Genetic Algorithm based Voltage Regulator Placement in Unbalanced Radial Distribution Systems

Ganesh VULASALA, Sivanagaraju SIRIGIRI and Ramana THIRUVEEDULA

Abstract: *In rural power systems, the Automatic Voltage Regulators (AVRs) help to reduce energy losses and to improve the energy quality of electric utilities, compensating the voltage drops through distribution lines. This paper presents selection of optimal locations and selection of tap setting for voltage regulators in Unbalanced Radial Distribution System (URDS). In this paper Genetic Algorithm (GA) is used for voltage regulator placement in an unbalanced radial distribution system. An algorithm makes the initial selection, installation and tap setting of the voltage regulators to provide a smooth voltage profile along the network. The effectiveness of the proposed method is illustrated with 19 bus and 25 bus unbalanced radial distribution systems.*

Keywords: *Power Flows, Genetic Algorithm, Voltage Regulator Placement, Loss Reduction*

1. INTRODUCTION

In distribution system operation, shunt capacitor banks and feeder regulators are necessary for providing acceptable voltage profiles to all end-use customers and reducing energy losses on large distribution systems. A Voltage Regulator [VR] is a device that keeps a predetermined voltage in a distribution line in despite of the load variations within its rated power. It mainly consists of an autotransformer able to increase or reduce its output voltage by means of automatic tap changing. The command of the commutation mechanism can be done automatically or by manual operation.

A voltage regulator is equipped with controls and accessories for its tap to be adjusted automatically under load conditions. These accessories are sensitive to voltage variations so as to keep the output voltage within a determined range. The operational voltage profile, in the design phase, can be improved by the use of analytical tools such as optimal power flow, voltage stability analysis, reliability analysis, etc. Moreover, it can be controlled by the installation of devices such as fixed and controlled capacitors banks, transformers with On-Load Tap Changers (OLTCs), and Automatic Voltage Regulators (AVRs) [2], [3]. However, the use of the AVR is constrained by its high investment cost. So, the optimal location of these becomes an important issue. For many years, researchers have worked to define the optimal number, location, and sizing of capacitors banks to achieve voltage control while all operational constraints are satisfied, at different loading levels. Many single-objective optimization techniques have been applied to this problem, including heuristic methods such as expert systems, simulated annealing, and artificial neural networks [4].

Recently, evolutionary algorithms [5]–[8] have also been used. In these cases, the objective function is defined by taking losses reduction into account, voltage constraints, and total cost Optimal power flow analysis is used to determine the optimal tap position and the ON/OFF state of the capacitor banks [9]. The same problem is solved in [10] using the losses equation as the objective function and voltage inequalities as constraints through the use of an artificial neural network. The works presented in [11] and [12] search the optimal location of OLTC and capacitor banks and also establish the optimal open/close state of sectionalizers in the system. In [13]–[15], the optimal number and location of AVRs are studied separately. In the work of Safigianni and Salis in [17], mentioned the number and location of AVRs are determined by using a sequential algorithm. In addition to this, the objective function is defined by using the AVR’s investment and maintenance costs and also the cost of the total energy losses.

J. Mendoza et. al in [18], developed a method for optimal location of AVRs in radial distribution networks using simple genetic algorithms was developed. However, there are only few publications that have treated the complex problem of the optimal location of the AVRs in distribution systems, despite the fact that the benefits of including AVR devices are well known in [1]. In [19], the authors explained an evolutionary multi-objective approach for voltage regulation and power loss minimization in distribution networks. Recently Genetic Algorithms have been used to solve the problems of distribution systems more efficiently. The use of analogies of natural behaviour

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led to the development of Genetic Algorithms (GA) [21].

In this paper, voltage drops are first fund at each branch and which branch having highest voltage drop will be picked as the best location for the Voltage Regulator placement. Genetic algorithm is used to find the selection of tap position of the voltage regulator. To obtain the tap position of the voltage regulators that maintains the voltages within the limits of the radial distribution systems so as to maximize an objective function, which consists of capital investment and capitalized energy loss costs.

2. PROBLEM FORMULATION AND PROPOSED SOLUTION

In this paper the optimization problem has been separated into two sub problems. a) Locating the AVRs on the network and b) the selection of the tap position of AVRs.

