG converged power in this case is justified since the DG units are operating in an uncontrolled way with no upper limit to their injected power that is set by the system operator.

Fig. 6 shows the hourly DG convergence power, where the value at each hour is the result obtained from the proposed Monte Carlo-based algorithm discussed in Section II-B for all experiments, such as those shown in Fig. 4. It is to be mentioned that the converged value for overnight loading condition has approximately the same value independent of the system loading. Similarly, DG penetration values for the day loading condition have approximately the same value.

The hourly DG penetration level with respect to the corresponding hourly loading of the primary distribution system is shown in Fig. 7. The highest obtained percentage is found to be 33.96% for a primary distribution system loading of 30.41 MW at 6 a.m. This value is considered beyond the acceptable limit of currently available DG systems [16]. The lowest DG power contribution calculated is 9.96% for a primary distribution system loading of 118.04 MW at 6 p.m. The results obtained from Figs. 6 and 7 are considered to be primary results. However, to represent realistic system operating conditions, a limit for the total DG penetration level has to be included in the model to relate it to the actual hourly system loading levels. The

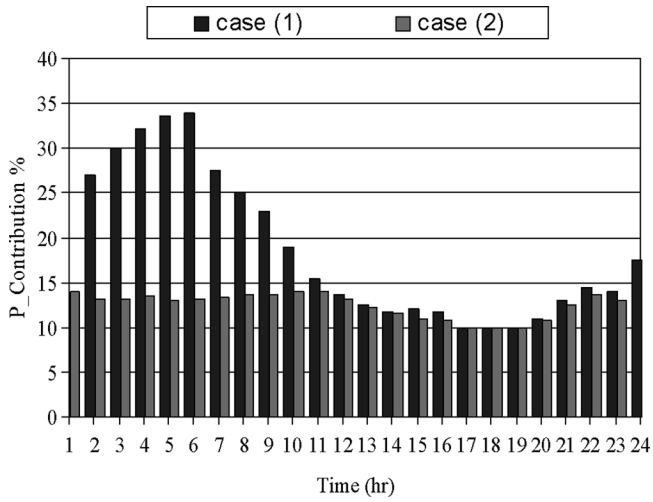


Fig. 7. The percentage of DG hourly penetration level.

allowable DG penetration limit has to be based on the system historical data.

*2) Penetration Level Constraint Case:* In this case, a practical limit is imposed during the Monte Carlo simulation on the total amount of power that can be exported from the DG units to the system. The proposed algorithm is executed using (1)–(8) and a power constraint for each hour in the day. This DG power contribution limit in this paper is assumed to be 25% for each created experiment of the total individual hourly load to be served in the primary distribution system where DG units are installed.

Performing the proposed algorithm discussed in Section II-B while the Monte Carlo simulation omits the experiments that violated the power limit, the total DG converged exported power are calculated. At 6 a.m., the converged power value was 4 MW in a total number of 697 successful experiments that were generated randomly and satisfy the assumed DG power contribution. It is to be noted that at overnight loads, the load level is low, and therefore, the probability of obtaining random DG exported power experiments number that exceeds the total load level is high. For example, at 6 a.m., the total primary distribution system is 30.41 MW. Therefore, a 25% total DG power contribution limit in each individual experiment (in Fig. 4) is constrained by a value of 7.6025 MW. As shown in Fig. 4, the number of individual experiments that exceed that limit is high. Applying Monte Carlo can obtain the converged total DG exported power that satisfies the assumed limits to imitate the operating condition reality.

For daytime loading level, for example, at 6 p.m., the total primary distribution system loading is 118.04 MW, which will limit the DG contributed power with an assumed 25% total DG power limit to 29.51 MW for each individual experiment shown in Fig. 4. The results obtained showed that the total converged DG contributed power reaches 11.62 MW in approximately 800 experiments

The hourly DG convergence power, taking into consideration the assumed 25% DG power limit, is depicted in Fig. 6. The comparison between this case and the constraint free case (case 1) shows the hourly variation of the solution according to the