2.1. Optimal location of automatic voltage regulators

To obtain the optimal location for placing voltage regulators that maintains the voltages within the limits of the radial distribution systems. The location of AVR is chosen as the one that gives the best voltage drop of the branch in the given system. The best location is selected as the branch with highest voltage drop.

To study the effect of placement of VR on isolated Distribution systems, the same sample system is assumed to be isolated. The location of branch at which voltage regulator is placed is varied from 2 to 19(except source node) for the sample 19 bus test feeder whose single line diagram is shown in Fig.1.

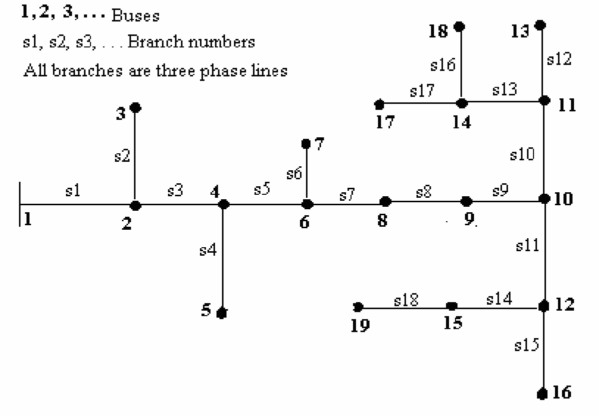

Fig. 1. Single line diagram of 19-bus URDS.

The voltage drop for different branches is shown in Fig. 2 for best location of voltage regulator.

The best location for voltage regulator placement is identified as 2^{nd} branch, which shows maximum voltage drop.

2.2. Selection of tap position

The determination of tap position of each voltage regulator is essential for solving the localization problem. In this tap adjustment, via successive displacement, can drive to inadequate solutions. For this reason a forward-backward sweep load flow algorithm, modelling the tap position as state variable is used. This improves the performance of the optimization process and genetic algorithm is used to find the selection of tap position of the voltage regulator.

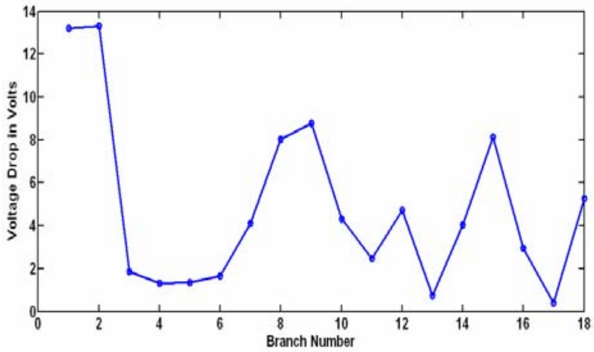

Fig. 2. Voltage drop for 19 bus URDS – Before Placing Voltage regulator.

By finding the optimal number and location of VR then tap positions of VR is to be determined as follows.

In general, VR position at bus ‘j’ can be calculated as

$$V_j^1 = V_j \pm tap \times V_{rated} \quad (1)$$

Where

V_j^1: Voltage at bus ‘j’ after VR installation at this bus in p.u

V_j : Voltage at bus ‘j’ before a VR installation at this bus p.u.

Tap position (tap) can be calculated by comparing voltage obtained before VR installation with the lower and upper limits of voltage

‘+’ for boosting of voltage
‘-’ for bucking of voltage

The bus voltages are computed by load flow analysis for every change in tap setting of VR’s, till all bus voltages are within the specified limits. Then obtain the net savings, with above tap settings for VR’s.

3. AGORITHM FOR FINDING THE TAPPINGS OF A REGULATOR

Step1. Read the given data of regulator

Step2. Read the branch current in which regulator is inserted from the backward sweep.

Step3. Find the CT ratio for three phases as

$$CT = \frac{CT_p}{CT_s} \quad \text{Where as} \quad CT_s = 5 \text{ Amps}, \quad (2)$$

Step4. Convert the R and X values from volts to ohms as

$$(R - jX)_{ohms} = \frac{(R - JX)_{volts}}{CT_s} \quad (3)$$

Step5. Calculating current in the compensator

$$I_{comp} = \frac{\text{current in the branch}}{\text{CT ratio}} \tag{4}$$

Step6. Calculate the Input voltage to the compensator as

$$V_{reg} = \frac{\text{Voltage at the sending end of the branch}}{\text{PT ratio}} \tag{5}$$

Step7. Voltage drop in the compensator circuit is

$$V_{drop} = \left(R + jX\right)_{ohms} I_{comp} \tag{6}$$

Step8. Voltage across the voltage relays in three phases

$$V_R = V_{reg} - V_{drop} \tag{7}$$

Step9. Finding the tapping of the regulator

$$Tap = \frac{\text{(lower limit of the voltage) - V}_\text{R}}{\text{change in voltage for a step change of the regulator}} \tag{8}$$

Step10. Voltage output of the regulator

$$V_{ro} = \text{voltage of the sending end of the branch} \pm \text{Tap} \times (0.00625) \tag{9}$$

‘+’ For raise
‘_’ For lower

Step11. End

3.1. GA Based Method

In this section, GA is applied to calculate the optimum size of voltage regulator required to be placed on an unbalanced radial distribution system under assumption that load is constant so as to minimize the total power loss in the system, while keeping the voltages at all the nodes within the limits.

3.2. Evaluation of fitness function

The fitness function should be capable of reflecting the objective and directing the search towards optimal solution. For each population or string size, the calculated voltage regulator are placed at the nodes and the load flow method is run and the losses are calculated and these losses become the fitness function of the GA (as total power loss has to be minimized).

3.3. Genetic Operations

In the proposed algorithm, roulette-wheel selection methods are employed. In this method, the diversity of population can be maintained and the best individuals can survive in new generation. Cross over and mutation has been done on the best fitness individuals. After all the genetic operations are performed, then chromosomes are selected for new generation.

3.4. Terminating Rule

The process of generating new trials with the best fitness will be continued until the predefined maximum generation is reached.

3.5. Algorithm for GA based voltage regulator tap-setting

The GA based voltage regulator tap-setting algorithm is given below

Step1. Generate the random population for size(s) of **voltage regulator** for Gen = 1

Step2. Perform load flows corresponding to the **voltage regulator** setting of the genetic algorithm string and determine various node voltages, active power losses.

Step3. Obtain the fitness value of each string.

Step4. Select parent strings by roulette wheel production process

Step5. Perform cross over and mutation on the selection strings and obtain new strings for next generation

Step6. Repeat steps 2 to 5 until Generation= max. Generation.

Step7. Stop.

4. RESULTS AND ANALYSIS

Example 1: 19-Bus unbalanced radial distribution system

The 11 kV, 19-bus unbalanced radial distribution system is shown in Fig. 1. The line, load and tie switch data are given in Appendix A1. The tap settings of the regulator are obtained with genetic algorithm. The GA control parameters selected are population size (20), cross over probability (0.9) and mutation probability (0.04).

Table 1. Voltage profile for the 19 bus unbalanced radial distribution system.

Bus No	Before Voltage Regulator Placement			After Voltage Regulator Placement		
	|Va|	|Vb|	|Vc|	|Va|	|Vb|	|Vc|
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.9875	0.9891	0.9880	1.0381	1.0333	1.0322
Reg	-	-	-	1.0500	1.0375	1.0375
3	0 .985	0.9887	0.9863	1.0362	1.0329	1.0307
4	0.9824	0.9839	0.9830	1.0333	1.0284	1.0275
5	0.9820	0.9837	0.9828	1.0330	1.0281	1.0273
6	0.9793	0.9808	0.9801	1.0304	1.0254	1.0247
7	0.9786	0.9803	0.9796	1.0298	1.0249	1.0242
8	0.9728	0.9738	0.9735	1.0243	1.0187	1.0184
9	0.9659	0.9660	0.9657	1.0178	1.0112	1.0110
10	0.9563	0.9555	0.9550	1.0086	1.0012	1.0007
11	0.9550	0.9543	0.9533	1.0075	1.0001	0.9991
12	0.9548	0.9538	0.9536	1.0073	0.9996	0.9994
13	0.9544	0.9534	0.9521	1.0069	0.9992	0.9980
14	0.9545	0.9539	0.9528	1.0070	0.9997	0.9986
15	0.9527	0.9512	0.9513	1.0053	0.9971	0.9971
16	0.9534	0.9515	0.9522	1.0059	0.9974	0.9980
17	0.9537	0.9534	0.9523	1.0062	0.9992	0.9982
18	0.9538	0.9532	0.9521	1.0063	0.9990	0.9979
19	0.9516	0.9498	0.9505	1.0042	0.9957	0.9964