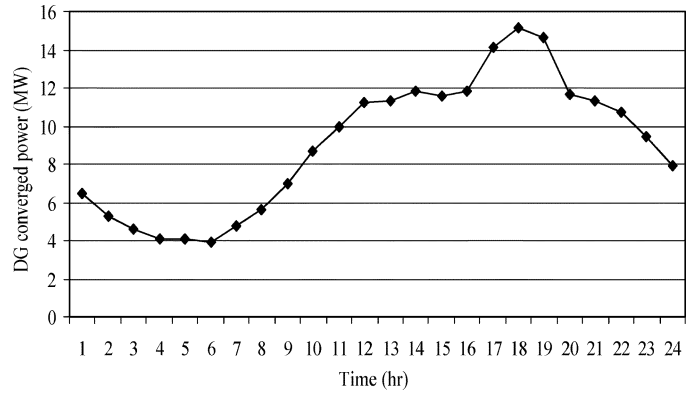


Fig. 8. DG Monte Carlo convergence of hourly penetration level.

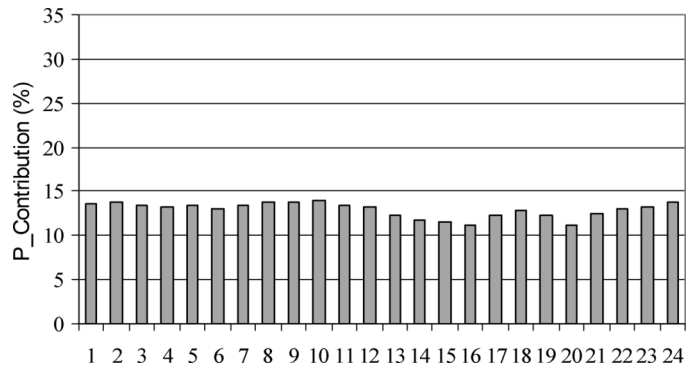


Fig. 9. Percentage of DG hourly penetration level.

system loading. The constraint case represents the practical operation of DG systems.

Fig. 7 indicates the converged DG penetration level as a percentage of the corresponding hourly loading of the primary distribution system. The highest percentage is found to be in the overnight loading hours with a converged value less than the assumed DG power contribution limit. The results obtained from Figs. 6 and 7 are considered to be replicating a daily load characteristic. However, introducing these values into a real system can cause feeders' congestion in some areas. Therefore, additional feeders' thermal capacity limit has to be included in the model in combination with the DG power limit, as will be discussed in the following case. The value of the feeders' thermal capacity is chosen based on the actual system load flow without taking the impact of DG injected power into consideration.

*3) Penetration Level and Feeder Operating Capacity Constraints Case:* A feeder capacity constraint of 20 MVA is implemented to the proposed algorithm combined with the DG penetration level constraint in this case. The Monte Carlo convergence DG contribution power for 6 a.m. and 6 p.m. are 3.94 MW (in 700 successful experiments) and 15.18 MW (in 1259 successful experiments), respectively.

The hourly convergence DG power shown in Fig. 8 has different values than those of Fig. 9. An increase in the amount of DG power exported to the primary distribution system is shown in Fig. 8 for 4–8 p.m. at day hours to meet the system demand, satisfy the 25% DG power contribution limit, and avoid feeders' contingences.

Fig. 9 provides the converged hourly percentage of DG penetration level. Comparing Figs. 7 and 9, it is clear that there is an

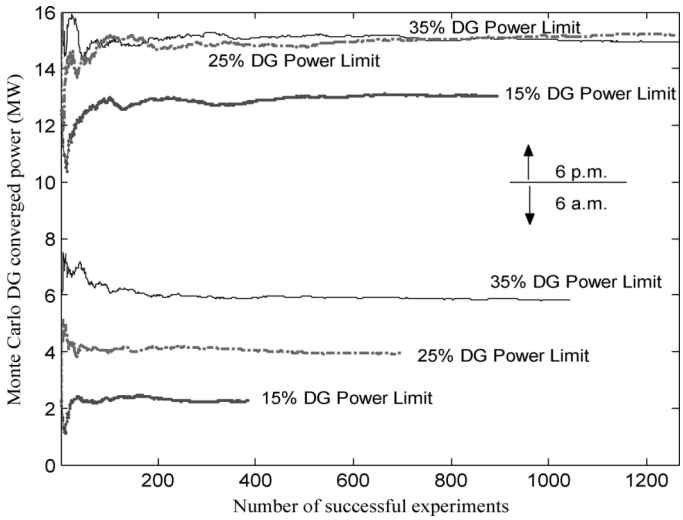


Fig. 10. DG converged penetration level final value.