From table 1 It has been observed that the minimum voltages in phases A, B, C are improved from 0.9516, 0.9498, 0.9505 p.u (without Regulator) to

1.0042, 0.9957, 0.9964 p.u (with Regulator) respectively. Hence, there is an improvement in the minimum voltage when with the before Regulator placement and after Regulator placement. Table 2 shows the summary of test results before and after Regulator placement. The contingency curve for power loss analysis after Regulator placement is shown in fig. 3.

Table 2. Summary of test result before and after voltage regulator placement of 19 bus URDS.

Description	Before Regulator Placement			After Regulator Placement		
	a	b	c	a	b	c
Voltage Regulator Tap position at 2 each bus	-	-	-	7	8	7
Minimum Voltage	0.9516	0.9498	0.9505	0.9975	1.0025	0.9964
Voltage regulation (%)	4.84	5.02	4.95	5.25	4.75	4.11
Improvement of Voltage regulation (%)	-	-	-	-8.4	5.37	16.96
Active Power Loss (kW)	4.45	4.45	4.56	4.06	3.99	4.17
Total Active Power Loss reduction (%)	-	-	-	8.7	10.33	8.5
Reactive Power Loss (kVAr)	1.94	1.89	1.959	0.57	0.51	0.34
Total Reactive Power Loss reduction (%)	-	-	-	70.61	73.01	82.65
Total Demand (kW)	126.33	116.24	123.27	125.94	115.78	122.88
Total Released Demand (kW)	-	-	-	0.39	0.46	0.39
Total Reactive Power Demand (kVAr)	61.23	56.34	59.7	59.86	54.96	58.08
Total Released Reactive Power Demand (kVAr)	-	-	-	1.37	1.38	1.619
Total Feeder Capacity (kVA)	140.38	129.17	136.96	139.44	128.16	135.91
Total Released Feeder Capacity (kVA)	-	-	-	0.94	1.01	1.04

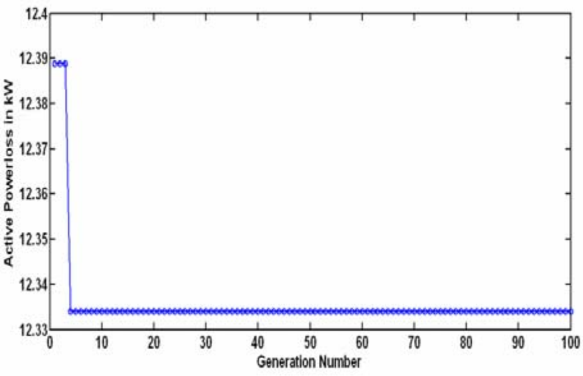

Fig. 3. Contingency curve for power loss analysis of 19 bus URDS.

Example 2: 25 bus unbalanced radial distribution system

The proposed algorithm is tested on 25 bus unbalanced radial distribution system as shown in Fig. 4. The line and load data are given in Appendix A2. The tap settings of the regulator are obtained with genetic algorithm. The GA control parameters selected are population size (20), cross over probability (0.9) and mutation probability (0.04). The voltage drops in the various branches of the system, before placement of voltage regulator are shown in fig.5.

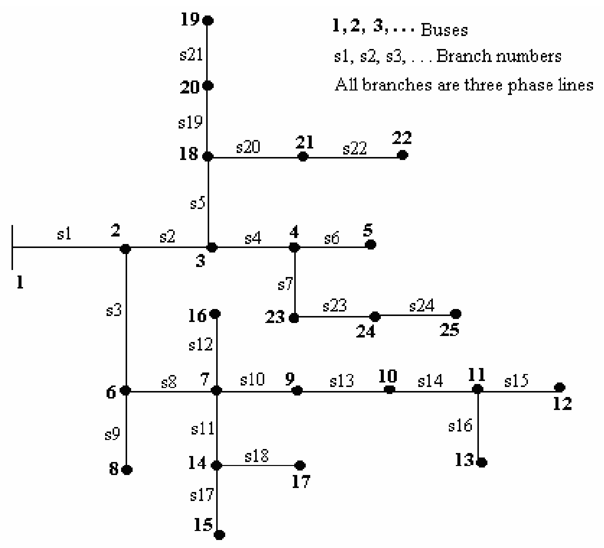

Fig. 4. Single line diagram of 25bus unbalanced radial distribution systems.

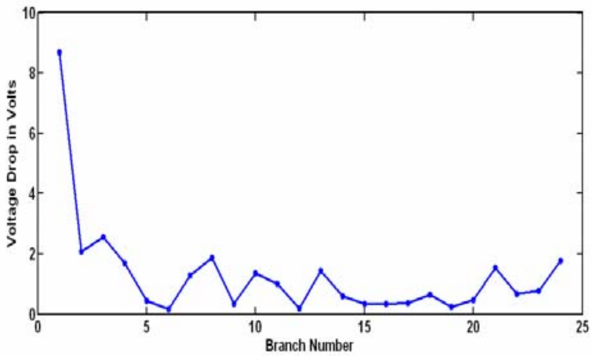

Fig. 5. Voltage drop for 25 bus URDS – Before Placing Voltage regulator.

Fig.5 shows the voltage drops in the 25 bus unbalanced radial distribution system before regulator is placed. Voltage limits for the voltages at the nodes of the branches is taken as ±5%. From the fig.5 it can be concluded that 1st branch having more drop than others. Therefore regulator should be placed in this branch.

Table 3. Voltage profile for the 25 bus unbalanced radial distribution system.

Bus No	Before Voltage Regulator Placement			After Voltage Regulator Placement		
	|Va|	|Vb|	|Vc|	|Va|	|Vb|	|Vc|
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Reg	-	-	-	1.0500	1.0500	1.0313
2	0.9702	0.9711	0.9755	1.0219	1.0226	1.0074
3	0.9632	0.9644	0.9698	1.0153	1.0162	1.0019
4	0.9598	0.9613	0.9674	1.0120	1.0132	0.9995
5	0.9587	0.9603	0.9664	1.0110	1.0123	0.9986
6	0.9550	0.9559	0.9615	1.0075	1.0081	0.9938
7	0.9419	0.9428	0.9492	0.9952	0.9958	0.9819
8	0.9529	0.9538	0.9596	1.0055	1.0062	0.9920
9	0.9359	0.9367	0.9438	0.9895	0.9900	0.9767
10	0.9315	0.9319	0.9395	0.9854	0.9854	0.9725

11	0.9294	0.9296	0.9376	0.9834	0.9833	0.9707
12	0.9284	0.9284	0.9366	0.9824	0.9821	0.9697
13	0.9287	0.9287	0.9368	0.9827	0.9824	0.9699
14	0.9359	0.9370	0.9434	0.9896	0.9903	0.9763
15	0.9338	0.9349	0.9414	0.9875	0.9882	0.9744
16	0.9408	0.9418	0.9483	0.9942	0.9948	0.9810
17	0.9347	0.9360	0.9420	0.9884	0.9893	0.9750
18	0.9573	0.9586	0.9643	1.0097	1.0107	0.9965
19	0.9524	0.9544	0.9600	1.0050	1.0067	0.9923
20	0.9548	0.9563	0.9620	1.0073	1.0086	0.9943
21	0.9537	0.9549	0.9605	1.0063	1.0072	0.9929
22	0.9518	0.9525	0.9585	1.0044	1.0049	0.9909
23	0.9565	0.9584	0.9648	1.0089	1.0105	0.9970
24	0.9544	0.9565	0.9631	1.0070	1.0087	0.9953
25	0.9520	0.9547	0.9612	1.0047	1.0070	0.9935

From table 3 It has been observed that the minimum voltages in phases A , B, C are improved from 0.9284, 0.9284, 0.9366 p.u (without Regulator) to 0.9824, 0.9821, 0.9697p.u (with Regulator) respectively. Hence, there is an improvement in the minimum voltage when compared with the before regulator placement and after Regulator placement.