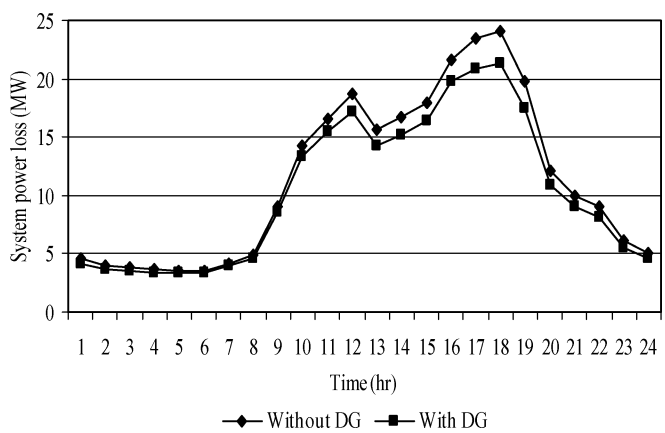


Fig. 11. Daily converged system power loss.

increase in the DG exported power to the system for four hours (4–8 p.m.) under the DG power constraint to reduce the feeder's power flow and satisfy the feeder's thermal capacity constraint. The results obtained from Figs. 8 and 9 are well thought-out to be a fine imitation of real performance of DG connected system.

By executing the proposed algorithm for different DG penetration allowance level, 15%, 25%, and 35%, we obtain different estimation of the total DG power exported to the system under their random operation. Fig. 10 shows a comparison among these DG penetration levels at 6 a.m. and 6 p.m. It is clear that as the level of penetration allowance increases, the value of converged DG contribution increases, which means that the system is in need and at the same time is capable of absorbing the DG power generated irrespective of random DG units operation.

### B. Total System Power Losses

By implementing the proposed algorithm based on Monte Carlo convergence, the total system power losses of the system under study constrained by the DG penetration level (25%) and the feeders' capacity constraint (20 MVA) is calculated. The hourly system power loss can be estimated irrespective of DG operation uncertainty. Fig. 11 depicts a comparison between the

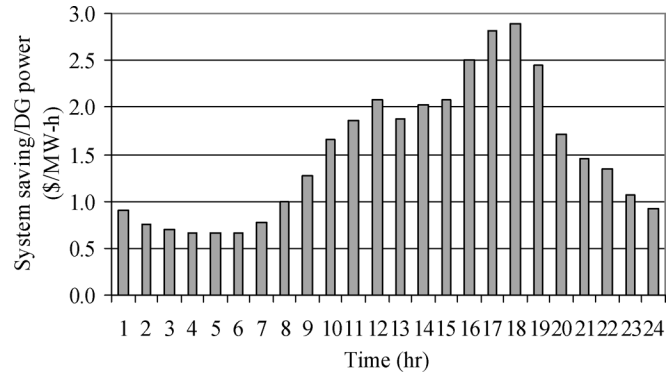


Fig. 12. Daily system power loss saving per MW DG contribution.

TABLE II
POWER LOSS SAVINGS

| Allowable DG Penetration Level | Power loss saving/day | |
|---|---|---|
| | MW/day | $/day |
| 15 % | 17.70 | 280.47 |
| 25 % | 24.11 | 381.79 |
| 35 % | 25.82 | 408.75 |

Monte Carlo convergences of the system power losses for allowable 25% DG penetration levels and feeders' capacity limit with respect to a case where DG contribution to the system is not taken into consideration.