Table 4 shows the summary of test results before and after regulator placement. The contingency curve for power loss analysis after regulator placement is shown in fig 6.

Table 4. Summary of test result before and after voltage regulator placement of 25 bus URDS.

Description	Before Regulator Placement			After Regulator Placement		
	a	b	c	a	b	c
Voltage Regulator Tap position at each bus 2	-	-	-	8	6	8
Minimum Voltage	0.9284	0.9284	0.9366	0.9822	0.9720	0.9902
Voltage regulation (%)	7.16	7.16	6.34	6.78	6.55	5.98
Improvement of Voltage regulation (%)	-	-	-	5.30	8.51	5.67
Active Power Loss (kW)	52.82	55.44	41.86	47.47	51.04	37.32
Total Active Power Loss reduction (%)	-	-	-	9.18	7.93	10.84
Reactive Power Loss (kVAr)	58.32	53.29	55.69	52.16	49.31	49.90
Total Reactive Power Loss reduction (%)	-	-	-	10.56	7.46	10.39
Total Demand (kW)	1126.12	1138.74	1125.16	1121.27	1134.34	1120.62
Total Released Demand (kW)	-	-	-	4.85	4.4	4.54
Total Reactive Power Demand (kVAr)	850.32	854.29	855.69	844.16	850.39	849.9
Total Released Reactive Power Demand (kVAr)				6.16	3.98	5.79
Total Feeder Capacity (kVA)	1411.09	1423.57	1413.57	1403.51	1417.7	1406.45
Total Released Feeder Capacity (kVA)	-	-	-	7.58	5.87	7.12

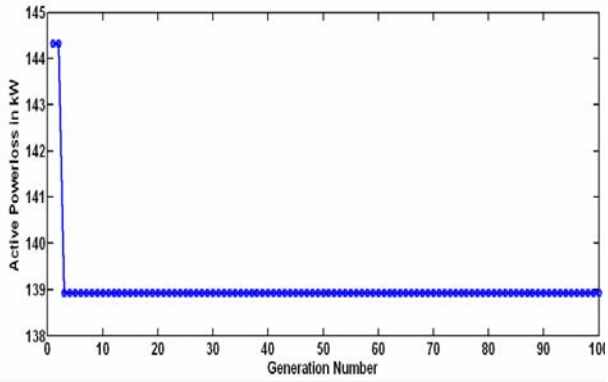

Fig. 6. Contingency curve for power loss analysis of 25 bus URDS.

5. CONCLUSIONS

This paper presents a methodology for solving the location and tap setting of voltage regulator problem in unbalanced radial distribution systems through voltage drop analysis and Genetic Algorithm. The effectiveness of the GA was demonstrated and tested. The tap setting of voltage regulator obtained with the view of objective function of reducing power losses by using GA. The proposed GA based methodology was successfully applied to 19 bus and 25 bus URDS test feeders. The obtained solution has succeeded in reducing total active power losses 8.43% in 19 bus system and 9.1% in 25 bus URDS. Thus the proposed method based on GA is efficient for solving voltage regulator placement and tap settings in unbalanced radial distribution systems.

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APPENDIX A1

Base kV: 11.00, Base kVA: 1000

Table A1. Load data and line connectivity of 19-bus unbalanced system.

branch	Sending End	Receiving End	Conductor type	Length, km	Receiving end load in kVA		
					A phase	B phase	C phase
1	1	2	1	3.0	10.38 + j5.01	5.19 + j2.52	10.38 + j5.01
2	2	3	1	5.0	11.01 + j5.34	5.19 + j2.52	9.72 + j4.71
3	2	4	1	1.5	4.05 + j 1.95	5.67 + j2.76	6.48 + j3.15
4	4	5	1	1.5	6.48 + j3.15	5.19 + j2.52	4.53 + j2.19
5	4	6	1	1.0	4.20 + j2.04	3.09 + j1.50	2.91 + j1.41
6	6	7	1	2.0	9.72 + j4.71	8.10 + j3.93	8.10 + j3.93
7	6	8	1	2.5	7.44 + j3.60	5.34 + j2.58	3.39 + j1.65
8	8	9	1	3.0	12.3 + j5.97	14.91 + j7.23	13.29 + j6.42
9	9	10	1	5.0	3.39 + j1.65	4.20 + j2.04	2.58 + j1.26
10	10	11	1	1.5	7.44 + j3.60	7.44 + j3.60	11.01 + j5.34
11	10	12	1	1.5	9.72 + j4.71	8.10 + j3.93	8.10 + j3.93
12	11	13	1	5.0	4.38 + j2.13	5.34 + j2.58	6.48 + j3.15
13	11	14	1	1.0	3.09 + j1.50	3.09 + j1.50	4.05 + j1.95
14	12	15	1	5.0	4.38 + j2.13	4.86 + j2.34	6.96 + j3.36
15	12	16	1	6.0	7.77 + j3.78	10.38 + j5.01	7.77 + j3.78
16	14	17	1	3.5	6.48 + j3.15	4.86 + j2.34	4.86 + j2.34
17	14	18	1	4.0	5.34 + j2.58	5.34 + j2.58	5.52 + j2.67
18	15	19	1	4.0	8.76 + j4.23	10.05 + j4.86	7.14 + j3.45

Type		Impedance in ohms/km		
		a	b	c
	a	1.5609 + j0.67155	0.5203 + j0.22385	0.5203 + j0.22385
1	b	0.5203 + j0.22385	1.5609 + j0.67155	0.5203 + j0.22385
	c	0.5203 + j0.22385	0.5203 + j0.22385	1.5609 + j0.67155

APPENDIX A2

Base kV: 4.16, Base MVA: 30

Table A2. Load data and line connectivity of 25-bus unbalanced system.

branch	Sending End	Receiving End	Conductor type	Length, ft	Receiving end load in kVA		
					A phase	B phase	C phase
1	1	2	1	1000	0	0	0
2	2	3	1	500	35 + j25	40 + j30	45 + j32
3	2	6	2	500	40 + j30	45 + j32	35 + j25
4	3	4	1	500	50 + j40	60 + j45	50 + j35
5	3	18	2	500	40 + j30	40 + j30	40 + j30
6	4	5	2	500	40 + j30	40 + j30	40 + j30
7	4	23	2	400	60 + j45	50 + j40	50 + j35
8	6	7	2	500	0	0	0
9	6	8	2	1000	40 + j30	40 + j30	40 + j30
10	7	9	2	500	60 + j45	50 + j40	50 + j35
11	7	14	2	500	50 + j35	50 + j40	60 + j45
12	7	16	2	500	40 + j30	40 + j30	40 + j30
13	9	10	2	500	35 + j25	40 + j30	45 + j32
14	10	11	2	300	45 + j32	35 + j25	40 + j30
15	11	12	3	200	50 + j35	60 + j45	50 + j40
16	11	13	3	200	35 + j25	45 + j32	40 + j30
17	14	15	2	300	133.3 + j100	133.3 + j100	133.3 + j100
18	14	17	3	300	40 + j30	35 + j25	45 + j32
19	18	20	2	500	35 + j25	40 + j30	45 + j32
20	18	21	3`	400	40 + j30	35 + j25	45 + j32
21	20	19	3	400	60 + j45	50 + j35	50 + j40
22	21	22	3	400	50 + j35	60 + j45	50 + j40
23	23	24	2	400	35 + j25	45 + j32	40 + j30
24	24	25	3	400	60 + j45	50 + j30	50 + j35

Type		Impedance in ohms/mile		
		a	b	c
1	a	0.3686 + j0.6852	0.0169 + j0.1515	0.0155 + j0.1098
	b	0.0169 + j0.1515	0.3757 + j0.6715	0.0188 + j0.2072
	c	0.0155 + j0.1098	0.0188 + j0.2072	0.3723 + j0.6782
2	a	0.9775 + j0.8717	0.0167 + j0.1697	0.0152 + j0.1264
	b	0.0167 + j0.1697	0.9844 + j0.8654	0.0186 + j0.2275
	c	0.0152 + j0.1264	0.0186 + j0.2275	0.9810 + j0.8648
3	a	1.9280 + j1.4194	0.0161 + j0.1183	0.0161 + j0.1183
	b	0.0161 + j0.1183	1.9308 + j1.4215	0.0161 + j0.1183
	c	0.0161 + j0.1183	0.0161 + j0.1183	1.9337 + j1.4236