The final values of power loss saving were minimum at 6 a.m. with a value of 0.166 MW and maximum value at 6 p.m. with a value of 2.76 MW. The total power loss saving due to taking the impact of DG in the network into consideration is 24.11 MW/day, which is $381.79/day. It is to be mentioned that the loss saving is calculated based on (9) [17], as we consider this saving as the difference in the centralized power generation cost with and without including the converged Monte Carlo DG penetration level into calculation

$$\text{Cental power generation}\cos t(\$)=\sum_{H=1}^{24}\left(AP_g^2+BP_g+C\right) \tag{9}$$

where A, B, and C are generator constants.

The estimation of the converged power loss saving value can benefit the ISO/IMO to estimate the amount of power required to be purchased from the Genco as an ancillary service to overcome the system power loss. Based on the hourly saving calculated in Fig. 11, a post analysis can be carried out to evaluate the hourly system saving per each estimated DG MW obtained from the converged Monte Carlo proposed algorithm. Fig. 12 estimates the system savings per a converged DG MW. It is to be mentioned that during the daytime loading, although the system loading increases, the DG penetration level rises. Therefore, the saving in power loss increases, and as a result, the system saving in power loss per DG MW moves up during the period 9 a.m.–10 p.m.

By carrying out the saving in power loss for different DG penetration allowance level as mentioned before, 15%, 25%, and 35%, we obtain different values, as shown in Table II.

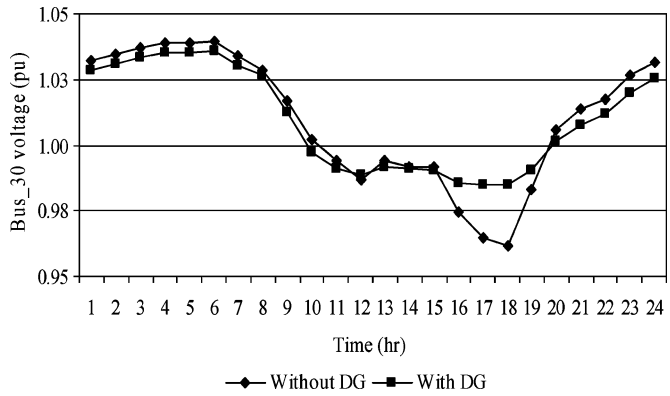


Fig. 13. Bus 30 voltage profile.

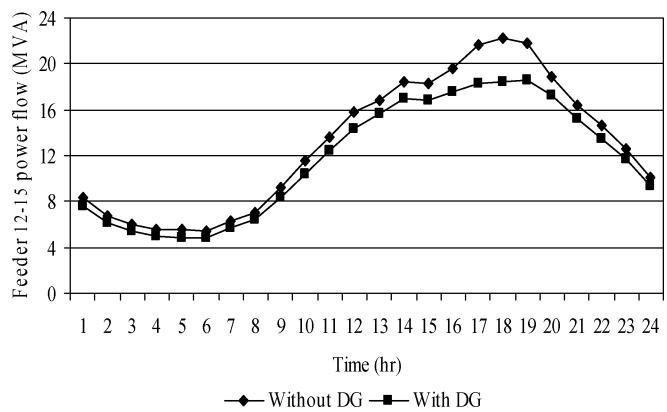


Fig. 14. Feeder 12–15 power flow.

### C. System Bus Voltage

The converged Monte Carlo daily voltage at any bus can be evaluated each hour. Fig. 13 shows the results of Monte Carlo convergences of the system at bus 30, which represents the far end user in the network. The voltage fluctuation found to be 1.04–0.96 p.u. in case of not taking DG impact into consideration, while it is between 1.036–0.985 p.u. in case of constraint operation (25% DG power constraint and feeders' capacity constraint). It is clear that taking DG into consideration during calculating the steady-state bus voltage shows that the voltage fluctuation is relatively small with respect to its value while considering the system alone.

These results can be used to estimate the value of the short-circuit current at each bus and predict the power quality problems, which might exist at the system buses.

### D. Feeder Power Flow

Fig. 14 shows the daily power flow in feeder 12–15 (see Fig. 2). It compares the converged feeder's power flow obtained from the proposed algorithm in case of taking DG exported power into consideration, constrained by the feeder's capacity limit and the DG penetration level constraint, and in the case of not considering DG power. It is clear that in case of not including DG power in calculating the system parameters, there is an over loading for four hours (4–8 p.m.). However, in case of implementing the proposed algorithm with the 25% DG penetration level and feeders' capacity limit, we do not have this violation.

TABLE III
FEEDER 12–15 POWER FLOW

| Allowable DG Penetration Level | Power Flow in Feeder 12-15 (MVA) | |
|---|---|---|
| | Minimum | Maximum |
| 15 % | 5.10 | 18.87 |
| 25 % | 4.87 | 18.58 |
| 35 % | 4.62 | 18.56 |

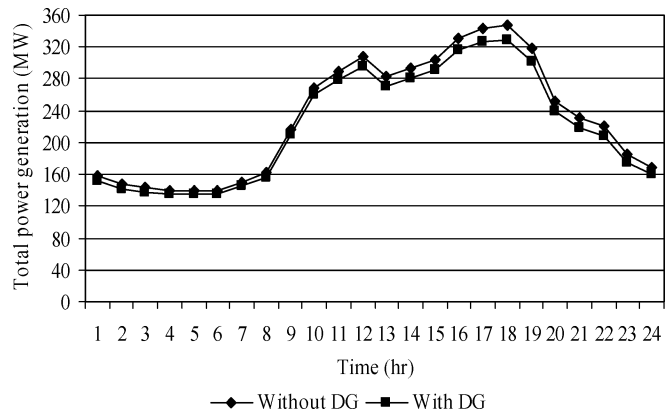


Fig. 15. Total system power generation.

TABLE IV
GENERATION REDUCTION AND LOSS OF REVENUE

| Allowable DG Penetration Level | Power reduction and revenue loss/day | |
|---|---|---|
| | MW/day | $/day |
| 15 % | 167.04 | 3086.34 |
| 25 % | 241.366 | 4435.46 |
| 35 % | 270.799 | 4952.16 |

By performing the calculation for different DG penetration allowance levels, we obtain different Feeder 12–15 flow, as shown in Table III.

### E. System Generation

Due to the existence of customer DGs in the network, the total amount of generated power required by the main central generators is reduced. Fig. 15 shows a comparison between the hourly system central generated power without and with including DG exported power to the system into the load flow model. The results show that the sum of power differences (241.366 MW/day) represents a loss of revenue from the Genco point of view of an amount of $4435.46/day. As mentioned in Section V, these converged values can asset Genco to estimate new generation-cost bidding based on the existence of DG power and open a new direction for Genco to join the ancillary service market to overcome their revenue loss.

By executing the proposed algorithm for different DG penetration allowance level, 15%, 25%, and 35%, we obtain different loss of revenue as shown in Table IV.

## V. CONCLUSION

A novel algorithm that incorporates the deterministic and the stochastic natures of the newly structured DG systems in analyzing their steady-state performance is devised and examined.

The main sources of uncertainty in the operation of such systems were thoroughly investigated. The algorithm employs the Newton–Raphson method as a powerful power flow solution in conjunction with Monte Carlo simulation technique to cover all possible operation conditions of the systems. The algorithm provides the hourly systems' centralized power generation, the total DG penetration level, the reduction in the centralized power generation, the system power loss saving, feeders' flow, and bus voltages. Three comprehensive cases are investigated using the proposed algorithm application, namely, unbounded DG operation, constrained DG exported power, and combined DG power constraint with feeders' thermal capacity constraint to emulate a real system operation. The results obtained showed how the random operation of DG units can dynamically affect the overall performance of the underlying system.

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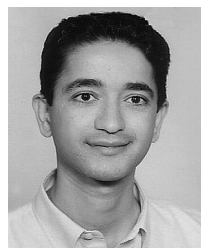


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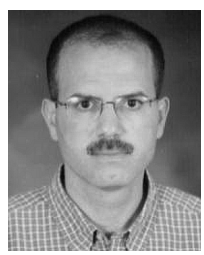


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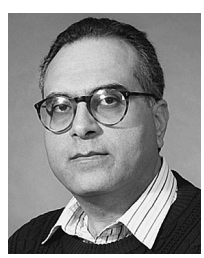


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